Resources placement in the 4-dimensional fault-tolerant hypercube processors network

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***Abstract*—In the paper some properties of the perfect resources placement in the 4-dimensional hypercube network with soft degradation are considered. The methods of determining some kind of perfect placement are discussed. Two algorithms for the resources deployment are proposed. Moreover, some characteristics of the network degradation are determined. The average number of processors working of degraded network with given order for a specific type of resource placement is determined. This value characterizes the loss of the network computing capabilities resulting from the increase of the degree of network degradation. Also, the probability distribution that for degraded logical structures of the hypercube with given order there exists a specific type of resources placement is evaluated.**

I. Introduction

A REGULAR structure of processors network as torus or hypercube ([1],[2]) could be implemented in many kind of networks. An interconnection network with the hypercube logical structure is a well-known interconnection model for multiprocessor systems. Such networks possess already numerous applications in critical systems ([3], [4]) and still they are the field of interest of many theoretical studies concerning system level diagnostics ([5], [6], [7], [8], [9]) or resource placement problem, which has intensively been studied in [10]-[16].

We could observe the usage processors networks in critical application areas (military, aerospace or medical systems etc.). Such networks are (mostly) used in real-time systems, which require a very high data reliability processing throughout the life of the network, which is achieved with the use of real-time system diagnostic techniques. It causes increase of specific requirements concerning the dependability of such systems. The increase of the dependability of a processors network could be achieved by using effective methods of diagnosis and reconfiguration after the faulty processor identification. Designing and exploitation of such networks is a comprehensive project that requires addressing a number of theoretical and practical problems. One of the problem is skillful resources deployment in the network and modification of resource deployment after each phase of the network degradation. Wherein by the resource is to be understood for example: database, I/O port, data files etc.

We investigate the case when network of processors has the logical structure of 4-dimensional hypercube. A topology of hypercube may be represented by an ordinary consistent graph whose nodes are described by 4-dimensional binary vectors such that the Hamming distance between vectors (labels) of the adjacent nodes equals one.

In networks with soft degradation a processor identified as faulty is not repaired (or replaced) but access to it is blocked. New (degraded) network continues work under the condition that it meets specific requirements [17]-[19]. The method of a hypercube network reconfiguration after identifying faulty processors, based on the determination of the set of fit coherent sub-cubes of hypercube with maximum dimensions was presented by Chen and Tzeng in [11]. In the work [20] an analysis of the different patterns of reconfiguration in networks with soft degradation was conducted, these schemes were divided into software and hardware.

We assume that processors of a network could be divided into processors working, and resources processors (e.g. distributed database, I/O devices). The execution of some tasks by a processor working requires an access to resources, also some results obtained by processors working must be submitted to the resources [21].

A generalized cost of a network traffic with a specified resources deployment and workload of network are usually tested with experimental methods or simulation. We try to express it analytically for a given perfect deployment, which is defined as the minimum number of resources processors which are accessible by processors working. Each processor working has access to the at least *m* resources processors at a distance of not more than *d*. Such an approach is known in relation to the network of regular logical structure [12], [13], [14], [15], [16]. The definition *(m,d)*-perfect deployment is a characteristic of a value of the generalized cost of information traffic in the network at a given load of tasks.

The goal of this paper is to apply the above presented approach to a processors network of a 4-dimensional cube type logical structure along with its soft degradation process.

The paper consists of four sections and a summary. The second section provides a basic definitions and properties of working structures which are induced by the network in the process of its degradation. There are defined a concept of perfect deployment of processors with resources in the working structure of network G (definition 3). In the third section are considered the conditions of existence of a specific type of perfect deployments and the algorithms for determining them for working structures. In the fourth section the characteristics of the network degradation as the probability distribution of the number of consecutive hypercube processors failures, after which it loses possibility of perfect resources deployment of type $(m,1)$.

In summary there are formulated conclusions from the results presented in the paper.

II. THE BASIC DEFINITIONS AND PROPERTIES

Definition 1. The logical structure of processors network we call the structure of 4-dimensional hypercube if it is described by coherent ordinary graph $G = < E, U >$, (E-set of processors, U-set of bi-directional data transmission lines), which nodes can be described (without repetitions) by 4-dimensional binary vectors (labels) in the following way

$$[H(\varepsilon(e'), \varepsilon(e'')) = 1] \Leftrightarrow [(e', e'') \in U], \qquad (1)$$

where $H(\varepsilon(e'), \varepsilon(e''))$ is Hamming distance between the labels of nodes e' and e''.

