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A hybrid approach to supply chain modeling and optimization

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***Abstract*—The paper presents the concept and an outline of the implementation of a hybrid approach to supply chain modeling and optimization. Two environments mathematical programming (MP) and logic programming (LP) were integrated. The strengths of integer programming (IP) and constraint logic programming (CLP), in which constraints are treated in a different way and different methods are implemented, were combined to use the strengths of both. The proposed approach is particularly important for the decision models with an objective function and many discrete decision variables added up in multiple constraints. In order to verify the proposed approach, the optimization models were presented and implemented in both traditional mathematical programming and the hybrid environment.**

I. Introduction

A SUPPLY chain is referred to as an integrated systems which synchronizes a series of related business processes in order to acquire raw materials and parts, transform them into finished products and distribute to customers and retailers. The supply chain plays an important role in the automotive, electronics and food industries.

Huang et al. [1] studied information sharing in supply chain management. They considered and proposed four classification criteria: supply chain structure, decision level, modeling approach and shared information.

Supply chain structure: It defines the way various organizations within the supply chain are arranged and related to each other.

Decision level: Three decision levels may be distinguished in terms of the decision to be made: strategic, tactical and operational, with their corresponding period, i.e., long-term, mid-term and short-term.

Supply chain analytical modeling approach: This approach focuses on type of representation, in this case, mathematical relationships, and the aspects to be considered in the supply chain. The literature mostly describes and discusses mathematical programming-based modeling: linear programming, integer programming or mixed integer linear programming models [2]–[6]. Minimization of integrated costs is the main purpose of the models presented in the literature [6]–[10]. Maximization of revenues or sales is considered to a lesser extent [4], [11].

Shared information: Information is shared between network nodes determined by the model. This enables production, distribution, inventory and transport planning, depending on the purpose. The information sharing process is a vital aspect in an effective supply chain. The following groups of parameters are taken into account: resources, inventory, production, transport, demand, time, etc.

This paper focuses on the modeling approach to optimization problems in supply chain. A type of representation together with aspects to consider in the supply chain makes up a modeling approach. The vast majority of the works reviewed have formulated their models as linear programming (LP), integer programming (IP) and mixed integer linear programming (MILP) problems and solved them using the Operations Research methods. Nonlinear programming, multi-objective programming, fuzzy programming with stochastic programming are used much less frequently [12].

Problems related to the design, integration and management of the supply chain affect many aspects of production, distribution, warehouse management, supply chain structure, transport modes etc. Those problems are usually closely related to each other, some may influence one another to a greater or lesser extent. Because of the interconnectedness and a very large number of different constraints: resource, time, technological, and financial, the constraint–based environments are suitable for producing "natural" solutions for highly combinatorial problems. In the literature, references to modeling and optimizing supply chain problems using constraint–based environments are relatively few in number [11], [12].

A. Constraint-based environments

Constraint satisfaction problems (CSPs), constraint programming (CP) and constraint logic programming (CLP) [13]–[15] offer a very good framework for representing the knowledge and information needed to deal with supply chain problems.

Constraint satisfaction problems (CSPs) are mathematical problems defined as a set of elements whose state must satisfy a number of constraints. CSPs represent the entities in a problem as a homogeneous collection of finite constraints over variables, which are solved by constraint satisfaction methods. CSPs are the subject of intense study in both artificial intelligence and operations research, since the regularity in their formulation provides a common basis

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to analyze and solve problems of many unrelated families [13]. Formally, a constraint satisfaction problem is defined as a triple (X,D,C), where X is a set of variables, D is a domain of values, and C is a set of constraints. Every constraint is in turn a pair (t,R) (usually represented as a matrix), where t is an n-tuple of variables and R is an n-ary relation on D. An evaluation of the variables is a function from the set of variables to the domain of values, $v{:}X\rightarrow D$. An evaluation v satisfies constraint $((x_1,\ldots,x_n),R)$ if $(v(x_1),..v(x_n))\in R$. A solution is an evaluation that satisfies all constraints.

Constraint satisfaction problems on finite domains are typically solved using a form of search. The most used techniques are variants of backtracking, constraint propagation, and local search. Our experience as well as that of other researchers, confirms that constraint propagation is central to the process of solving a constraint problem [13], [14], [16]. Constraint propagation embeds any reasoning that consists in explicitly forbidding values or combinations of values for some decision variables of a problem because a given subset of its constraints cannot be satisfied otherwise.

CSPs are often used in constraint programming. Constraint programming is the use of constraints as a programming language to encode and solve problems. Constraint logic programming is a form of constraint programming, in which logic programming is extended to include concepts from constraint satisfaction. A constraint logic program is a logic program that contains constraints in the body of clauses. Constraints can also be present in the goal. These environments are declarative.

B. Organization and structure of the paper

In this paper, we focus on the problem of hybrid modeling and optimization of the supply chain problems in the hybrid environment. We propose a novel approach to supply chain modeling and optimization by developing integrated models and methods using the complementary strengths of MILP and CP/CLP (II, III). In this approach, both the hybrid model (V) and the hybrid framework (IV) to its efficient solution were developed.

