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Comparison of Selected Textural Features as Global Content-Based Descriptors of VHR Satellite Image – the EROS-A Study

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Absrtact—**Texture is considered as one of the most crucial image features used commonly in computer vision. It is important source of information about image content, especially for single-band images. In this paper we present the results of research carried out to assess the usefulness of selected textural features of different groups in panchromatic very high resolution (VHR) satellite image classification. The study is based on images obtained from EROS A satellite. The aim of our tests was to estimate and compare the accuracy of main land cover types classification, with a particular focus on determining usefulness of textural features based on multifractal formalism.**

Presented research confirmed that it is possible to use the textural features as efficient global descriptors of VHR satellite image content. It was also prove that multifractal parameters should be considered as valuable textural features in the context of land cover classification.

I. Introduction

TEXTURE as a primary factor of visual perception is a necessary feature of image description. It is usually easy to recognize texture, but it is more difficult to define it, because texture, in contrast to colour, is not determined by a single point, but involve neighbouring area and can be related to a direction or a scale. It was created large spectrum of parameters, due to a lot of possible textural descriptors, to help to extract information about texture (also in the context of satellite images [1]). Textural characteristic can be calculated based on the entire image (global features), fragments of this image delineated by segmentation results or small clusters of pixels formed by moving windows [2], [3], [4]. Different texture analysis techniques, such as Markovian analysis (including Haralick measures), spatial autocorrelation, multi-scale autoregressive models, wavelet transforms or fractals have been successfully used to describe the content of the images [2], [3], [5]. They are considered especially important in case of single-band images, like medical ones.

The textural analysis becomes also an important component of the process of information extraction from satellite images, especially in Object-Based Image Analysis approach where textural features supplement the set of typical characteristics obtained from histogram features and image objects' shape. However, it is even more valuable tool when single-band panchromatic images of very high spatial resolution are considered. Textural analysis may facilitate information extraction from such images by enabling the automatic classification of their content. It is also especially important from the content-based image retrieval (CBIR) point of view. Due to the increase of high resolution remote sensing imagery, the developments in this direction are particularly desirable.

In our work we propose multifractal formalism, as a generalization of the fractal geometry, in order to more complete analyze the texture of satellite images [6]. In our previous study [7] we compared efficiency of selected textural features as global content-based descriptors of panchromatic WorldView2 Very High Resolution satellite images. We wanted to investigate how accurately remotely sensed image can be automatically classified to the broad land cover types such as agriculture, urban areas, water bodies, and forest, based on textural information derived from the entire image. We were able to construct decision trees capable for very accurate classification of images from our test image database into these main landuse categories. The research proved that degree of multifractality can be considered as important global image characteristic.

However, tested WorldView2 images were characterised by very homogeneous landuse – over 90% of the image was in the dominating landuse category – and high radiometric quality. In case of the present study we applied the same methodology of analysis to panchromatic EROS A satellite images. This is also a VHR sensor, although older then WorldView2 and acquiring images with a little bit lower spatial resolution (2 m in case of EROS A vs. 0.5 m in case of WorldView2) and higher level of noise. Moreover, images in our EROS A test image database are not such homogeneous regarding the landuse of imaged terrain – the dominat-

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ing landuse category covers over 50% of the image. Our previous work [6] showed that selected multifractal parameters can be used as features describing the content of these images. The aim of the present study is to compare their efficiency with other textural parameters. We also intend to carry out experiment to determine the parameters most appropriate as classification features.

II. Data

The test data used in the experiment are the same like used in [6] and consist of two partially overlapping EROS-A scenes (panchromatic high-resolution images, ground resolution – 2 m) of Krakow area (south Poland) acquired on 10th and 15th October 2007. Images were acquired using similar pointing angles (29 and 27.5 degrees) and ground sample distances (2.20 and 2.15 m respectively). The images were orthorectified using orbital model.

We created our testing image database from the 512x512 pixel orthoimage tiles. To make more image tiles available for the study, the ortophotomaps were cut into tiles twice. The second set of tiles was cut with the origin of tiles shifted 256 pixels east and 256 pixels north. Every image in the created database was labeled according to its prevailing land cover category (agriculture, forest, urban), based on photo interpretation done in other studies for the purpose of landscape ecological research. Only images where dominating land cover class covered over 50% of imaged area were used for analysis.

