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Fuzzy Multi-attribute Evaluation of Investments

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***Abstract*—Most companies have a large number of projects that they would like to do for various reasons. However, usually there is never enough time and money available to complete all of them. Selecting a portfolio from available project proposals is crucial for the success of each company. This paper proposes a practical framework for modelling projects portfolio selection problem with fuzzy parameters resulting from uncertainty associated with decision makers' judgment. A fuzzy multi-attribute decision-making approach is adopted. A two-step evaluation model that combines fuzzy AHP (*Analytic Hierarchy Process*) and fuzzy TOPSIS (*Technique for Order Preference by Similarity Ideal Solution*) methods is used to rank potential projects. The proposed approach is illustrated by an empirical study of a real case from steel industry involving five criteria and ten projects.**

I. Introduction

DECISIONS on investment projects have a direct impact on a company's success. They are, however, particularly difficult, because of the ubiquitous uncertainty associated with any business activity. This causes that the project portfolios selection (PPS) becomes an increasingly complex decision task, which in turn motivates managers to utilise modern techniques and tools to optimise capital allocation.

At present, there are a lot of methods that can be applied to solve PPS problems, including Economic Analysis, Decision Theory, Optimisation and Multi-criteria methodologies. In order to deal with both financial and non-financial project attributes, the multi-criteria decision making (MCDM) analysis is a preferred approach. The goal of the multi-criteria decision making analysis is to "provide a set of attributes aggregation methodologies that enable the development of models considering the decision makers' (DMs') preferential system and judgement policy" [7]. In general, MCDM methods may be divided into two groups: multi-objective decision making (MODM) and multi-attribute decision making (MADM). The latter have been used to solve problems with discrete decision choices and a predetermined or limited number of alternative choices. A comparative study on various MCDM methods is presented, e.g., in [1] and [8].

In this paper, an MADM approach to project portfolio selection is applied. The classical approach is expanded to deal with uncertainty expressed in the form of fuzzy numbers. There is a range of scientific publications which develop very sophisticated methods for describing uncertainty. Meanwhile, according to the survey of Hubbard [9], modern enterprises still asses and mitigate risk using old fashioned methods which have not evolved much for several decades. This paper attempts to fill out the gap between the theory and practise. A practical framework to deal with the PPS problem is developed. The reminder of this paper is organised as follows. Section II briefly describes problems with modelling uncertainty in PPS. Section III presents methodology used to solve the PPS problem. The proposed framework is described in Section IV. Numerical example is shown in Section V. The paper ends with concluding remarks.

II. Risk and uncertainty in project portfolio selection

There is no universally accepted definition of business risk and uncertainty, but in the PPS context they may be understood as potential problems with availability and certainty of information, and also imprecise choices. To deal with uncertainty in PPS, it must be first noted that PPS usually consists of two stages. In the first stage, projects are selected on the basis of the threshold criteria which are determined by decision-makers. In order that a project can pass to the next stage, it must strictly fulfil these criteria. The selected projects are input for an MADM method, which usually first calculates the weights of criteria, and then determines the ranking of potential projects. Each part of an MADM method is associated with different type of uncertainty. The main source of uncertainty in determination of criteria weights is imprecision of expert judgements. Due to cognitive biases, decisions may be deviated from a standard of rationality or good judgement. To take into account these systematic errors, fuzzy numbers are used instead of crisp numbers.

In this paper, fuzzy criteria weights are obtained using a fuzzy AHP method. Some researchers believe that classical Saaty's AHP method has some weaknesses which are connected with uncertainty. In [18] authors points out that mapping experts judgement to crisp numbers and cognitive biases generates uncertainty which is not taken into account by the classical AHP method and may have huge impact on the results. To deal with this problem, some researches fuzzified AHP (e.g., [4] has considered trapezoidal fuzzy intervals for comparison ratios in AHP and [5] has proposed approach for triangular case).

Some criticise the fuzzy AHP and argue that it does not give much different results then the crisp version. However, it is important to note that the main criticism is based on assumption that comparison ratios are based on expert consensus. In practise, comparison ratios are usually averages of expert ratios. As long as there are no agreement between experts, everyone of them interprets linguistic variables in different ways. In this situation, expert verbal possibilities should better be translated into fuzzy than crisp numbers.

The second aspect of representing model uncertainty by fuzzy numbers concerns the type of fuzzy numbers that should be used. Generally, there are two approaches to fuzzification of comparison ratios – fuzzy numbers [19], [21] and fuzzy intervals [13]. The empirical study shows [3] that membership functions of numerical equivalents of linguistic terms are similar to fuzzy numbers, which are not distributed equidistantly along the possibility scale and which vary considerably in symmetry and vagueness.