In order to verify the proposed approach, the optimization model in mixed linear integer programming (MILP) was created and implemented in traditional (IP) and hybrid approaches. Finally, the hybrid model was optimized in the hybrid framework (V).

II. Motivation

Based on [1], [2], [13], [15], [16] and our previous work [14], [17], [18] we observed some advantages and disadvantages of these environments.

An integrated approach of constraint programming (CP) and mixed integer programming (MIP) can help to solve optimization problems that are intractable with either of the two methods alone [20]–[22]. Although Operations Research (OR) and Constraint Programming (CP) have different roots, the links between the two environments have grown stronger in recent years.

Both MIP/MILP/IP and finite domain CP/CLP involve variables and constraints. However, the types of the variables and constraints that are used, and the way the constraints are solved, are different in the two approaches [23], [24].

III. State Of The Art

As mentioned earlier, the vast majority of decision-making models for the problems of production, logistics, supply chain are formulated in the form of mathematical programming (MIP, MILP, IP).

Due to the structure of these models (adding together discrete decision variables in the constraints and the objective function) and a large number of discrete decision variables (integer and binary), they can only be applied to small problems. Another weakness is that only linear constraints can be used. In practice, the issues related to the production, distribution and supply chain constraints are often logical, nonlinear, etc. For these reasons the problem was formulated in a new way

In our hybrid approach to modeling and optimization supply chain problems, we proposed the environment, where:

- knowledge related to supply chain can be presented in a linear and logical constraints (implement all types of constraints of previous MILP/MIP models [18], [19] and introduce new types of constraints (logical, nonlinear, symbolic etc.));
- the optimization model solved by using the framework can be formulated as a pure MILP/MIP model, a CP/ CLP model or as a hybrid model;
- the novel method of constraint propagation is introduced (obtained by transformation of the optimization model to explore its structure (feasible routes, capacities, etc.));
- constrained domains of decision variables, new constraints and values for some variables are transferred from CP/CLP to MILP/MIP;
- the efficiency of finding solutions to the problems of larger sizes is increased.

As a result, we obtained the hybrid optimization environment that ensures a better and easier way of modeling and optimization, and more effective search solution for a certain class of optimization problems. This class includes quantitative models related to costs, customer service and inventories. Models of this class are characterized by adding up many discrete decision variables in both constraints and the objective function.

IV. Hybrid optimization environment

In order to implement all the assumptions and requirements outlined in the previous chapter, both constraint logic programming (CLP) and integer programming (MILP/MIP) had to be combined and linked.

The hybrid environment consists of MILP/CLP/Hybrid models and hybrid optimization framework to solve them (Fig. 1). The concept of this framework and its phases (P1 .. P5, G1..G3) are presented in Fig. 2.

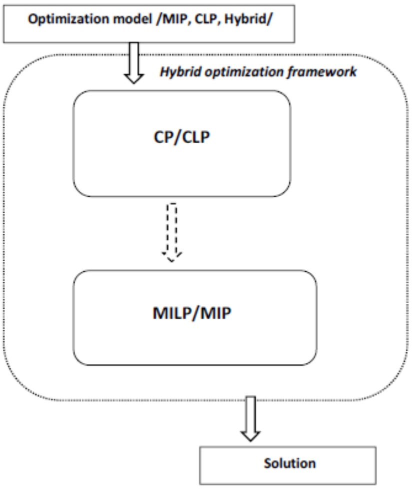

Fig. 1 Scheme of the hybrid optimization environment

The details of the hybrid environment have been discussed in [24]. The motivation was to offer the most effective tools for model–specific constraints and solution efficiency.

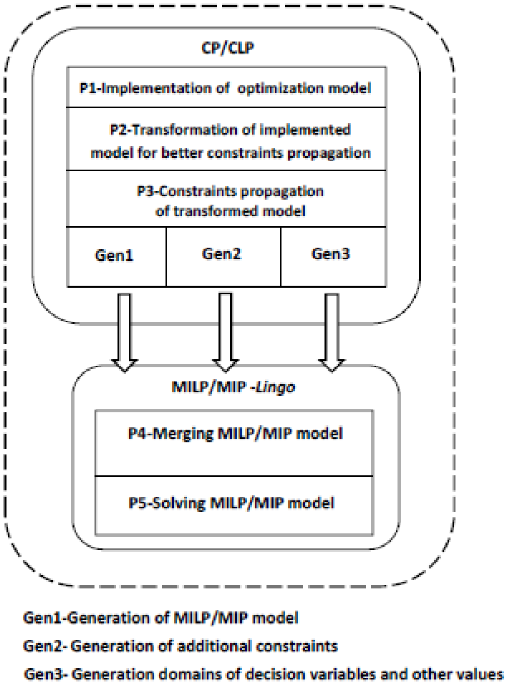

Fig. 2 Scheme of the hybrid optimization framework

The constraints propagation of the transformed model (phase–P3) largely affected the efficiency of the solution. Therefore phase P2 was introduced. During this phase, the transformation was performed using the structure and properties of the model. The details of this transformation are described in the following chapter. From a variety of tools for the implementation of the CP/CLP environment, ECLiPSe software [25] was selected. ECLiPSe is an open-source software system for the cost–effective development and deployment of constraint programming applications. Environment for the implementation of MILP/MIP was LINGO by LINDO Systems. LINGO Optimization Modeling Software is a powerful tool for building and solving mathematical optimization models [26].