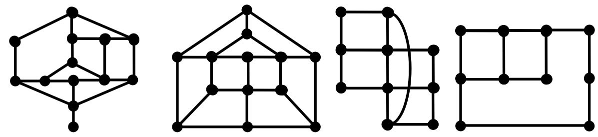
If $|E| = 2^4$ and $|U| = 2|E|$ then graph of type of 4-dimensional cube we denote by H^4 and we call it (non labeled) 4-dimensional the cube.

Let $\tilde{H}^4 = \langle H^4; \{\varepsilon(e) : e \in E(H^4)\}\rangle$ indicates 4-dimensional cube with labeled nodes, and $\breve{G}_p^C(H^4)$ and $\breve{G}_p^C(\tilde{H}^4)$ denote coherent sets of subgraphs of graphs H^4 and $\tilde{H}^4$ of order p.

Let $\nu(G)\ (G \in \breve{G}_p(H^4), p \geq 6)$ indicates the number of ways to assign labels for nodes of graph G satisfying the formula 1 and thus the number of subgraphs of graph $\tilde{H}^4$ which after removal of the labels of the nodes are isomorphic with graph G.

The graph G is a regular graph of degree 4 with the degree $\mu(e) = |E(e)|$ ($E(e)$ - a set of nodes adjacent to a node $e \in E$) -of each node of the graph G is equal to 4.

Geometric forms of working structures $G \in \breve{G}_p(H^4)$ of order $p \geq 6$ induced by the network in the process of its degradation are presented in the form of a possibly minimum number of intersecting edges ([9], [22], [23]). It should be noticed that this form of presentation makes it easier to analyze the structures properties. The numbers $\nu(G)$ are determined by the method of structures composition [22][23]. Some examples of such structures are presented in Fig. 1.



32 32 128 96

Fig. 1 Examples of geometric forms of working structures of the network (specified the numbers $\nu(G)$) [9]

From many properties of working structures of the network we will discuss only properties of a cycle.

The cycle will be treated as a subgraph of graph G and not as a cyclical chain [24], [25].

Definition 2. A cycle C in the graph G we call such a coherent subgraph of G that $\forall_{e \in E(C)} : \mu(e|C) = 2$.

Property 1. $\forall_{C \in C(G)} : |E(C)| \in \{4, 6, 8\}$ ($C(G)$ -the set of cycles in the graph G), because the graph C is a graph of class H^4 and thus (in accordance with definition 1)

$$\forall_{e \in E(C)} : [\{e', e''\} = E(e)] \Rightarrow$$
$$\Rightarrow [(H(\varepsilon(e), \varepsilon(e')) = H(\varepsilon(e), \varepsilon(e'')) = 1) \wedge$$
$$\wedge (\forall_{e''' \in \{E(C) \setminus \{E(e) \cup \{e\}\}\}} : H(\varepsilon(e), \varepsilon(e''')) > 1)]$$

($H(\varepsilon(e), \varepsilon(e'))$ -Hamming distance between the labels of nodes e and e'), what is satisfied only if the string lengths of the cycle binary labels are equal to 4, 6 or 8 (there is no need to justify that).

Property 2.

$$[\exists_{\{C', C''\} \subseteq C(G)} : \{E(C') \cap E(C'')\} = \varnothing] \Rightarrow$$
$$\Rightarrow [\neg \exists_{u \in U(G)} : |\{E(u) \cap E(C')\}| =$$
$$= |\{E(u) \cap E(C'')\}| = 1].$$

The deployment of data resources in the processors network of 4-dimensional cube type logical structure and soft degradation we will regard as a labeled graph $\langle G; \dot{E}\rangle$ ($G \in \breve{G}_p(H^4)$, $p \geq 6$, $\dot{E} \subset E(G)$, $\dot{E}$ - set of database processors, $\{E(G) \setminus \dot{E}\}$ - set of the working processors of the network G) [26].