Usually, after the first stage, the calculated criteria weights are defuzzified. In the proposed approach, fuzzy weights are passed to the second stage. This guaranties that uncertain judgements of decision-makers are taken into account also during the second phase of PPS.

The second phase of PPS determines the ranking of potential projects based on the weights obtained in the first stage. In this phase, uncertainty concerns attributes of alternatives. The attributes are divided into two groups: objective (numerical) and subjective (linguistic). A majority of authors argue that only subjective criteria should be described in terms of fuzzy numbers. In the proposed approach it is assumed that quantification of financial attributes of investment projects should be modelled as mixture of possibility and probability distribution.

III. Methodology

The proposed methodology of selecting an efficient portfolio of investment projects consists of the following steps. First, multiple criteria that are considered in the decision-making process are identified. Then, criteria weights are calculated according to the fuzzy AHP methodology. After constructing the relationship of a criteria decision matrix, the fuzzy TOPSIS approach is used to achieve the final ranking results.

The AHP method ([16]) is a flexible MCDM tool for complex problems where both qualitative and quantitative aspects need to be considered ([2]). The AHP integrates different measures into a single overall score for ranking alternatives. By reducing complex decisions to a series of simple, pairwise comparison judgements, then synthesising the results, the AHP not only helps the analysts to arrive at the best decision, but also provides a clear rationale for the choices made [5]. The fuzzy AHP [5] is the fuzzy extension of AHP to deal with the fuzziness of the data involved in the decision making process. Fuzzy AHP enables decision makers to specify preferences in the form of natural language expressions about the importance of each performance attribute.

TOPSIS [10] is another popular approach to MCDM. The main idea is that the best alternative should have the shortest distance from the (positive) ideal solution and the farthest distance from the negative ideal solution. The TOPSIS method has also been extended in different ways to deal with fuzzy numbers. The simplest one is to change fuzzy MCDM into a crisp one by using defuzzification. This approach, however, can lead to the loss of information. Another approach is to define a crisp Euclidean distance between fuzzy numbers. An approach based on α-cuts can also be found in the literature [20].

IV. Proposed framework for project portfolio selection

Based on the methodology described in Section III, a new approach to project portfolio selection problem is proposed. It consists of four stages described in the following subsections.

A. Identification of available investment projects and criteria

First, a committee of decision-makers who come from different managerial levels is formed. They identify m potential investment projects $A_1,\ldots,A_m$ and n criteria $C_1, \ldots, C_n$ that each project must fulfil. To properly assess a project, many factors should be considered [14], [15]. McKown and Mohamed [12] presents multi-criteria project selection where uncertainty of profitability parameters is described by fuzzy numbers. They point out that the selection of investment projects should consist of two kinds of parameters – financial (e.g., net present value) and non-financial (e.g., social, environmental, strategic an organisational). The method proposed in this paper allows to aggregate financial and non-financial indicators. First, the criteria are divided into two groups – objective and subjective. Objective criteria are described by fuzzy numbers which usually result from fuzzy modelling or aggregation of historical data. Subjective criteria are qualitative criteria with values that are specified by decision makers in the form of linguistic variables. Here, linguistic variables are transformed into fuzzy numbers (triangular or trapezoidal). This simplifies further ranking of projects.

B. Calculation of synthetic importance weights

Obviously, the problem of calculating the importance weights of the criteria is a typical multi-variable and multi-objective optimisation problem. To calculate importance weights of the criteria the fuzzy AHP is used. To make a pairwise comparison, a linguistic scale is developed. Table I provides summary of translation developed based on [3]. The final scores of criteria are also represented by fuzzy numbers.

C. Development of performance ratings for projects

The performance ratings of objective and subjective parameters are calculated. At the end of this stage the threshold selection is made. Only those projects which have passed the threshold selection are taken into account in the next stage of the PPS.

D. Calculating hierarchy of projects using fuzzy TOPSIS

The hierarchy of projects is established. Then, the overall ranking of projects is calculated. The ranking allows a decision-maker to select the most appropriate investment option.