V. EXAMPLES OF SUPPLY CHAIN OPTIMIZATION

The proposed HSE environment was verified and tested on two models.

First model was formulated as a mixed linear integer programming (MILP) problem [18], [19] under constraints (2) .. (23) in order to test the proposed environment (Fig. 1) against the classical integer programming environment [26]. Then the hybrid model (1) .. (25) was implemented and solved. Indices, parameters and decision variables used in the models together with their descriptions are summarized in Tab. 1. The simplified structure of the supply chain network for this model, composed of producers, distributors and customers is presented in Fig. 3.

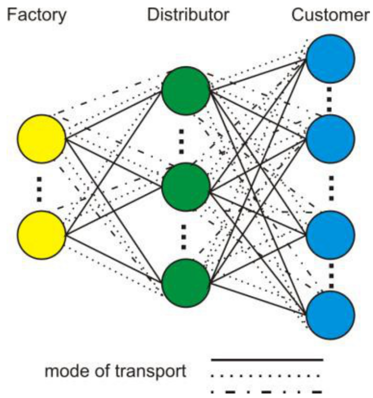

Fig. 3 The simplified structure of the supply chain network

Both models are the cost models that take into account three other types of parameters, i.e., the spatial parameters (area/volume occupied by the product, distributor capacity and capacity of transport unit), time (duration of delivery and service by distributor, etc.) and the transport mode.

The main assumptions made in the construction of these models were as follows:

- the shared information process in the supply chain consists of resources (capacity, versatility, costs), inventory (capacity, versatility, costs, time), production (capacity, versatility, costs), product (volume), transport (cost, mode, time), demand, etc;
- part of the supply chain has a structure as in Fig. 3.;
- transport is multimodal (several modes of transport, a limited number of means of transport for each mode);
- the environmental aspects of use of transport modes are taken into account;
- different products are combined in one batch of transport;
- the cost of supplies is presented in the form of a function (in this approach, linear function of fixed and variable costs);

- the models have linear or linear and logical constraints.

TABLE I
INDICES, PARAMETERS AND DECISION VARIABLES

Symbol	Description
Indices	
k	product type (k=1..O)
j	delivery point/customer/city (j=1..M)
i	manufacturer/factory (i=1..N)
s	distributor /distribution center (s=1..E)
d	mode of transport (d=1..L)
N	number of manufacturers/factories
M	number of delivery points/customers
E	number of distributors
O	number of product types
L	number of mode of transport
Input parameters	
F_s	the fixed cost of distributor/distribution center s
P_k	the area/volume occupied by product k
V_s	distributor s maximum capacity/volume
$W_{i,k}$	production capacity at factory i for product k
$C_{i,k}$	the cost of product k at factory i
$R_{s,k}$	if distributor s can deliver product k then $R_{s,k}$=1, otherwise $R_{s,k}$=0
$Tp_{s,k}$	the time needed for distributor s to prepare the shipment of product k
$Tc_{j,k}$	the cut-off time of delivery to the delivery point/customer j of product k
$Z_{j,k}$	customer demand/order j for product k
Zt_d	the number of transport units using mode of transport d
Pt_d	the capacity of transport unit using mode of transport d
$Tf_{i,s,d}$	the time of delivery from manufacturer i to distributor s using mode of transport d
$K1_{i,s,k,d}$	the variable cost of delivery of product k from manufacturer i to distributor s using mode of transport d
$R1_{i,s,d}$	if manufacturer i can deliver to distributor s using mode of transport d then $R1_{i,s,d}$=1, otherwise $R1_{i,s,d}$=0
$A_{i,s,d}$	the fixed cost of delivery from manufacturer i to distributor s using mode of transport d
$Koa_{s,j,d}$	the total cost of delivery from distributor s to customer j using mode of transport d
$Tm_{s,j,d}$	the time of delivery from distributor s to customer j using mode of transport d
$K2_{s,j,k,d}$	the variable cost of delivery of product k from distributor s to customer j using mode of transport d
$R2_{s,j,d}$	if distributor s can deliver to customer j using mode of transport d then $R2_{s,j,d}$=1, otherwise $R2_{s,j,d}$=0
$G_{s,j,d}$	the fixed cost of delivery from distributor s to customer j using mode of transport d
$Kog_{s,j,d}$	the total cost of delivery from distributor s to customer j using mode of transport d
Od_d	the environmental cost of using mode of transport d
Decision variables	
$X_{i,s,k,d}$	delivery quantity of product k from manufacturer i to distributor s using mode of transport d
$Xa_{i,s,d}$	if delivery is from manufacturer i to distributor s using mode of transport d then $Xa_{i,s,d}$=1, otherwise $Xa_{i,s,d}$=0
$Xb_{i,s,d}$	the number of courses from manufacturer i to distributor s using mode of transport d
$Y_{s,j,k,d}$	delivery quantity of product k from distributor s to customer j using mode of transport d
$Ya_{s,j,d}$	if delivery is from distributor s to customer j using mode of transport d then $Ya_{s,j,d}$ =1, otherwise $Ya_{s,j,d}$ =0
$Yb_{s,j,d}$	the number of courses from distributor s to customer j using mode of transport d
Tc_s	if distributor s participates in deliveries, then Tc_s=1, otherwise Tc_s=0
CW	Arbitrarily large constant