Let $d(e', e''|G)$ denotes the distance between nodes e' and e'' in a coherent graph G, that is the length of the shortest chain (in the graph G) connecting node e' with the node e''.

Of course $d(e', e''|\tilde{H}^4) = H((\varepsilon(e'), \varepsilon(e''))$.

Definition 3. We say ([12], [13], [14], [15], [16]) that the labeled graph $\langle G; \dot{E}\rangle$ ($|\dot{E}| \geq 1$) is $(m, d\,|\,G)$ **-perfect placement** ($m \in \{1, \ldots, \mu(G)\}$, $d \in \{1, \ldots, D(G)\}$, $D(G)$ - diameter of the graph G) a set $\dot{E}$ of resources processors in the network G if there exists set $\dot{E}$ of minimum cardinality satisfying the following condition

$$[\forall_{e\in\{E(G)\setminus\dot{E}\}} : |\{e'\in\dot{E} : d(e,e'|G)\le d\}|\ge m]\wedge$$
$$\wedge[\forall_{\{e^*,e^{**}\}\subset\dot{E}} : d(e^*,e^{**}|G)>d]\wedge \quad (2)$$
$$\wedge[(\mu(e''|G)=1)\Rightarrow(e''\notin\dot{E})].$$

$(m,d|G)$- **perfect placement** will be denoted by $(m,d|G)d$ to distinguish from the placement of type $(m,d|G)$ in which the set $\dot{E}$ does not need to be of minimum cardinality.

Note that (according to the definition 3) a set $\dot{E}$ in deployment $(m,d|G)$ is a particular kind of an externally stable set.

Let $F_{(m,d)d}(G)$ denote the set of $(m,d|G)$-perfect deployment in network G.

We are interested in the conditions of existence perfect deployments of type $(m,1|G)$, the algorithms for their determination and the average number $\Re_{(m,d|G)d}(p)$ of working processors of the network of order p with $(m,d|G)$-perfect deployment of resources. The average number $\Re_{(m,d|G)d}(p)$ characterize the loss of the network computing potential (the number of working processors) together with increasing its degradation with the use of the specified $(m,d|G)$-perfect deployment.

III. Perfect Placements in the Network Working Structures

Let $F_{(m,d)}(G)$ denotes the set of $(m,d|G)$-perfect deployments in the network G and $\dot{E}(f)$-set of resources processors of the network G for the placement $f\in F_{(m,d)}(G)$.

Let us consider the methods and the algorithms for determining perfect deployments $(1,1|G)$ for $G\in\breve{G}_p(H^4)$ $(p\ge 6)$ and we determine (for these deployments) the average number

$$\Re_{(m,d|G)d}(p)=\left|\breve{G}_p(H^4)\right|^{-1}\sum_{G\in\breve{G}_p(H^4)}\nu(G)(p-\left|\dot{E}_{(m,d)d}(G)\right|)$$
$$(p\ge 6)$$

of working processors of the network of order p with $(m,d|G)$-perfect deployment of data resources (with distinction of cyclic and acyclic structures).

We denote $E^{(d)}(e|G)=\{e'\in E(G): d(e,e'|G)\le d\}$ $(d\in\{1,\ldots,D(G)\})$ and

$\hat{E}^1(G)=\{e\in E(G): (\exists_{e'\in E(e)} : \mu(e')=1)\}$.

Let us consider $(1,1|G)$-perfect deployment.

Note that $[f\in F_{(1,1)}(C)]\Rightarrow[\left|\dot{E}(f)\right|=\left\lceil 3^{-1}|E(C)|\right\rceil]$ $(C\in C(G))$, and cyclic sequences $\lambda_{(1,1)}(|E(C)|)$ of numbers $d(e',e''|C)$ $(\{e',e''\}\subseteq\dot{E}(f))$ have the form of $\lambda_{(1,1)}(4)=(2,2)$, $\lambda_{(1,1)}(6)=(3,3)$ and the $\lambda_{(1,1)}(8)=(3,3,2)$.

