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A computational support for the group consensus reaching process in the fuzzy environment

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***Abstract*—In this paper we present an intelligent consensus reaching support system within the group of individuals under fuzzy preferences and fuzzy majority. Our solution is based on the idea of soft degree of consensus proposed by Fedrizzi, Kacprzyk, Nurmi and Zadrożny, which is meant as the statement: "most of the individuals agree with the most of the options". Our new comprehensive model provides an effective support for the discussion guidance in the form of quantitative indices, i.e. sensitivity of individuals, option consensus degree and the cost of preference's changes. This additional measures support and simplify consensus reaching process and improve the degree of total agreement among decision makers.**

I. Introduction

CURRENTLY, any activity that a human being does involves solving problems, making choices, thus in general, involves some decisions. The essence of decision making is unified and short: there are some options to choose between and only one has to be chosen [2].

We accept the statement that the goal-directed decisions are difficult to make alone. Thus, we assume a session with a group of individuals and make the *group decision making process* the groundwork of our further consideration [3]. What matters here is respecting the preferences of all decision makers and arriving at a joint solution meant as an agreement of individuals as to the final decision. This interactive and iterative process is meant in the literature as a *consensus reaching process* and it requires: time, active participating of all individuals, creative thinking and being open-minded, active listening, etc [1]. The model of consensus reaching process is manageable only if individuals are able to negotiate and change their preferences.

Consensus reaching support system is commonly known as an intelligent, computer-based system that helps a team of decision makers solve problems and make choices [10]. The main role of this computer-based system plays *moderator*. His most important task is to support the discussion, i.e. he stimulates the exchange of knowledge, encourages appropriate individuals to change their opinions, focus the discussion on the relevant issues, etc. This is repeated until the group gets close to acceptable consensus or until some time limit is reached [6].

All of these features of consensus reaching process developed a need for a modern computer-based support with sophisticated tools which simplify this dynamic process and allow to achieve consensus in a more efficient way. There are many different methods that facilitate multi-stage consensus reaching process, but in this paper we show the implementation of the one of computer-based support systems. We consider either the improvement of consensus achieved or the cost of entire decision making process meant as a cost of total changes of individuals preferences.

II. Framework of the consensus reaching model

A. Fuzzy Preference Relations

The core of our system is a human consistent representation of preferences. Preference relation is a very useful tool that gives relevant information about the comparison of options in decision making process [11].

Formally, there is a finite set of $n \geq 2$ options, $S = \{s_1, s_2, ..., s_n\}$, and a finite set of $m \geq 2$ individuals, $E = \{e_1, e_2, ..., e_m\}$. Each individual $k \in E$ presents his opinion as to the particular pairs of options in S. These testimonies are assumed to be individual *fuzzy preference relation* R_k defined over the set of options S (i.e. in $S \times S$) [3].

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An individual fuzzy preference relation of expert k, R_k, is given by its membership function $\mu_{R_k}: S \times S \to [0,1]$. Namely, $\mu_{R_k}(s_i, s_j) > 0{,}5$ denotes that the alternative s_i is preferred to the alternative s_j, $\mu_{R_k}(s_i, s_j) < 0{,}5$ indicates that the option s_j is preferred to the option s_i and $\mu_{R_k}(s_i, s_j) = 0{,}5$ denotes that there is no difference between two considered options s_i and s_j [11].

We assume cardinality of S to be small enough to allow us to represent individual fuzzy preference relation R_k by a $n \times n$ matrix $R_k = [r_{ij}^k]$, such that $r_{ij}^k = \mu_{R_k}(s_i, s_j)$, *i,j=1,...,*n; *k=1,...,m*. R_k is also assumed to be reciprocal, i.e. $r_{ij}^k + r_{ji}^k = 1$, moreover, $r_{ii}^k = 0$, for all i, j, k [5].