A. Objective function

The objective function (1) defines the aggregate costs of the entire chain and consists of five elements. The first element comprises the fixed costs associated with the operation of the distributor involved in the delivery (e.g. distribution centre, warehouse, etc.). The second element corresponds to environmental costs of using various means of transport. Those costs are dependent on the number of courses of the given means of transport, and on the other hand, on the environmental levy, which in turn may depend on the use of fossil fuels and carbon-dioxide emissions.

The third component determines the cost of the delivery from the manufacturer to the distributor. Another component is responsible for the costs of the delivery from the distributor to the end user (the store, the individual client, etc.). The last component of the objective function determines the cost of manufacturing the product by the given manufacturer.

Formulating the objective function in this manner allows comprehensive cost optimization of various aspects of supply chain management. Each subset of the objective function with the same constrains provides a subset of the optimization area and makes it much easier to search for a solution.

$$\sum_{s=1}^{E} F_s \cdot Tc_s + \sum_{d=1}^{L} Od_d \Big(\sum_{i=1}^{N}\sum_{s=1}^{E} Xb_{i,s,d} + \sum_{s=1}^{E}\sum_{j=1}^{M} Yb_{j,s,d}\Big) + \sum_{i=1}^{N}\sum_{s=1}^{E}\sum_{d=1}^{L} Koa_{i,s,d} + \sum_{s=1}^{E}\sum_{j=1}^{M}\sum_{d=1}^{L} Kog_{s,j,d} + \sum_{i=1}^{N}\sum_{k=1}^{O}\Big(C_{ik} \cdot \sum_{s=1}^{E}\sum_{d=1}^{L} X_{i,s,k,d}\Big) \quad (1)$$

B. Constraints

The model was based on constraints (2) .. (24) Constraint (2) specifies that all deliveries of product k produced by the manufacturer i and delivered to all distributors s using mode of transport d do not exceed the manufacturer’s production capacity.

Constraint (3) covers all customer j demands for product k ($Z_{j,k}$) through the implementation of delivery by distributors s (the values of decision variables $Y_{i,s,k,d}$). The flow balance of each distributor s corresponds to constraint (4). The possibility of delivery is dependent on the distributor’s technical capabilities - constraint (5). Time constraint (6) ensures the terms of delivery are met. Constraints (7a), (7b), (8) guarantee deliveries with available transport taken into account. Constraints (9), (10), (11) set values of decision variables based on binary variables Tc_s, $Xa_{i,s,d}$, $Ya_{s,j,d}$. Dependencies (12) and (13) represent the relationship based on which total costs are calculated. In general, these may be any linear functions. The remaining constraints (14)..(23) arise from the nature of the model (MILP).

Constraint (24) allows the distribution of exclusively one of the two selected products in the distribution center s. Similarly, constraint (25) allows the production of exclusively one of the two selected products in the factory i.

Those constraints result from technological, marketing, sales or safety reasons. Therefore, some products cannot be distributed and/or produced together. The constraint can be re-used for different pairs of product k and for some of or all distribution centers s and factories i. A logical constraint like this cannot be easily implemented in a linear model. Only declarative application environments based on constraint satisfaction problem (CSP) make it possible to implement constraints such as (24), (25).

The addition of constraints of that type changes the model class. It is a hybrid model.

$$\sum_{s=1}^{E}\sum_{d=1}^{L} X_{i,s,k,d} \cdot R_{s,k} \le W_{i,k} \text{ for } i=1..N, k=1..O \quad (2)$$

$$\sum_{s=1}^{E}\sum_{d=1}^{L} (Y_{s,j,k,d} \cdot R_{s,k}) \ge Z_{j,k} \text{ for } j=1..M, k=1..O \quad (3)$$

$$\sum_{i=1}^{N}\sum_{d=1}^{L} X_{i,s,k,d} = \sum_{j=1}^{M}\sum_{d=1}^{L} Y_{s,j,k,d} \text{ for } s=1..E, k=1..O \quad (4)$$

$$\sum_{k=1}^{O}(P_k \cdot \sum_{i=1}^{N}\sum_{d=1}^{L} X_{i,s,k,d}) \le Tc_s \cdot V_s \text{ for } s=1..E \quad (5)$$

$$Xa_{i,s,d} \cdot Tf_{i,s,d} + Xa_{i,s,d} \cdot Tp_{s,k} + Ya_{s,j,d} \cdot Tm_{s,j,d} \le Tc_{j,k}$$
$$\text{for } i=1..N,\ s=1..E, j=1..M,\ k=1..O, d=1..L \quad (6)$$

$$R1_{i,s,d} \cdot Xb_{i,s,d} \cdot Pt_d \ge X_{i,s,k,d} \cdot P_k$$
$$\text{for } i=1..N, s=1..E, k=1..O, d=1..L \quad (7a)$$