Property 3. $[G\in\breve{G}^C_p(H^4)(p\ge 6)]\Rightarrow[F_{(1,1)d}(G)\ne\varnothing]$ because $[G\in\breve{G}^C_p(H^4)(p\ge 6)]\Rightarrow[F_{(1,1)}(G)\ne\varnothing]$ because such a placement $\langle\tilde{H}^4;\dot{E}\rangle$ that $\dot{E}=\{e\in E(\tilde{H}^4):\delta(\varepsilon(e))\in\{0,2,4\}\}$ ($\delta(\varepsilon(e))$- is the number of ones in the binary label $\varepsilon(e)$ of a node $e\in E(\tilde{H}^4)$) is placement of type $(1,1|\tilde{H}^4)$, and thus the placement $\langle G;E(G)\cap\dot{E}\rangle$ is an element of the set $F_{(1,1)}(G)$ although it does not need to be an element of the set $F_{(1,1)d}(G)$ (see Fig. 2).

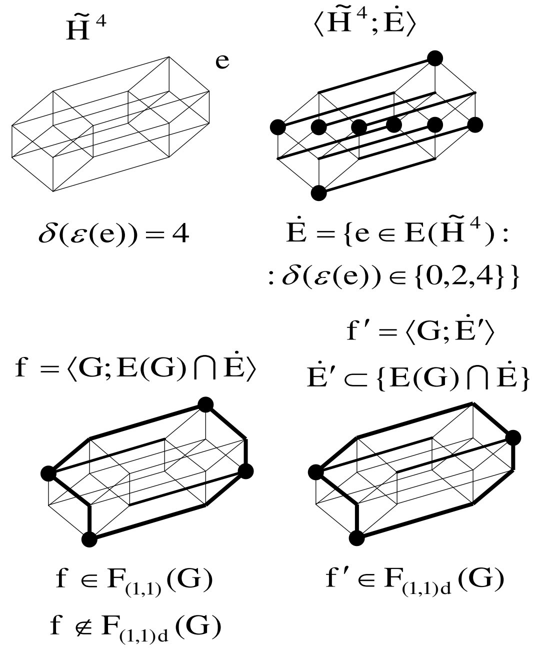


Fig. 2 Illustration of the property 3

Property 4. $[F_{(1,1)}(G)=\varnothing]\Leftrightarrow[\exists_{\{e',e''\}\subset E(G):\mu(e')=\mu(e'')=1} : (E(e'),E(e''))\in U(G)]$ because (according to the formula 2) nodes $E(e')$ and $E(e'')$ must belong to the set $\dot{E}$, which contradicts that it is an externally stable set.

The methods of determining $(1,1|G)$-perfect deployment:

1. If $G\in\breve{G}^C(H^4)$ then as the first node of a set to be found $\dot{E}(f)$ $(f\in F_{(1,1)d}(G))$ we choose such a node $e_i\in E(G)$ with the greatest degree that the subgraph $\overline{G}^{(1)}(G,e_i)=\langle\{E(G)\setminus E^{(1)}(e_i)\}\rangle_G$ has the smallest number of components of coherence and we determine a placement for every of these components of coherence,

wherein if a component of coherence is one-node it belongs to the set $\dot{E}(f)$.

2. If $G \in \breve{G}^C(H^4)$ then we determine the subgraph of G which is a cycle (in sense of definition 2) C of the greatest order and so we choose nodes of set $\dot{E}(C)$ so that they form a cyclic sequence $\lambda_{(1,1)}(|E(C)|)$ and the expression $\sum_{e \in \dot{E}(C)} \mu(e|G)$ reached the maximum value. If $\exists_{e' \in \{E(G) \setminus E(C)\}} : \{E(e'|G) \cap \dot{E}(C)\} = \varnothing$ then $e' \in \dot{E}(G)$.

A. The algorithm I - determining the resource placement based on the first method

Step 1.

Choose a node $e_i \in E(G)$ such that:

- the degree of $\mu(e_i) = \max_{e \in E(G)} \mu(e)$;
- subgraph $\overline{G}^{(1)}(G, e_i)$ has the smallest number of components of coherence.

Add the node e_i to the set $\dot{E}(f)$.

Step 2.

Check if a component of coherence of the subgraph $\overline{G}^{(1)}(G, e_i)$ is one-node or $\overline{G}^{(1)}(G, e_i) = \varnothing$.