B. Fuzzy Majority and Fuzzy Linguistic Quantifiers

An important part of our consensus reaching model is a fuzzy majority in the sense of fuzzy linguistic quantifiers, i.e. most, almost all etc. It is represented by the fuzzy logic-based calculus of linguistically quantified statements due to Zadeh [12].

A linguistically quantified statement is understood as "most individuals are satisfied" which can be written as

$$Qy\text{'s are } F \tag{1}$$

where Q is a linguistic quantifier (e.g., most), $Y = \{y\}$ is a set of objects (e.g., individuals) and F is a property (e.g., satisfied).

Our task is to find the degree of truth value of this linguistically quantified statement (1). First, a fuzzy linguistic quantifier is equated with a fuzzy set in [0,1]. For instance, "most" may be given as

$$\mu_{\text{"most"}}(x) = \begin{cases} 1 & \text{for } x > 0.8 \\ 2x - 0.6 & \text{for } 0.3 \le x \le 0.8 \\ 0 & \text{for } x < 0.3. \end{cases} \tag{2}$$

Property F is defined as a fuzzy set in the set of objects Y, and if $Y = \{y_1, \dots, y_p\}$, then we suppose that truth value $(y_i \text{is} F) = \mu_F(y_i), i = 1, \dots p$. The degree of statement (1), that is, truth value (*Qy's are F*), is now calculated in two steps:

$$r = \frac{1}{p} \sum_{i=1}^{p} \mu_F(y_i) \tag{3}$$

$$\text{truth value}(Qy\text{'s are }F) = \mu_Q(r)\,. \tag{4}$$

C. Soft Degree of Consensus

Here, we define a consensus measure which indicates the agreement between decision makers' opinions. We consider a "soft" degree of consensus as proposed by Kacprzyk and Fedrizzi [4]. In our context it is meant as the statement that: *"most of the* individuals agree in their preferences to most of the options.*"* Except of total agreement or disagreement between individuals as to the final decision, this approach allows some partial, acceptable consistency in the range [0,1].

The "soft" degree of consensus in the above sense is now obtained in three steps [5]:

1) for each pair of individuals we indicate a degree of agreement as to their preferences between all the pairs of options,

2) we aggregate these degrees to derive a degree of agreement of each pair of individuals as to their preferences between Q_1 (a linguistic quantifier as, e.g., "most") pairs of options,

3) we combine these degrees to obtain a degree of agreement of Q_2 (a linguistic quantifier similar to Q_1) pairs of individuals as to their preferences between Q_1 pairs of options and this is meant to be the degree of consensus.

We start with the degree of a sufficient agreement (at least to degree $\alpha \in [0,1]$) of individuals k_1 and k_2 as to their preferences between options s_i and s_j defined by

$$v_{ij}^{\alpha}(k_1, k_2) = \begin{cases} 1 & \text{if} \quad | r_{ij}^{k_1} - r_{ij}^{k_2} | \le 1 - \alpha \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

where, $k_1 = 1, \dots, m-1$, $k_2 = k_1 + 1, \dots, m$, $i = 1, \dots, n-1$, $j = i+1, \dots, n$.

Then, the degree of agreement between individuals k_1 and k_2 as to their preferences between all the pairs of options is:

$$v(k_1, k_2) = \frac{2}{n(n-1)} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} v_{ij}^{\alpha}(k_1, k_2)\,. \tag{6}$$

Next, the degree of agreement between individuals k_1 and k_2 as to their preferences between Q_1 pairs of options is:

$$v_{Q_1}(k_1, k_2) = \mu_{Q_1}(v(k_1, k_2))\,. \tag{7}$$

The degree of agreement of all the pairs of individuals as to their preferences between Q_1 pairs of options is:

$$v_{Q_1} = \frac{2}{m(m-1)} \sum_{k_1=1}^{m-1} \sum_{k_2=k_1+1}^{m} v_{Q_1}(k_1, k_2)\,. \tag{8}$$