The final database consisted from following sets of images:

- Image set EROS1 – 262 image tiles cut from Scene 1 (agriculture – 199, urban – 40, forest - 23);
- Image set EROS1s – 259 image tiles cut from Scene 1 with shifted origins (agriculture – 204, urban – 35, forest - 20);
- Image set EROS2 – 344 image tiles cut from Scene 2 (agriculture – 298, urban – 25, forest - 21);
- Image set EROS2s – 349 image tiles cut from Scene 1 with shifted origins (agriculture – 308, urban – 24, forest – 17).

III. Methods

The analytical approach adopted in the study is the same as in [7].

A. Textural parameters

Chosen global textural characteristics were calculated for every image chip. As the result every image in the database was described by 295 attributes, which may be grouped into 9 attribute groups (AG):

- AG1: the label (land cover class);
- AG2: four histogram-based characteristics (mean, variance, skewness and kurtosis);
- AG3: six multifractal parameters (Δ^{SUM}, Δ^{MAX}, Δ^{BCD}, Δ_p^{SUM}, Δ_p^{MAX}, Δ_p^{BCD}); Δ stands for the degree of multifractality and Δ_p for the degree of multifractality for $q > 0$; SUM, MAX and BCD are three different measure types (measure SUM takes sum of pixel values on a given box; measure MAX choses maximum value of pixels in a given box; measure BCD takes deviation of gray levels in a box) [8], [6].
- AG4: 220 co-occurrence matrix-based parameters [9], [10]: angular second moment, contrast, correlation, sum of squares, inverse difference moment, sum average, sum variance, sum entropy, entropy, difference variance, difference entropy; these parameters were computed 20 times, for $(d,0)$, $(0,d)$, (d,d), $(d,-d)$ where the distance d can take values of 1, 2, 3, 4, and 5;
- AG5: 20 run length matrix-based parameters [9], [11]: run length nonuniformity, gray level nonuniformity, long run emphasis moment, short run emphasis inverse moment, fraction of image in runs; these parameters were computed 4 times (for vertical, horizontal, 45-degree and 135-degree directions);
- AG6: 5 absolute gradient-based parameters [9]: mean absolute gradient, variance of absolute gradient, skewness of absolute gradient, kurtosis of absolute gradient, percentage of pixels with nonzero gradient;
- AG7: 5 autoregressive model parameters [12], [13]: θ_1, θ_2, θ_3, θ_4, σ;
- AG8: 20 parameters derived from wavelet analysis [14], [15], [16];
- AG9: fractal dimension determined by using differential boxing-counting (DBC) method [17].
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Features from attribute groups AG4 – AG8 were obtained using MaZda software [16]. Histogram-based features, multifractal parameters and fractal dimension were calculated in MatLab.

B. Fractal Dimension and Multifractal Parameters

There are several methods for estimating a fractal dimension (FD) in an image [18], [19]. In the most commonly used Box-Counting methods fractal dimension is calculated by covering an object with boxes of varying size l and is given by the relation:

$$D_F=\lim_{l\to 0}\frac{\ln N(l)}{\ln 1/l} \tag{1}$$

where $N(l)$ denotes the number of boxes of size l needed to cover considered object. Methods differ mainly in the ways they approximate the quantity $N(l)$. Most of them are applied to images that must be turned into binary images.

In our work we calculate fractal dimension by using differential box-counting (DBC) method [20], [17]. This method, proposed by Sarkar and Chaudhuri [20], allows working directly on grey-scale images and thus the binarization process is avoided.

In DBC algorithm an image of size $M \times M$ is considered as a three-dimensional spatial surface, where (x,y) denotes pixel position and the third coordinate (z) denotes pixel gray level. The (x,y) plane is partitioned into grids of size $s \times s$, where $M/2 \geqslant s > 1$ and s is an integer. On each grid there is a column of boxes of size $s \times s \times s'$, where s' is the height of each box, $G/s' = M/s$, and G is the total number of gray levels. Let the minimum and maximum gray level of the image in (i,j)-th grid fall into the kth and lth boxes, respectively [20]. Then the contribution of $N(l)$ in the pixel (i,j) of the grid is $n_l(i,j)=l-k+1$. Taking contributions from all grids, we have

$$N(l)=\sum_{i,j} n_l(i,j) \tag{2}$$

Then $N(l)$ is computed to different values of l. Finally, the fractal dimension D_F is estimated from the least square linear fit of log $(N(l))$ against log $(1/l)$ (see Equation 1). It is worth noting that presented DBC methods was compared with other four methods proposed by Peleg [21], Pentland [22], Gangepain and Roques-Carmes [23], and Keller et al. [24], respectively. The DBC method was considered as a better method, as was also supported by the other investigation [25]. Moreover, some modifications of DBC method have been lately proposed [26].

