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Real life cable constraints in designing Passive Optical Network architecture

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***Abstract*—Fiber To The Home (FTTH) deployment is crucial for telecommunication operators for both economical and quality of service reasons. This paper deals with a real-life Passive Optical Network (PON) design problem focusing on optical cabling constraints. This decision problem is formulated as an integer linear program (ILP) and several solving approaches are designed. Tests performed on real instances assess the efficiency of the proposed solution algorithms.**

***Index Terms*—Fiber optics, cables, integer programming.**

I. Introduction

INNOVATIVE bandwidth-requiring services lead telecommunication operators to the renewal of their fixed copper access networks, with the introduction of optical fibers. Among the optical fiber-based architectures, most telecommunication companies favor the PON architecture, appearing the best long-term technological solution [2]. In this paper, we thus focus on the design of FTTH-PON access network as experienced by field deployment teams from France Telecom-Orange. This decision problem arises as a joint optimization problem of optical splitters location, and cable routing and dimensioning.

Access network design problems have been intensively studied in the past decades. For a relevant survey, the reader can refer to [1]. To our knowledge, most papers related to PON design focus on fiber-oriented models i.e. skipping thus the cabling issues for later considerations (see. [3], [4]). Despite these problems being highly combinatorial, strengthened formulations of fiber-oriented PON design problems prove to be fairly tractable in practice (see. [4]). With regards to cabling issues, closest related work remains from Kim et al. [5] who propose ILP formulations for a global PON with cables design problem in a tree graph and a two-dimensional heuristic. The real-life PON deployment problem which is here dealt with, has at least two major differences: first no assumption is made on the underlying existing infrastructure except being with sufficient capacity (i.e. uncapacitated), and second, we exclude the possibility of gathering fibers with different types of end points (splitters or demand points) into the same cable, as well as fibers with different direction. Notice that this last possibility may appear as our underlying graph is not a tree. As far as the numerical experiment is concerned, the size of the instances and the set of available cables are as well very different in Kim et al. work compared to the present article. Those differences seem to us major requirements of an operational cabling policy. Formulated as an ILP, this problem proves intractable due to the specific cabling constraints. Therefore, we aim at proposing branch and bound-based algorithm based on its ”fiber-oriented” relaxation, taking benefit from the practical tractability of the latter.

The remaining of this paper is organized as follows. Section II is dedicated to the modeling of the problem, proposing ILP formulations for both the real-life problem and its ”fiber-oriented” relaxation. Section III reports for numerical tests performed on 4 real instances, before concluding in Section IV.

II. Problem modeling

In this section, we introduce a model for the PON deployment with cable constraints problem. Then, we show our decomposition approach to solve it more efficiently.

A. Main model and Real life constraints

Let $G = (V, E)$ be an undirected graph representing the infrastructure where the PON shall be deployed. Give orientation to each edge of G to obatin the directed graph $\overrightarrow{G} = (V, A)$ where for each edge $ij \in E$ define two reverse arcs ij and $ji \in A$. This architecture is modelled as in [4] under the form of an integer multiflow. Every edge has a length D_{ij} and every node has a demand a_i. We define 2 kinds of demand : a_i^h for ”high” ones and a_i^l for ”low” ones. High (respectively low) demands are defined as such if they exceed (respectively do not exceed) a given threshold t. For high demands, splitters of capacity m must be put directly on the demand node. Then, we have to transform the initial client demand into the last level splitter demand for the same node. For a given node $i \in V$, if $a_i < t$ (resp. $a_i \geq t$), then $a_i^h = \lceil \frac{a_i}{m} \rceil$ and $a_i^l = 0$ (resp. $a_i^h = 0$ and $a_i^l = a_i$. Low

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demands are to be filled with fibers coming from an optical splitter. Splitters have a given capacity m, cost C_s and can be put on every nodes and are denoted by s_i^l. Fibers are coming out from a single node called the Optical Line Termination (OLT), of index 0. For the low demands architecture, there are 2 fiber levels. The level 1 fibers denoted by $f_{ij}^{l_1}$ (for all arcs $(i,j) \in A$), are used by splitters to produce level 2 fibers, denoted by $f_{ij}^{l_2}$ (for all arcs $(i,j) \in A$). For high demand nodes, technological concerns impose to put splitters on the demand node. It implies that there are only level 1 fibers to route for high demand nodes. We denote these fibers by f_{ij}^{h} (for all $(i,j) \in A$). Fibers are aggregated within cables of respective capacities $q \in Q$ (given in decreasing order, with $q_0 = \max_Q q$). We denote by $c_{ij}^{k,q}$ the number of cables of level $k \in \{1,2\}$ of capacity $q \in Q$ routed along the arc $(i,j) \in A$. Cables of capacity q cost C_c^q. Finally, we introduce the boolean variable b_{ij}^k which controls whether the arc $(i,j) \in A$ is used or not.