Finally, according to the third step, the degree of agreement of Q_2 pairs of individuals as to their preferences between Q_1 pairs of options, called the degree of consensus, is:

$$\text{con}(Q_1, Q_2) = \mu_{Q_2}(v_{Q_1})\,. \tag{9}$$

III. Indices Of Consensus

A. Sensitivity of Individuals

The above definition of the "soft" consensus concerns most individuals as to most options without any distinguish between the individuals or the options. In this paper, we adopt a more flexible concept of a consensus reaching process which takes into account a sensitivity of individuals. This important component of the decision-making process is defined by the perturbation of every particular fuzzy preference relation matrix R_k.

If the fuzzy preference relation matrix is defined as $R_k = [r_{ij}^k]$, then the perturbed fuzzy preference relation matrix may be identified by $R_k^p = [r_{ij}^{kp}]$, such that: $[r_{ij}^{kp}] = [r_{ij}^k \pm a], a \in (0,1)$, for $i = 1,\ldots,n-1;\ j = i+1,\ldots,n$, $k = 1,\ldots,m$. We also assume that $[r_{ji}^{kp}] = 1 - [r_{ij}^{kp}]$.

After matrix perturbation, we compute the degree of consensus for each individual which is now denoted as $con_k^p(Q_1, Q_2)$. Then, the measure of distance between $con(Q_1, Q_2)$ and $con_k^p(Q_1, Q_2)$ is obtained as:

$$d_k = |con(Q_1, Q_2) - con_k^p(Q_1, Q_2)| \qquad (10)$$

where $k = 1,\ldots,m$.

It is relevant for which individual small changes in fuzzy preference relation matrix cause the biggest change in the consensus degree. We obtain an ordered argument vector B where the b_i is the $i-th$ largest element (the most sensitive individual) among $\{d_1,\ldots,d_m\}$. B is called an ordered argument vector if each $b_i \in [0,1]$, and $j > i$ implies $b_i \ge b_j$, $i - 1,\ldots,m$.

B. Option Pair Related Consensus Degree

Calculating the degree of "soft" consensus might derive additionally some partial indicators of consensus, like e.g. the option consensus degree which points out to the most controversial or popular options. Thus, this indicator facilitates the work of moderator by providing him with some hints as to the most promising directions of a further discussion.

The option pair related consensus degree [7] for options s_i and s_j, $OCD(s_i, s_j) \in [0,1]$, is the degree of truth value: "most pairs of individuals agree in their preferences in respect to the pairs of options s_i and sj ." It may be formally defined as:

$$OCD(s_i, s_j) = \mu_{Q_2}\left(\frac{2}{m(m-1)} \sum_{k_1=1}^{m-1} \sum_{k_2=k_1+1}^{m} v_{ij}^{\alpha}(k_1, k_2)\right). \qquad (11)$$

C. Cost of Changes

The cost of the entire consensus reaching process may be defined as the sum of absolute values of all changes in decision makers' preferences until the session ends [9], i.e.

$$\cos t_{ij}^k(q) = \sum_{q=1}^{s} | r_{ij}^k(q) - r_{ij}^k(q-1) | \qquad (12)$$

where q denotes the iteration, $q \in [0, t]$.

IV. Consensus Reaching Support System

To clarify, initially preferences of the decision makers are far away from each other and this system aims at minimizing these distances [3]. Therefore, the moderator measures distances between individuals on each stage of the process and checks whether the consensus is reached (and process can be stopped)

$$con(Q_1, Q_2) \ge \beta \qquad (13)$$

where β indicates the acceptable degree of the consensus.

If the consensus level is not acceptable the moderator encourages appropriate individuals to update their preferences in order to improve the level of total agreement.