$$R2_{s,j,d} \cdot Yb_{s,j,d} \cdot Pt_d \ge Y_{s,j,k,d} \cdot P_k$$
$$\text{for } s=1..E,\ j=1..M,\ k=1..O, d=1..L \quad (7b)$$

$$\sum_{i=1}^{N}\sum_{s=1}^{E} Xb_{i,s,d} + \sum_{j=1}^{M}\sum_{s=1}^{E} Yb_{j,s,d} \le Zt_d \text{ for } d=1..L \quad (8)$$

$$\sum_{i=1}^{N}\sum_{d=1}^{L} Xb_{i,s,d} \le CW \cdot Tc_s \text{ for } s=1..E \quad (9)$$

$$Xb_{i,s,d} \le CW \cdot Xa_{i,s,d} \text{ for } i=1..N, s=1..E, d=1..L \quad (10)$$

$$Yb_{s,j,d} \le CW \cdot Ya_{s,j,d} \text{ for } s=1..E, j=1..M, d=1..L \quad (11)$$

$$Koa_{i,s,d} = A_{i,s,d} \cdot Xb_{i,s,d} + \sum_{k=1}^{O} K1_{i,s,k,d} \cdot X_{i,s,k,d}$$
$$\text{for } i=1..N, s=1..E, d=1..L \quad (12)$$

$$Kog_{s,j,d} = G_{s,j,d} \cdot Yb_{j,s,d} + \sum_{k=1}^{O} K2_{s,j,k,d} \cdot Y_{s,j,k,d}$$
$$\text{for } s=1..E, j=1..M, d=1..L \quad (13)$$

$$X_{i,s,k,d} \ge 0 \text{ for } i=1..N, s=1..E, k=1..0,\ d=1..L \quad (14)$$

$$Xb_{i,s,d} \ge 0 \text{ for } i=1..N, s=1..E, d=1..L, \quad (15)$$

$$Yb_{s,j,d} \ge 0 \text{ for } s=1..E, j=2..M,\ d=1..L, \quad (16)$$

$$X_{i,s,k,d} \in C \text{ for } i=1..N, s=1..E,\ k=1..0, d=1..L, \quad (17)$$

$$Xb_{i,s,d} \in C \text{ for } i=1..N, s=1..E, d=1..L \quad (18)$$

$$Y_{s,j,k,d} \in C \text{ for } s=1..E, j=1..M,\ k=1..0, d=1..L \quad (19)$$

$$Yb_{s,j,d} \in C \text{ for } s=1..E, j=1..M,\ d=1..L, \quad (20)$$

$$Xa_{i,s,d} \in \{0,1\} \text{ for } i=1..N, s=1..E, d=1..L, \quad (21)$$

$$Ya_{s,j,d} \in \{0,1\} \text{ for } s=1..E, j=2..M,\ d=1..L, \quad (22)$$

$$Tc_s \in \{0,1\} \text{ for } s=1..E \quad (23)$$

ExclusionD($X_{i,s,k,d}$, $X_{i,s,l,d}$, s) for k≠l , s=1..S (24)

ExclusionP($X_{i,s,k,d}$, $X_{i,s,l,d}$, i) for k≠l , i=1..N (25)

C. Model transformation

Due to the nature of the decision problem (adding up decision variables and constraints involving a lot of variables), the constraint propagation efficiency decreases dramatically. Constraint propagation is one of the most important methods in CLP affecting the efficiency and effectiveness of the CLP and hybrid optimization environment (Fig. 1). For that reason, research into more efficient and more effective methods of constraint propagation was conducted. The results included different representation of the problem and the manner of its implementation. The classical problem modeling in the CLP environment consists in building a set of predicates with parameters. Each CLP predicate has a corresponding multi-dimensional vector representation. While modeling both problems, (1) .. (23) and (1) .. (25), quantities i, s, k, d and decision variable $X_{i,s,k,d}$ were vector parameters. The process of finding the solution may consist in using the constraints propagation methods, labeling of variables and the backtracking mechanism. The quality of constraints propagation and the number of backtrackings are affected to a high extent by the number of parameters that must be specified/labeled in the given predicate/vector. In both models presented above, the classical problem representation included five parameters: i, s, k, d and $X_{i,s,k,d}$. Considering the domain size of each parameter, the process is complex and time-consuming. Our idea was to transform the problem by changing its representation without changing the very problem. All permissible routes were first generated based on the fixed data and a set of orders, then the specific values of parameters i, s, k, d were assigned to each of the routes. In this way, only decision variables $X_{i,s,k,d}$ (deliveries) had to be specified. This transformation fundamentally improved the efficiency of the constraint propagation and reduced the number of backtracks. A route model is a name adopted for the models that underwent the transformation.

D. Decision-making support

The proposed models can support decision-making in the following areas:

- the optimization of total cost of the supply chain (objective function, decision variables-Appendix A2);
- the selection of the transport fleet number, capacity and modes for specific total costs;
- the sizing of distributor warehouses and the study of their impact on the overall costs (Appendix A3, Fig. 4, Fig. 5, Tab. V);
- the selection of transport routes for optimal total cost.