Projects	Capital investment	C1			C2					C3		C4				C5
	000's PLN	C1.1	C1.2	C1.3	C2.1	C2.2	C2.3			C3.4	C3.5	C4.1	C4.2	C4.3	C4.4	C5.1
							C2.3.1	C.2.3.2	C.2.3.3							
P1	(135,150,165)	(-66 083.9, 14 376.1, 127 533.0,225 122.1)	(-5.1,4.5, 9.3,12.1)	2.1	middle	stability	average	average	external	widely used	neutral	without growth	no impact	positive	avaliable	one
P2	(216,240,264)	(-1 334 440.0, 628 247, 493 130.0, 1 757 191.0)	(-4.3,-1.0, 3.3,7.4)	4.5	big	stability	average	average	well-developed	widely used	increase	moderate growth	no impact	nautral	need to train	one
P3	(1 270,1 430,1 590)	(-1 00 0694.0, -24 717, 659 279.1, 1779 179.1)	(-2.3,-0.9 , 2.3,5.9)	5.2	big	stability	average	average	well-developed	widely used	increase	moderate growth	no impact	positive	need to train	more than two
P4	(1 600,1 780 ,1 995)	(-258 273.0, -111 259.0, 202 888.0, 5 023 315.0)	(-1.1,-0.8 , 1.3,6.2)	4.6	big	stability	average	average	well-developed	widely used	large increase	moderate growth	no impact	positive	need to train	more than two
P5	(375, 410, 450)	(-356 417.0, -140 463.0, 291 199.4, 785 944.0)	(-3.3,-0.8 , 2.4, 7.9)	3.2	big	growth	best	below average	well-developed	widely used	large increase	moderate growth	improve condition	neutral	need to train	more than two
P6	(465, 515, 565)	(-368 432.0, -102371.0, 560 595.0, 1 123 142.0)	(-3.0,-0.5 , 2.9, 7.8)	3.1	big	growth	average	average	well-developed	widely used	increase	moderate growth	degradation	very positive	need to train	more than two
P7	(125, 138 , 150)	(-102 722, -50 513.0, 245 279.2, 391 245.1)	(-1.7 ,-0.8, 5.9, 9.1)	2.1	big	growth	best	average	well-developed	highest	increase	moderate growth	degradation	very positive	need to train	more than two
P8	(170, 190, 210)	(-144 482, -70 168.0, 235 800.5, 448 462.5)	(-2.8, -0.6 , 4.9, 7.9)	2.3	big	growth	best	average	well-developed	highest	increase	moderate growth	degradation	very positive	need to train	more than two
P9	(250, 320, 350)	(-137 252.0, 23 797.3, 129 599.0, 301 074.2)	(-3.8, 1.4 , 5.9, 6.9)	3.3	small	stability	best	below average	no network	highest	large increase	moderate growth	degradation	positive	need to train	more than two
P10	(20, 22, 24)	(-12 344.0, 4 322.1, 16 123.0, 28 144.0))	(-4.1, 2.2, 5.9, 7.6)	4.1	big	small	average	lower	no network	widely used	neutral	moderate growth	no impact	neutral	difficulties in recruiting	one

Fig. 1. Description of the projects alternative

TABLE I
COMPARISON OF RELATIVE IMPORTANCE OF CRITERIA FOR FUZZY AHP

Linguistic terms	**Crisp intensity of importance**	**Fuzzy intensity of importance**
Equally important	1	$(1, 1, 1)$
Moderately more important	3	$(1, 3, 5)$
Strongly more important	5	$(2, 5, 6)$
Very strongly more important	7	$(6, 7, 8)$
Extremely more important	9	$(8, 9, 9)$

TABLE II
PAIRWISE COMPARISON MATRICES

	Criteria				
	C1	C2	C3	C4	C5
C1	(1,1,1)	(2,5,6)	(8,9,9)	(8,9,9)	(8,9,9)
C2	(0.17,0.2,0.5)	(1,1,1)	(1,3,5)	(1,3,5)	(1,3,5)
C3	(0.11,0.11,0.13)	(0.2,0.34,1)	(1,1,1)	(1,1,1)	(1,1,1)
C4	(0.11,0.11,0.13)	(0.2,0.34,1)	(0.2,1,1)	(1,1,1)	(1,1,1)
C5	(0.11,0.11,0.13)	(0.2,0.33,1)	(0.2,1,1)	(0.2,1,1)	(1,1,1)

V. NUMERICAL EXAMPLE

The proposed approach was applied for PPS in steel industry. There are five criteria $C1, \ldots, C5$ – financial, market, technology and environment, staff and compliance with the company's strategic objective. Each of them is divided into subcriteria. The objective ones are NPV, IRR, Pay-back period, the rest is subjective. There is also the third level of subcriteria for the $C2$ criterion. They are called attributes.

To calculate weights of criteria, a team of decision makers make pairwise comparison. The results of this comparison are presented in Tables II, III and IV. Then, using the fuzzy AHP global priorities are obtained (Table V). The priorities are presented in terms of fuzzy numbers. It can be noticed that the higher hierarchy of the criteria is, the wider fuzzy number are. For example, fuzzy weight $C2.3.1$ range between 0 to nearly 0.3. This illustrates the well-known phenomenon of accumulation of uncertainty. That is why in next step the consistency degree should be used (e.g., fuzzy preference