YES

If $\overline{G}^{(1)}(G, e_i) \neq \varnothing$ add all nodes of $\overline{G}^{(1)}(G, e_i)$ to the set $\dot{E}(f)$.

Go to step 3.

NO

Assume that the $\overline{G}^{(1)}(G, e_i)$ is a new graph G.

Return to step 1.

Step 3.

The end of the algorithm.

An illustration of the algorithm work is presented in Fig. 3. One of the possible $(1,1|G)$-perfect deployment for the structure G (chosen by the algorithm) is shown in Fig. 3a.

B. The algorithm II - determining the resource placement based on the second method

Step 1.

Determine the subgraph of G which is a cycle C such that: $|E(C)| = \max_{C' \subset G} |E(C')|$. If a cycle C not exist in G go to step 3.

Step 2.

Choose nodes of set $\dot{E}(C)$ such that:

- the nodes form a cyclic sequence $\lambda_{(1,1)}(|E(C)|)$ ($\lambda_{(1,1)}(4) = (2,2)$, $\lambda_{(1,1)}(6) = (3,3)$, $\lambda_{(1,1)}(8) = (3,3,2)$).
- the expression $\max(\sum_{e \in \dot{E}(C)} \mu(e|G))$.

Step 3.

The end of the algorithm.

The algorithm working correctly for the structure G which is a cyclic graph. An illustration of the algorithm work is presented in Fig. 4. One of the possible $(1,1|G)$-perfect deployment for the structure G (chosen by the algorithm) is shown in Fig. 4a.

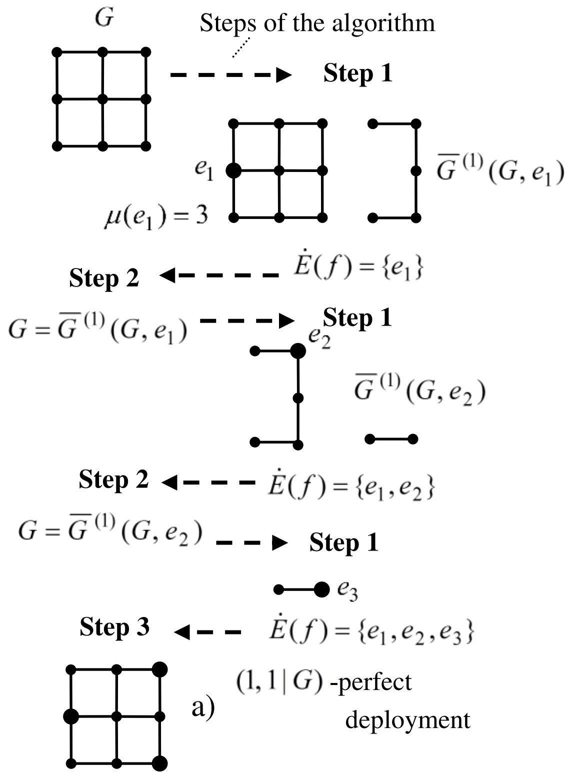


Fig. 3 An illustration of the algorithm I steps

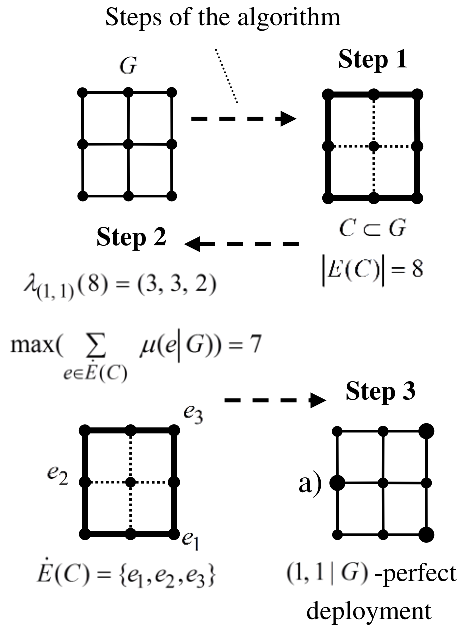


Fig. 4 An illustration of the algorithm II steps

IV. CHARACTERISTICS OF THE NETWORK DEGRADATION

A degraded structure G' of the structure $G \in \breve{G}^{C}_{p}(\tilde{H}^{4})$, $(p > 6)$, after a processor $e \in E(G)$ failure is the most numerous (in terms of number of nodes) such a cyclic consistent component $G' = \langle E(G) \setminus \{e\} \rangle_{G}$ of the graph G that there exists a placement $(m, 1 | G')$ for $m \geq 1$. If the graph G' does not exist, then processor e is called a critical processor of the structure G [18]. The critical processor failure causes the unfitness of the network for further usage. A structure, in which a critical processor exists, is called a critical structure.