$$z_{cable} = \min_{c_{ij}^{k,q}, s_i} \sum_{ij \in E} (D_{ij} \cdot \sum_{\substack{k \in \{1;2\} \\ q}} C_c^q \cdot c_{ij}^{k,q}) + \sum_{i \in V} C_s \cdot s_i \quad (1)$$

$$c_{ij}^{k,q} \in \mathbb{N}, f_{ij}^{l_1} \in \mathbb{N}, f_{ij}^{l_2} \in \mathbb{N}, f_{ij}^{h} \in \mathbb{N}, s_i \in \mathbb{N}, b_{ij}^k \in \mathbb{N} \quad (2)$$

$$\forall i \in V \setminus \{0\} : s_i = \sum_{j \neq i} f_{ji}^{l_1} - \sum_{j \neq i} f_{ij}^{l_1} \quad (3)$$

$$\forall i \in V : a_i^l \leq \sum_{j \neq i} f_{ji}^{l_2} - \sum_{j \neq i} f_{ij}^{l_2} + m s_i \quad (4)$$

$$\forall i \in V : a_i^h = \sum_{j \neq i} f_{ji}^{h} - \sum_{j \neq i} f_{ij}^{h} \quad (5)$$

$$\forall k \in \{1;2\}, \forall i \in V : \sum_{j \neq i} b_{ji}^k \leq 1 \quad (6)$$

$$\forall k \in \{1;2\}, \forall ij \in A : b_{ij}^k \geq \frac{\sum_q c_{ij}^{k,q}}{N} \quad (7)$$

$$\forall ij \in A : \sum_{q \in Q} q \cdot c_{ij}^{1,q} \geq f_{ij}^{l_1} + f_{ij}^{h} \quad (8)$$

$$\forall ij \in A : \sum_{q \in Q} q \cdot c_{ij}^{2,q} \geq f_{ij}^{l_2} \quad (9)$$

$$\forall k \in \{1;2\}, \forall ij \in A : \sum_{q \in Q \setminus Q_0} c_{ij}^{k,q} \leq 1 \quad (10)$$

The objective function is denoted by z_{cable}. In the model presented above, constraints (3)-(5) ensure flow conservation for all level of fibers, according to the number of splitters and demand. Constraints (6) and (7) ensure that only one edge incident to a node will be used, in order for the deployed network to have tree properties. Constraints (8) and (9) allow aggregation of fibers within cables. Constraints (10) ensure that only one cable is routed through each edge (except for the biggest cables q_0).

B. Decomposing and using a warm start

The $\mathcal{P}_c$ model proves intractable on real-size instances (refer to section III), but it can be decomposed so that we obtain a model easier to solve. A fiber-based model, denoted by $\mathcal{P}_f$, can be derived from $\mathcal{P}_c$ as follows:

1) discard variable $c_{ij}^{k;q}$ describing the cables,
2) discard inequalities (8) to (10),
3) optimize along the objective function z_{fiber} instead of z_{cable}, that is replace equation (1) by equation (11) with C_f^k the cost[1] of fiber f^k for all $k \in \{h; l_1; l_2\}$.

$$z_{fiber} = \min_{f_{ij}^k, s_i} \sum_{ij \in E} (D_{ij} \cdot \sum_{k \in \{h; l_1; l_2\}} C_f^k \cdot f_{ij}^k) + \sum_{i \in V} C_s \cdot s_i \quad (11)$$

Solving $\mathcal{P}_f$ allows us to get values for fibers and splitters variables, so a solution of $\mathcal{P}_f$ is almost a feasible solution of $\mathcal{P}_c$ in a sense that only cable variables remain to be set. Therefore we design two solution algorithms based on feasible solution of $\mathcal{P}_f$.

- $\mathcal{P}_f + \mathcal{H}_c$: given any feasible solution of $\mathcal{P}_f$, we set cable constaints by use as much maximum capacity cables as necessary and cover the remaining fibers with the smallest cable whose capacity is greater than the number. $\mathcal{H}_c$ is delailed in Algorithm 1.
- $\mathcal{P}_f + \mathcal{P}_c$: given any feasible solution of $\mathcal{P}_f$, we set arbitrarily large values to c^k variables to satisfy the cable constraints (8) to (10) and be able to set a warm start to $\mathcal{P}_c$.