After calculating the consensus indicators (10) and (11), the moderator has to suggest the most sensitive decision makers to change their preferences in the most promising direction for a further discussion. Among the selected group of the most sensitive individuals the moderator finds the "typical preference relation" equited with their preference relations with respect to the pairs of options pointed out in (11). The "typical preference relation" is calculated by:

$$r_{ij}^c = \frac{\sum_{k=1}^{m} r_{ij}^k}{\sum_{k=1}^{m} r_{ij}^k + \sum_{k=1}^{m} (1 - r_{ij}^k)}. \qquad (14)$$

Then the moderator checks the relation:

$$| r_{ij}^k - r_{ij}^c | \le \delta \qquad (15)$$

If the inequality (14) is not fulfilled then the new value of preference relation for each individual k is defined as a mean value between the typical preference relation and the former value of the preference relation of individual k, i.e., as an arithmetic average:

$$r_{ij}^k(q+1) = \frac{r_{ij}^k(q) + r_{ij}^c(q)}{2}. \qquad (16)$$

V. Application Of Proposed COmputer-Based System

The parameters applied to the group consensus reaching support system are:

a) $M = 10, N = 10$

b) Acceptable degree of the soft consensus (13) is: $con(Q_1, Q_2) > 0.7$

c) $\alpha = 0.8$ in $v_{ij}^{\alpha}(k_1, k_2)$ in (5)

d) $\delta = 0.1$ in (15) which denotes that almost everyone is supposed to update his opinion even with the small step.

The initial degree of consensus was equal to $con(Q_1, Q_2) = 0.38$ which was definitely below the acceptable agreement.

The sensitivity of individuals calculated in (10) was: $B = \{0.026; 0.024; 0.021; 0.01; 0.01, 0.006; 0.006; 0.004; 0.004, \ldots$ $\ldots 0.002\}$.

Hence the ordered argument vector of the most sensitive individuals was: $E = \{e_5, e_2, e_8, e_6, e_3, e_4, e_{10}, e_1, e_9, e_7\}$.

After calculating the indicator (11) for each pair of options, we discovered that there were almost no difference between the ordering options from the most preferred (promising) to the worst and inversely. This dependency is exemplified on Fig.1.

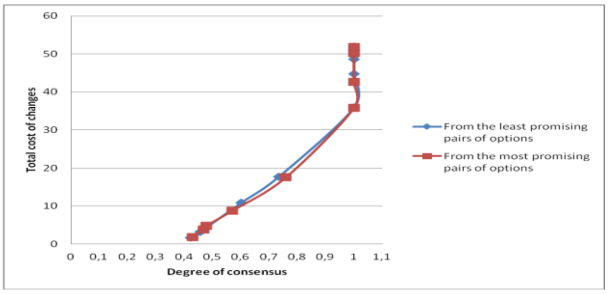

Fig. 1 Comparison of the degree of consensus and the total cost of changes for different direction of option consensus degree for 10 decision makers

However, for a smaller group of individuals (3,4 or 5) this exponential growth has differences in favor of the direction from the most promising pairs of options. TableI presents the maximum degree of the consensus obtained by a different number of sensitive individuals during the update process.

TABLE I.
MAXIMUM DEGREE OF CONSENSUS OBTAINED BY A DIFFERENT NUMBER OF SENSITIVE INDIVIDUALS COMPARED WITH THE TOTAL COST

Number of most sensitive individuals	Degree of Consensus	Total cost
3	0,61	16,86
4	0,67	22,13
5	0,77	26,73
6	0,87	32,63
7	0,99	37,23

Clearly, we can easily see an improvement in the value of the degree of consensus achieved in the group of individuals. It is also noticeable that at least the group of 5 allows us to obtain an acceptable agreement among decision makers.

VI. CONCLUSION

In this paper we proposed a new method for improving the degree of total agreement among decision makers in a consensus reaching process. We applied different procedures to find some useful indicators which allow us to run the process in the more efficient way. These procedures are to be further extended so that the improvement might take into account many aspects of this multi-criteria problem, e.g. the new optimization methods to find a best solution in the sense of aggregation either the improvement of the final degree of consensus or the total cost of changes between the decision makers during the process.

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