It could be easily observed (on the base of properties 3-4) that verifying the condition of the existence of placement $(m, 1 | G')$ for degraded structure G' could be reduced to checking the following condition: $(G' \in \breve{G}^{C}(H^{4})) \wedge (|E(G')| \geq 6)$.

Let us determine

$$p^{i}_{j}(G) = \left|\{e \in E(G) : \langle E(G) \setminus \{e\} \rangle_{G} \in \breve{G}^{C}_{j}\,(j \geq 6)\}\right| \; |E(G)|^{-1}$$

$$G \in \breve{G}^{C}_{i}(H^{4}), i > j$$

and

$$p^{i}_{\bullet}(G) = \left|\{e \in E(G) : \langle E(G) \setminus \{e\} \rangle_{G} \notin \breve{G}^{C}_{j}\,(j \geq 6)\}\right| \; |E(G)|^{-1}.$$

Degradation characteristics $\pi^{C}(G)$ of structure $G \in \breve{G}^{C}_{i}(H^{4})$ is called a pair $\langle p^{i}(G); p^{i}_{\bullet}(G) \rangle$, where $p^{i}(G) = (p^{i}_{i-1}(G), p^{i}_{i-2}(G), ..., p^{i}_{6}(G))$, $7 \leq i \leq 16$, whereas $p^{6}_{\bullet}(G) = 1$.

For example, structure G' in Fig. 5 Examples of specifying degradation characteristics of structure is not a critical structure and has a degraded characteristic $\pi^{C}(G')$ $= \langle$ (2/10, 4/10, 0, 4/10); 0 $\rangle$, and structure G'' is a critical structure with four (indicated) critical processors as well as $\pi^{C}(G'') = \langle$ (0, 4/8); 4/8 $\rangle$.

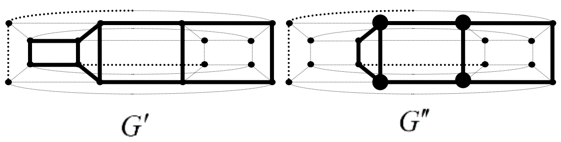


Fig. 5 Examples of specifying degradation characteristics of structure ($\pi^{C}(G') = \langle$ (2/10, 4/10, 0, 4/10); 0 $\rangle$ and $\pi^{C}(G'') = \langle$ (0, 4/8); 4/8 $\rangle$) (critical nodes of the structures were indicated)

The set $\{\langle p^{i}(\tilde{H}^{4}); p^{i}_{\bullet}(\tilde{H}^{4}) \rangle : i \in \{7, ..., 16\}\}$, $(p^{i}(\tilde{H}^{4}) = (p^{i}_{i-1}(\tilde{H}^{4}), ..., p^{i}_{6}(\tilde{H}^{4})))$, is called a degradation characteristic of a processor network, whereas

$$p^{i}_{j}(\tilde{H}^{4}) = \left|\breve{G}^{C}_{i}(\tilde{H}^{4})\right|^{-1} \sum_{G \in \breve{G}^{C}_{i}(H^{4})} p^{i}_{j}(G)\,\nu(G) \qquad (3)$$

$$p^{i}_{\bullet}(\tilde{H}^{4}) = \left|\breve{G}^{C}_{i}(\tilde{H}^{4})\right|^{-1} \sum_{G \in \breve{G}^{C}_{i}(H^{4})} p^{i}_{\bullet}(G)\,\nu(G) \qquad (4)$$

Notice that if the random variables describing the reliability state of each network processor are mutually independent as well as the probability of damage of any processor is the same, then $p^{i}_{j}(\tilde{H}^{4})$ as well as $p^{i}_{\bullet}(\tilde{H}^{4})$ indicate the possibility of occurrences that the structure $G \in \breve{G}^{C}_{i}(H^{4})$ with order i will be degraded (after the processor failure of this structure) to the structure with order j and that the network would become unfit for further usage accordingly.