Detailed studies of these topics are being conducted and will be described in our future articles. We use the hybrid approach to both modeling and solving.

VI. NUMERICAL EXPERIMENTS

In order to verify and evaluate the proposed approach, many numerical experiments were performed. All the examples relate to the supply chain with two manufacturers (i=1..2), three distributors (s=1..3), five customers (j=1..5), three modes of transport (d=1..3), and ten types of products (k=1..10). Experiments began with three examples of P1, P2, P3 for the optimization MILP model (1) .. (23). The examples differ in terms of capacity available to the distributors s (V_s), the number of transport units using the mode of transport d (Zt_d) and the number of orders (No). The first series of experiments was designed to show the benefits and advantages of the hybrid approach. For this purpose the model (1) .. (23) was implemented in both the hybrid and integer programming environments. In addition, hybrid implementation of the transformed model was performed with and without constraint propagation, (MILPT2) and (MILPT1), respectively. The experiments that follow were conducted to optimize examples P4, P5, which are implementations of the model (1) .. (25) for the hybrid approach. Examples P4, P5 were obtained from P1, P3 by the addition of logical constraints (24), (25).

Numeric data of input parameters for examples P1, P2, P3, P4, P5 are shown in Appendix A1. The results in the form of the objective function and the computation time are shown in Table II. Other results including the decision variables for the optimal value of the objective function are given in Appendix A2.

TABLE II
THE RESULTS OF NUMERICAL EXAMPLES FOR BOTH APPROACHES

P(No)	Hybrid				Integer Programming	
	MILPT1		MILPT2		MILP	
	Fc	T	Fc	T	Fc	T
P1(10)	22401*	600**	22394^{O}	18	22404*	600**
P2(10)	21167*	600**	21142^{O}	150	21343*	600**
P3(20)	45654^{O}	95	45654^{O}	8	45710*	600**

P(No)	MH	
	Fc	T
P4(10)	22397^{O}	255
P5(20)	46419^{O}	43

Fc	the value of the objective function
T	time of finding solution (in seconds)
o	the optimal value of the objective function after the time T
*	the feasible value of the objective function after the time T
**	calculation was stopped after 600 s
MILP	MILP model implementation in the IP environment.
MILPT1	MILP model after transformation - implementation in the hybrid optimization framework without phase P3
MILPT2	MILP model after transformation-implementation in the hybrid optimization framework.
MH	Hybrid model after transformation-implementation in the hybrid optimization framework.

For each example the solution for the MILPT2 implementation was found faster than that for the MILP implementation. Moreover, for examples P1 .. P3, the traditional approach based on integer programming gives only feasible solution (calculation was stopped after 600 s) despite using highly efficient LINGO solvers. It is obvious that the solution of the hybrid model (MH) was, due to its nature, only possible using the hybrid environment. Also, the proposed environment brought the expected results. The results were obtained in only a slightly longer period of time than that necessary for examples P1 and P3.

VII. CONCLUSION

The efficiency of the proposed approach is based on the reduction of the combinatorial problem. This means that using the hybrid approach practically for all models of this class, the same or better solutions are found faster (the optimal instead of the feasible solutions). Another element contributing to the high efficiency of the method is a possibility to determine the values or ranges of values for some of the decision variables (phase P3). All effective LINGO solvers can be used in the hybrid method.

Therefore, the proposed solution is highly recommended for all types of decision problems in supply chain or for other problems with similar structure. This structure is characterized by the constraints of many discrete decision variables and their summation. Furthermore, this method can model and solve problems with logical constraints.

Further work will focus on running the optimization models with non-linear and other logical constraints, multi-objective, uncertainty etc. in the hybrid optimization framework.

APPENDIX A1

TABLE III
DATA FOR COMPUTATIONAL EXAMPLES P1, P2, P3,P4,P5

k	V_k
P1	1
P2	1
P3	3
P4	2
P5	3
P6	1
P7	1
P8	3
P9	2
P10	3

j
1
2
3
4
5

i
F1
F2

s	V_s			F_s
	P_1,P_4	P_2,	P_3,P_5	
C1	200	300	800	600
C2	200	300	1000	700
C3	200	400	1000	900

d	Pt_s	Zt_s			Od_s
		P_1,P_4	P_2	P_3,P_5	
S1	10	30	50	60	125
S2	20	20	35	35	180
S3	40	10	20	20	240

i	s	d	$K_{i,s,d}$	$T_{i,s,d}$
F1	C1	S2	2	3
F1	C1	S3	4	4
F1	C2	S2	4	2
F1	C2	S3	8	3
F1	C3	S2	6	2
F1	C3	S3	8	3
F2	C1	S2	5	4
F2	C1	S3	7	4
F2	C2	S2	2	6
F2	C2	S3	4	7
F2	C3	S2	2	6
F2	C3	S3	3	6

i	k	$W_{i,k}$	$C_{i,k}$
F1	P1	350	10
F1	P2	300	40
F1	P3	500	30
F1	P4	600	40
F1	P5	400	50
F1	P6	300	60
F2	P5	400	50
F2	P6	300	60
F2	P7	400	70
F2	P8	500	80
F2	P9	600	90
F2	P10	650	90