Algorithm 1 Heuristic H_c

Initialize $C_{ij}^{k,q} \leftarrow 0 \; \forall ij \in A; \forall k \in 1;2; \forall q \in Q$
for all arc $ij \in A$ **do**
  Set $f := f_{ij}^k$
  while $f \geq 288$ **do**
    $C_{ij}^{k,288} := C_{ij}^{k,288} + 1$
    $f := f - 288$
  end while
  choose $\min q$ s.t. $q \geq f$
  Set $C_{ij}^{k,q} := 1$
  Set $C_{ij}^{k,288} \leftarrow \lfloor f_{ij}^k 288 \rfloor$
  Set $C_{ij}^{k,\tilde{q}} \leftarrow 1$ where $\tilde{q} \leftarrow \min q$ s.t. $q \geq f_{ij}^k - \lfloor f_{ij}^k / 288 \rfloor$
end for

[1]Note that this cost is "virtual" in a sense as it should depend on the size of the cable which will be used for each level of fiber. In practice we will assume that capacity of cables are decreasing with the level of fiber and, given the concave cost structure of the cables, we will have decreasing fiber cost (with respect to their level)

III. Numerical results

Our goal is to be efficient on our very specific operational data. That is the reason why we have not conducted any experiments on any public Network or MIP library. In this context, we present numerical results from experiments on 4 data sets, named $Data1$ the smallest one, to $Data4$ the biggest one. The underlying infrastructure G and the constants (such as cables costs C_c^q or demands a_i^h and a_i^l) have been set to their actual values; splitters capacity set to $m = 8$, and the cables capacities are chosen in $Q = \{288 = q_0; 144; 96; 72; 48; 36; 24; 12\}$, while N is set to 1000. The Linear Programming and the branch and bound were performed by CPLEX 12.2 running on an AMD Athlon II X3 powered by a Linux 2.6 kernel. Computation times were set to 1800 seconds for $\mathcal{P}_f$ and $\mathcal{P}_c$, meanwhile $\mathcal{H}_c$ runs quite instantly. Table (I) and (II) summaries some numerical experiments.

TABLE I
Experimentation $\mathcal{P}_f$, 1800 seconds

Data	$\lvert V\rvert$	$\lvert E\rvert$	Demand	Cost $\mathcal{P}_f + \mathcal{H}_c$
$Data1$	583	838	8285	45606
$Data2$	808	2528	46294	110269
$Data3$	1232	3119	28080	105062
$Data4$	1624	2711	23774	110554

TABLE II
Experimentation $\mathcal{P}_c$, 1800 seconds

Data	Cost $\mathcal{P}_c$	Gap	Cost $\mathcal{P}_f + \mathcal{P}_c$	Gap	Δ
$Data1$	34624	23.95	34348	22.42	-24%
$Data2$	N/A	N/A	60202	27.94	-45%
$Data3$	N/A	N/A	71668	34.06	-32%
$Data4$	N/A	N/A	106372	52.46	-3%

Let us explain some results shown in table (I). $\mathcal{P}_c$ finds a feasible solution only on $Data1$, the smallest. Hence, by solving $\mathcal{P}_f$ first, we help to find a feasible solution even on biggest data sets. And then by solving $\mathcal{P}_c$ we lower the cost a lot more than by solving $\mathcal{H}_c$ for the three smallest data sets as shown in column Δ where Δ quantifies the difference between the cost of $\mathcal{P}_f + \mathcal{H}_c$ and $\mathcal{P}_f + \mathcal{P}_c$.

Moreover, since for $Data1$ the cost of $\mathcal{P}_c$ and $\mathcal{P}_f + \mathcal{P}_c$ are about the same, we think that the warm start helps to find a solution as good as the one that would have been found without it on other data sets.

Let us discuss about $Data4$. For this data set the gap is almost twice the gap on smaller data sets, and the value in column Δ is rather small. We think the number of vertices $|V|$ is too big for CPLEX to solve efficiently the branch and bound process. This thought is also supported by the fact that for model $\mathcal{P}_c$, if the computing time increases up to 10800 seconds, this does not achieve any improvement more than 1% gap. We expect to find a more efficient decomposition technique in order to lower the gap on big data sets. We should remark as well that for $\mathcal{P}_f$, gaps vary from 1% to 4%, which is very small.

IV. Conclusion

In the present article we have shown that by using a decomposition and a warm start, we make possible to solve a MIP model describing a PON access network design problem with cable constraints, even on very big data sets. We have shown that for some data sets, the entire model performs a good optimization once a warm start is given. The solutions found are admissible in an operational point of view, that means conform and checkable, fast to compute and easy deploy; moreover far cheaper than the best solution found by another mean.Notice that cable constraints (8) to (10) make the problem hard to solve on our large size of data sets. Notice that although it is easy to find a deployable, feasible solution without any computer assistance the price found by our model is around 50% of the best cost found by hand.

We think that the warm start could be improved by finding a good heuristic or by studying the link between objective function (1) and (11). To test wether this could help CPLEX to find a better solution on the entire problem is an open question. Some other constraints are post-processed after the MIP has been solved; we wish to encompass them inside the main model. Finally, since the gap for some data sets is quite large and that running CPLEX a long time doesn't lower it, we wish as well to better understand the decomposition used for the warm start in order to improve this step and achieve a better gap.

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