On the base of formulas 3 – 4 and results from section III as well as geometric forms of hypercube network working structures given in [9], the characteristics of degradation of the cyclic network with an initial H^{4}-type logical structure were calculated (Table 1).

Table 1 $\{\langle p^{i}(\tilde{H}^{4}); p^{i}_{\bullet}(\tilde{H}^{4}) \rangle : i \in \{5, ..., 13\}\}$

$i \setminus j$	$i-1$	$i-2$	$i-3$	$i-4$	$i-5$	$p^{i}_{\bullet}$
16	1					0
15	1					0
14	0,870	0,130				0
13	0,796	0,204				0
12	0,604	0,267	0,119	0,005	0,005	0
11	0,541	0,247	0,138	0,034	0,033	0,007
10	0,384	0,218	0,149	0,139		0,112
9	0,316	0,103	0,226			0,355
8	0,032	0,323				0,645
7	0,245					0,755
6						1

By using Table 1 it is easy to determine the probability distribution of $P_{H^{4}}\{\xi = k\}, (k \in \{1, 2, ...\})$ numbers of consecutive failures of the network processors with an initial logical structure H^{4}, after which in accordance with the accepted rule of degradation it ceases to be a cyclic network of at least six processors, with the assumption that processors failures are mutually independent random variables with the equal probability (see Fig. 6).

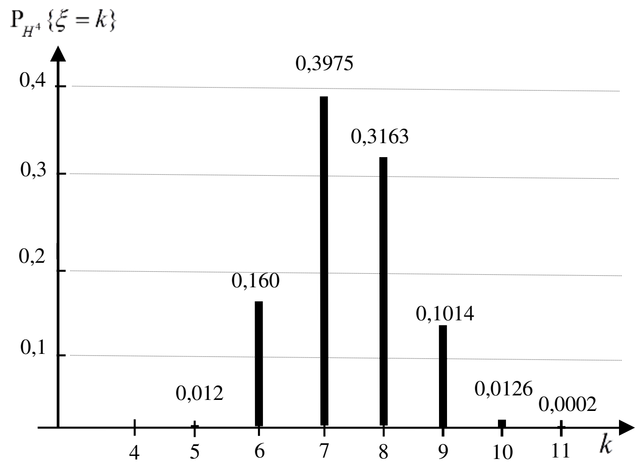


Fig. 6 Probability distribution of the number of consecutive hypercube processors failures, after which it ceases to be a cyclic network with at least six processors

Notice that to state the value of $P_{H^4}\{\xi = k\}$ it is sufficient to just calculate the probability sum of changes of the network structure from the initial state to the final state of all paths with the length of k.

V. Conclusion

On the base of the conditions of the existence of certain types of perfect deployments in hypercube structures established in Section III and the catalog of geometric forms of the hypercube network working structures given in [9], [22], [23] the average number of working processors for selected resources deployments and given degree of network degradation was determined. It characterizes of a loss of the computational potential of the degradable hypercube network for the specific perfect deployment along with the process of its degradation.

Obtained characteristics of the impact of network degradation on the average number of processors working in the network and the probability distribution of the number of consecutive processors failures, after which it loses possibility of perfect resources deployment of type $(m,1)$ indicates that 4-dimensional hypercube network characterized by a high resistance for processors failures. It turned out that if the number of faulty processors is less than 6, the probability that for the network exists the perfect placement $(m,1)$ is equal 0,998 and the average number of processors working is equal 8,17. (Table 2).

Table 2 The average numbers of working processors for $(1,1|G)$ -perfect placement

p	16	15	14	13	12	11	10	9	8	7
$\Re^{C}_{(1,1\|G)}(p)$	12	11	10	9,19	8,46	8,17	6,92	6	5	4,57

Generalized cost of information traffic in a network for a given deployment of resources processors depends on the nature of the tasks performed by the network. Such an analysis is not the subject of this paper. This is another problem which can be examined by using simulation methods for a specified (m,d)-perfect deployment and a particular type of task load of the network.

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