In our research we also consider one of the multifractal functions: generalized dimensions, D_q, as well quantitative parameter strictly connected with this function. The generalized dimensions D_q are calculated as a function of a continuous index q, where $-\infty < q < \infty$ (e.g., see [27], figure 3.1). Index q can be compared to a microscope for exploring different regions of the considered image.

As for FD estimation, many methods exist to obtain the multifractal functions [20]. Here, the Box-Counting based moment method has been applied [28]. In the first step of analysis an image is divided into boxes of size $\delta \times \delta$. Next, for each box following multifractal measure is calculated:

$$\mu_i(\delta)=\frac{p_i(\delta)}{\sum_{i=1}^{N(\delta)} p_i(\delta)}, \tag{3}$$

where i = 1, ..., $N(\delta)$ = 2^n labels the individual boxes of size δ. Here $p_i(\delta)$ denotes three different measures [29], [8]:

$$p_i^{\mathrm{SUM}}(\delta)=\sum_{(k,l)\in\Omega_i} g(k,l) \tag{4}$$

$$p_i^{\mathrm{MAX}}(\delta)=\max_{(k,l)\in\Omega_i} g(k,l) \tag{5}$$

$$p_i^{\mathrm{BCD}}(\delta)=\max_{(k,l)\in\Omega_i} |d(k,l)| \tag{6}$$

where $g(k,l)$ is a gray-scale intensity at point (k,l), Ω_i is a set of all pixels (k,l) in the ith box and $d(k,l)$ denotes the deviation of gray levels in box i.

In the next step of our analysis, a weighted summation is performed over all boxes in a particular grid returning the partition function of order q, i.e.

$$\chi(q,\delta)=\sum_{i=1}^{N(\delta)} (\mu_i(\delta))^q \tag{7}$$

which scales with the box length $\delta \to 0$ and $N(\delta) \to \infty$ according to:

$$\chi(q,\delta) \propto \delta^{D_q(q-1)} \tag{8}$$

From the Equation (8) we obtain generalized dimensions [30]

$$D_q=\lim_{\delta\to 0}\frac{\log(\chi(q,\delta))}{(q-1)\log(\delta)} \tag{9}$$

The difference of the maximum and minimum dimension D_q, associated with the least dense and the most dense regions in the considered measure, is given by

$$\Delta=D_{-\infty}-D_{+\infty} \tag{10}$$

and can be regarded as a degree of multifractality (e.g., [31], [27]). The degree of multifractality Δ is a measure of complexity of considered data; higher values of Δ inform us about greater non homogeneity on image and suggest that different fractals are needed for its full description. In particular, for monofractal scaling the degree of multifractality equals zero.

Finally, as a result of multifractal analysis performed for each image we obtain the following set of six parameters: the degree of multifractality (Δ) for measure SUM (Δ^{SUM}), MAX (Δ^{MAX}) and BCD (Δ^{BCD}), as well the degree of multifractality for positive values of index q (Δ_p^{SUM}, Δ_p^{MAX} and Δ_p^{BCD}). Presented parameters state quantitative and global characteristics used to compare complexity of images.

C. Classification

The decision (classification) tree approach was used for classification. We decided to use this method as it has good computational efficiency and the obtained tree can be presented as a set of easily interpretable rules. It has also been already successfully applied for the semantic labeling of satellite images [32], [33].

In our study, the classification was done using See5 software (Rel. 2.07). The software generates decision trees based on C5.0 algorithm, improved commercial version of well-known C4.5 [24].