TABLE III
PAIRWISE COMPARISON MATRICES - SUBCRITERIA

	Sub-criteria			
	C1.1	C1.2	C1.3	
C1.1	(1,1,1)	(1,1,1)	(6,7,8)	
C1.2	(1,1,1)	(1,1,1)	(6,7,8)	
C1.3	(0.13,0.14,0.17)	(0.12,0.14,0.17)	(1,1,1)	
	C2.1	C2.2	C2.3	
C2.1	(1,1,1)	(0.12,0.14,0.17)	(1,3,5)	
C2.2	(6,7,8)	(1,1,1)	(0.16,0.2,0.5)	
C2.3	(0.2,0.33,1)	(2,5,6)	(1,1,1)	
	C3.1	C3.2		
C3.1	(1,1,1)	(6,7,8)		
C3.2	(0.13,0.14,0.17)	(1,1,1)		
	C4.1	C4.2	C4.3	C4.4
C4.1	(1,1,1)	(0.17,0.2,0.5)	(0.2,0.33,1)	(1,3,5)
C4.2	(2,5,6)	(1,1,1)	(1,3,5)	(8,9,9)
C4.3	(1,3,5)	(0.2,0.333,1)	(1,1,1)	(8,9,9)
C4.4	(0.2,0.33,1)	(0.11,0.11,0.13)	(0.11,0.11,0.13)	(1,1,1)

TABLE IV
PAIRWISE COMPARISON MATRICES - ATTRIBUTES

	Attributes		
	C2.3.1	C2.3.2	C2.3.3
C2.3.1	(1,1,1)	(1,3,5)	(1,1,1)
C2.3.2	(0.2,0.333,1)	(1,1,1)	(1,3,5)
C2.3.3	(1,1,1)	(0.2,0.333,1)	(1,1,1)

TABLE V
IMPORTANCE WEIGHTS OF INDIVIDUAL REQUIREMENTS

	Weight		Weight
C1	(0.429,0.633,0.917)	C3.1	(0.039,0.056,0.124)
C2	(0.073,0.175,0.372)	C3.2	(0.006,0.008,0.018)
C3	(0.051,0.064,0.122)	C4.1	(0.003,0.007,0.039)
C4	(0.051,0.064,0.122)	C4.2	(0.013,0.036,0.145)
C5	(0.051,0.064,0.122)	C4.3	(0.008,0.019,0.085)
C1.1	(0.181,0.292,0.471)	C4.4	(0.001,0.003,0.013)
C1.2	(0.181,0.292,0.471)	C5.1	(0.051,0.064,0.122)
C1.3	(0.025,0.042,0.072)	C2.3.1	(0.002,0.027,0.313)
C2.1	(0.006,0.05,0.307)	C2.3.2	(0.001,0.02,0.264)
C2.2	(0.018,0.061,0.204)	C2.3.3	(0.002,0.015,0.127)
C2.3	(0.011,0.063,0.31)		

TABLE VI
FINAL RANKING OF PROJECTS

Project	Rank	Project	Rank	Project	Rank
P9	0.7154	P3	0.6817	P5	0.6770
P10	0.7101	P7	0.6789	P6	0.6714
P1	0.7095	P8	0.6782	P2	0.6615
P4	0.7011				

programming).

In the next step, evaluation matrix is created. Matrix consists of 15 criteria and 10 projects ($P1, \ldots, P10$). The objective criteria are characterised by fuzzy intervals. The level of subjective criteria are specified by experts. The subjective criteria are translated into triangular fuzzy numbers.

In the presented example, there are two kinds of subjective attributes – some of them describe patterns, and some of them judgements. Market size criterion $C2.1$ and prospects for market growth criterion $C2.2$ belong to first group. They describe the belief of decision maker that market for project i will behave in accordance with some pattern. For example, pattern *stable* means the dynamic of the market growth which may be described by the fuzzy number $(-1.02, 0, 1.02)$.

The second group that is subjective criteria represents judgements of experts. Therefore, they are treated as ordinal fuzzy variables. Since all of subjective criteria are ordinal (variable with order), thus fuzzy ordinal rank transformation is used. After translation of linguistic variables – the fuzzy TOPSIS is applied. The obtained final ranking of projects is presented in Table VI.

VI. CONCLUSION

The evaluation and selection of industrial projects is one of the most important aspects of PPS. This paper proposed a combined fuzzy MADM approach based on fuzzy AHP and fuzzy TOPSIS techniques. A real world case study from steel industry was presented to explain approach. The paper introduced fuzzy decision making concept, when some data is burden with uncertainty. It is argued that if a fuzzy MADM problem is defuzzified into crisp one to early, then the advantage of modeling uncertainty becomes negligible. The rational approach is to defuzzify imprecise values at the very end of methods. Based on this argument, we perform deffuzzification at the very end of MADM method during calculate weight of criteria.

More research is needed to examine projects interaction and dependency. Further research is also required with respect to the subjective criteria of project selection. The problem of quantifying the qualitative factors remains a difficult and sometimes controversial tasks.

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