s	j	d	$K_{s,j,d}$	$T_{s,j,d}$	s	j	d	$K_{s,j,d}$	$T_{s,j,d}$
C1	M1	S1	2	1	C2	M3	S2	6	2
C1	M1	S2	4	2	C2	M4	S1	3	1
C1	M2	S1	2	1	C2	M4	S2	6	2
C1	M2	S2	4	2	C2	M5	S1	3	1
C1	M3	S1	2	1	C2	M5	S2	6	2
C1	M3	S2	4	2	C3	M1	S1	4	1
C1	M4	S1	2	1	C3	M1	S2	8	2
C1	M4	S2	4	2	C3	M2	S1	4	1
C1	M5	S1	2	1	C3	M2	S2	8	2
C1	M5	S2	4	2	C3	M3	S1	4	1
C2	M1	S1	3	1	C3	M3	S2	8	2
C2	M1	S2	6	2	C3	M4	S1	4	1
C2	M2	S1	3	1	C3	M4	S2	8	2
C2	M2	S2	6	2	C3	M5	S1	4	1
C2	M3	S1	3	1	C3	M5	S2	8	2

k	i	k
P5	F1	P6
P5	F2	P6
P2	F1	P8
P2	F2	P8

k	s	k
P5	C1	P6
P5	C2	P6
P5	C3	P6
P2	C1	P8
P2	C2	P8
P2	C3	P8

s	k	$T_{s,k}$	s	k	$T_{s,k}$	s	k	$T_{s,k}$
C1	P1	2	C2	P1	1	C3	P1	3
C1	P2	2	C2	P2	1	C3	P2	3
C1	P3	2	C2	P3	1	C3	P3	3
C1	P4	2	C2	P4	1	C3	P4	3
C1	P5	2	C2	P5	1	C3	P5	3
C1	P6	2	C2	P6	1	C3	P6	3
C1	P7	2	C2	P7	1	C3	P7	3
C1	P8	2	C2	P8	1	C3	P8	3
C1	P9	2	C2	P9	1	C3	P9	3
C1	P10	2	C2	P10	1	C3	P10	3

Name	k	j	$T_{j,k}$	$Z_{j,k}$	Name	k	j	$T_{j,k}$	$Z_{j,k}$
Z_01	p1	m1	8	10	Z_11	p1	m3	8	15
Z_02	p2	m2	12	10	Z_12	p2	m4	12	20
Z_03	p3	m3	10	25	Z_13	p3	m5	10	25
Z_04	p4	m4	8	30	Z_14	p4	m1	8	30
Z_05	p5	m5	12	10	Z_15	p5	m2	12	30
Z_06	p6	m1	8	15	Z_16	p6	m3	8	15
Z_07	p7	m2	12	20	Z_17	p7	m4	12	20
Z_08	p8	m3	10	25	Z_18	p8	m5	10	25
Z_09	p9	m4	8	30	Z_19	p9	m1	8	30
Z_10	p10	m5	12	30	Z_20	p10	m2	12	35

APPENDIX A2

TABLE IV

RESULTS OF OPTIMIZATION FOR COMPUTATIONAL EXAMPLES P1, P2 (FULL) AND P3,P4,P5 (PART)

Example P1 Fc^{opt} = 22394

Name	i	k	s	j	d1	d2	X_{iskd1}	Y_{sjkd2}
Z_01	F1	P1	C1	M1	S3	S1	10.00	5.00
Z_01	F1	P1	C1	M1	S3	S2		5.00
Z_02	F1	P2	C1	M2	S3	S2	10.00	10.00
Z_03	F1	P3	C1	M3	S3	S1	25.00	5.00
Z_03	F1	P3	C1	M3	S3	S2		20.00
Z_04	F1	P4	C1	M4	S3	S2	30.00	30.00
Z_05	F2	P5	C2	M5	S3	S2	10.00	10.00
Z_06	F2	P6	C1	M1	S3	S2	15.00	15.00
Z_07	F2	P7	C1	M2	S3	S2	10.00	10.00
Z_07	F2	P7	C3	M2	S3	S1	10.00	10.00
Z_08	F2	P8	C1	M3	S3	S1	5.00	5.00
Z_08	F2	P8	C2	M3	S3	S2	20.00	20.00
Z_09	F2	P9	C2	M4	S2	S1	30.00	30.00
Z_10	F2	P10	C2	M5	S3	S2	10.00	10.00
Z_10	F2	P10	C3	M5	S3	S2	20.00	20.00

i	s	d	$Xb_{i,s,d}$	i	s	d	$Xb_{i,s,d}$
F1	C1	S3	4.00	F2	C2	S3	3.00
F2	C1	S3	1.00	F2	C3	S3	2.00
F2	C2	S2	3.00				

s	j	d	$Yb_{s,j,d}$	s	j	d	$Yb_{s,j,d}$
C1	M1	S1	1.00	C2	M3	S2	3.00
C1	M1	S2	1.00	C2	M4	S1	6.00
C1	M2	S2	1.00	C2	M5	S2	3.00
C1	M3	S1	3.00	C3	M2	S1	1.00
C1	M3	S2	3.00	C3	M5	S2	3.00
C1	M4	S2	3.00				