Classification was done based on different sets of features (classification features sets, CFS):

- CFS1: all classification features (AG2 – AG9);
- CFS2: all classification features apart from histogram-based ones (AG3 – AG9);
- CFS3: co-occurrence matrix-based features (AG4);
- CFS4: co-occurrence matrix-based and histogram-based features (AG2 and AG4);
- CFS5: run length matrix-based features (AG5);
- CFS6: run length matrix-based and histogram-based features (AG2 and AG5);
- CFS7: absolute gradient-based features (AG6);
- CFS8: absolute gradient-based and histogram-based features (AG2 and AG6);
- CFS9: autoregressive model parameters (AG7);
- CFS10: autoregressive model parameters and histogram-based features (AG2 and AG7);
- CFS11: parameters derived from wavelet analysis (AG8);
- CFS12: parameters derived from wavelet analysis and histogram-based features (AG2 and AG8);
- CFS13: fractal dimension and histogram-based features (AG2 and AG9);
- CFS14: histogram-based features (AG2);
- CFS15: multifractal parameters (AG3);
- CFS16: multifractal parameters and histogram-based features (AG2 and AG3);

Such approach enabled us both, to evaluate the individual performance of each group of textural characteristics (used alone and together with histogram-based features) and to assess the usefulness of combining of features from different groups.

Five approaches with different pruning and thresholding options as well as with or without winnowing of attributes were used for every set of classification features. Boosting with ten trials was used in every classification run.

In the area where satellite scenes overlap, existed some number of ‘twin’ tiles covering exactly the same area. To eliminate the possibility of using them as training and test data at the same time, in our study we have used the image data sets as shown in Table I.

The average overall classification accuracy was calculated for each classification approach and each set of tested classification features (CFS). The lowest of five classification errors was then assigned as a measure of classification quality for particular tested set of attributes.

TABLE I.
TRAINING AND TEST DATA SETS

	Training data set	Test data set
Classification 1	EROS1	EROS2s
Classification 2	EROS2s	EROS1
Classification 3	EROS1 and EROS2s	EROS1s and EROS2

IV. RESULTS AND DISCUSSION

The results of classification tests are shown in Table II. The best results for each of classification tests gave the classification accuracies in the range from 94 to 96 percent. In two classification tests the best results were obtained when all calculated textural parameters were included in classification feature set. However, in Classification 2 the best result was achieved using the classification feature set consisted only of absolute gradient-based and histogram-based features (CFS8). This kind of textural features is also the best one, when looking into the performance of particular texture attribute groups. This result is surprising as in the previous study for WorlView2 images [7], this attribute group gave rather poor results when compared to other ones. It should be noticed however, that the achieved level of accuracy was quite similar (93 – 95% in case of EROS A and 93% in case of WorldView2). The performance of other textural characteristics was much worse in the actual study. It is especially the case of autoregressive model parameters (CFS9), which performance for EROS A images classification can be pointed as the worst one. For WorldView2 images this group of attributes was between the best ones [7].

The results presented in Table II show that, in general, the classification performance increases when textural features are combined with histogram-based ones. This conclusion is consistent with the results of our previous studies [6], [7].

Multifractal parameters used together with histogram-based features gave the second-best result for Classification 2. However, in the two other tests their performance was rather average.

When features from all attribute groups are combined (CFS1 and CFS2) the classification tree built may be quite complex as many of available features can be used in classification process. In the classification method used in our study, the number of features used for classification may be reduced by using the winnowing approach. We compared the results of the overall accuracy achieved for both options (without winnowing and winnowed) in Table III.

It should be noted, that for two of three tests the classification performance of winnowed set is comparable to the full one and still better then performance of any other classification features set. In these cases the final set of winnowed attributes is the same and consists of two co-occurrence matrix-based parameters, multifractal parameter and skewness of absolute gradient. This result is very similar to achieved for WorldViev2 images, where also the result based on three winnowed parameters was better then any of the results of particular attribute groups [7]. In case of that study the set of winnowed attributes consisted of multifractal parameter (Δ_p^{-} ^{MAX,}) skewness of absolute gradient and the feature derived from autoregressive model (σ).

TABLE II.
CLASSIFICATION RESULTS

Classification 1		**Classification 2**		**Classification 3**	
Classification feature set	Overall clasification error [%]	Classification feature set	Overall clasification error [%]	Classification feature set	Overall clasification error [%]
CFS 1	5.5	CFS 8	5.8	CFS 2	3.8
CFS 2	5.5	CFS 16	6.9	CFS 1	4.4
CFS 8	7.0	CFS 7	9.3	CFS 3	4.9
CFS 3	7.3	CFS 2	9.7	CFS 11	4.9
CFS 11	7.3	CFS 6	9.7	CFS 8	5.1
CFS 4	7.8	CFS 12	9.7	CFS 12	5.1
CFS 13	7.8	CFS 1	10.0	CFS 4	5.4
CFS 12	8.7	CFS 3	10.0	CFS 10	5.9
CFS 15	8.7	CFS 4	10.0	CFS 6	6.1
CFS 16	8.7	CFS 14	10.0	CFS 13	6.4
CFS 7	9.3	CFS 15	10.4	CFS 16	6.9
CFS 6	9.9	CFS 13	12.2	CFS 15	7.4
CFS 10	10.2	CFS 5	13.5	CFS 7	7.7
CFS 14	11.0	CFS 11	16.6	CFS 5	8.2
CFS 9	14.0	CFS 10	21.6	CFS 14	10.6
CFS 5	14.5	CFS 9	43.6	CFS 9	13.7

TABLE III.
INFLUENCE OF ATTRIBUTES WINNOWING ON CLASSIFICATION RESULTS

	Overall classification error [%] without winnowing	Overall classification error [%] with winnowing	Winnowed attributes
Classification 1	5.5	5.5	S(5, 5) Contrast Δ_p^{SUM} S(1, 0) Difference Variance skewness of absolute gradient
Classification 2	9.7	12.0	S(3, -3) Difference Entropy S(0, 1) Entropy S(5, 0) Sum Entropy Horzl_ Long Run Emphasis Moment 45dgr_ Short Run Emphasis Inverse Moment
Classification 3	3.8	4.7	(5, 5) Contrast Δ_p^{SUM} S(1, 0) Difference Variance skewness of absolute gradient

V. CONCLUSIONS

The aims of the presented study were twofold: (i) to test the usefulness of the selected textural parameters as classification features of panchromatic VHR satellite images and (ii) to compare the efficiency of the multifractal parameters (which we propose for more complete description of the texture of remote sensing images) and other textural features in the context of land cover classification. The present study of EROS A satellite images was a continuation of the research done previously for WorldView2 data [7]. Some results confirmed, but partially the results of this research differ from the earlier ones.

In both studies we prove that for VHR satellite images it is possible to use the textural features as efficient global descriptors of image content. The observed in this study increase in classification accuracy when textural features are supplemented by histogram-based ones was also present in the results of our earlier experiments [6], [7]. Similarly, we noticed earlier the possibility of successful reduction of classification features. It is worth noting, that in both our experiments we were able to reduce the number of classification features from 295 to 3 or 4 with very limited (or even negligible) impact on the overall classification accuracy. This is very important, as calculating of textural parameters for VHR satellite images is very computationally expensive.

When comparing the classification efficiency of different groups of textural parameters, in the present study the best results were obtained for absolute gradient-based features. In the WorldView2 experiment done previously the best accuracy was achieved for multifractal parameters. It is interesting that in the case of the absolute gradient group the level of error obtained in both research is quite similar and for other kinds of textural features the errors are much higher for EROS A classification. In our opinion there are at least two possible sources of such results and much lower value of classification accuracy achieved for EROS A images in general (96.2% comparing to 99.6% for WorldView2).

First of all, the differences are present in the input images themselves. The WorldView2 images have higher spatial resolution (0.5 vs. 2.0 m) and are considered as better radiometrically. The higher level of noise potentially present in EROS A data may deteriorate the quality of textural measures derived from images. It is possible that absolute gradient-based features are less sensitive to the noise present in the imagery.

The second source of differences in the results may be in a different image content. In case of WorldView2 data used in previous experiment, the images were almost entirely (at least in 90%) covered by single land cover class. In present study the EROS A images were labeled based on the prevailing land cover category defined as covering over 50% of imaged area. This could result in much higher complexity of image content, and in turn, in lower classification accuracies.

Both possibilities should be taken into account during further research. Some noise may be added to WorldView2 images, as well as more homogenous EROS A images (or WV2 images of the areas having more complex land cover) may be used.

Presented research prove that for VHR satellite images multifractal parameters should be considered as valuable textural features. Based on this features the second-best classification result was obtained in the one of the three performed tests. For two other tests the multifractal feature (Δ_p^{SUM}) was in the set of the four winnowed attributes, enabling very efficient classification approach. It should be stressed that similar result was obtained also in previous WorldView2 experiment. In both cases, the very limited (and very efficient) sets of textural parameters were chosen by winnowing, containing the multifractal parameter (although not the same one) and the skewness of absolute gradient feature. However, the importance of these parameters for classification of VHR satellite images indicated in reported studies should be proven during further research extended for other VHR satellite sensors and images of different areas.

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