Example P2 Fc^{opt} = 2142

Name	i	k	s	j	d1	d2	X_{iskd1}	Y_{sjkd2}
Z_01	F1	P1	C1	M1	S2	S1	8.00	8.00
Z_01	F1	P1	C1	M1	S3	S1	2.00	1.00
Z_01	F1	P1	C1	M1	S3	S2		1.00
Z_02	F1	P2	C1	M2	S3	S2	10.00	10.00
Z_03	F1	P3	C1	M3	S3	S2	25.00	25.00
Z_04	F1	P4	C1	M4	S3	S2	30.00	30.00
Z_05	F1	P5	C1	M5	S2	S2	4.00	8.00
Z_05	F1	P5	C1	M5	S3	S2	4.00	
Z_06	F1	P6	C1	M1	S3	S1	1.00	1.00
Z_05	F2	P5	C1	M5	S3	S2	2.00	2.00
Z_06	F2	P6	C1	M1	S3	S2	14.00	14.00
Z_07	F2	P7	C1	M2	S3	S2	10.00	10.00
Z_07	F2	P7	C2	M2	S3	S1	10.00	10.00
Z_08	F2	P8	C2	M3	S3	S2	25.00	25.00
Z_09	F2	P9	C1	M4	S3	S2	30.00	30.00
Z_10	F2	P10	C1	M5	S3	S2	10.00	10.00
Z_10	F2	P10	C2	M5	S3	S2	20.00	20.00

i	s	d	$Xb_{i,s,d}$	i	s	d	$Xb_{i,s,d}$
F1	C1	S2	1.00	F2	C1	S3	3.00
F1	C1	S3	4.00	F2	C2	S3	4.00

s	j	d	$Yb_{s,j,d}$	s	j	d	$Yb_{s,j,d}$
C1	M1	S1	1.00	C1	M5	S2	3.00
C1	M1	S2	1.00	C2	M2	S1	1.00
C1	M2	S2	1.00	C2	M3	S2	4.00
C1	M3	S2	4.00	C2	M5	S2	3.00
C1	M4	S2	6.00				

Example P3 Fc^{opt} = 45654

Name	i	k	s	j	d1	d2	X_{iskd1}	Y_{sjkd2}
Z_01	F1	P1	C2	M1	S3	S1	25.00	10.00
Z_11	F1	P1	C2	M3	S3	S2		15.00
.....								
Z_20	F2	P10	C2	M2	S2	S2	1.00	35.00
Z_20	F2	P10	C2	M2	S3	S2	64.00	
Z_10	F2	P10	C2	M5	S3	S2		30.00

i	s	d	$Xb_{i,s,d}$	i	s	d	$Xb_{i,s,d}$
F1	C2	S2	1.00	F2	C2	S2	8.00
F1	C2	S3	8.00	F2	C2	S3	12.00

s	j	d	$Yb_{s,j,d}$	s	j	d	$Yb_{s,j,d}$
C2	M1	S1	15.00	C2	M4	S1	14.00
C2	M2	S1	11.00	C2	M4	S2	1.00
C2	M2	S2	6.00	C2	M5	S1	9.00
C2	M3	S1	8.00	C2	M5	S2	9.00
C2	M3	S2	5.00				

Example P4 Fc^{opt} = 22397

Name	i	k	s	j	d1	d2	X_{iskd1}	Y_{sjkd2}
Z_01	F1	P1	C1	M1	S3	S1	10.00	10.00
Z_02	F1	P2	C1	M2	S3	S1	10.00	10.00
Z_03	F1	P3	C1	M3	S3	S2	20.00	20.00
Z_03	F1	P3	C2	M3	S2	S1	5.00	4.00
Z_03	F1	P3	C2	M3	S2	S2		1.00
....								
Z_10	F2	P10	C2	M5	S3	S2	10.00	10.00
Z_10	F2	P10	C3	M5	S3	S2	20.00	20.00

i	s	d	$Xb_{i,s,d}$	i	s	d	$Xb_{i,s,d}$
F1	C1	S3	4.00	F2	C2	S2	2.00
F1	C2	S2	1.00	F2	C2	S3	3.00
F2	C1	S3	1.00	F2	C3	S3	2.00

s	j	d	$Yb_{s,j,d}$	s	j	d	$Yb_{s,j,d}$
C1	M1	S1	3.00	C2	M3	S2	3.00
C1	M2	S1	1.00	C2	M4	S1	2.00
C1	M3	S2	3.00	C2	M5	S2	3.00
C1	M4	S1	2.00	C3	M2	S2	1.00
C1	M4	S2	4.00	C3	M5	S2	3.00
C2	M3	S1	3.00				

Example P5 Fc^{opt} = 46419

Name	i	k	s	j	d1	d2	X_{iskd1}	Y_{sjkd2}
Z_01	F1	P1	C1	M1	S2	S1	10.00	10.00
Z_11	F1	P1	C1	M3	S3	S1	15.00	15.00
Z_01	F1	P2	C1	M2	S3	S2	30.00	10.00
Z_12	F1	P2	C1	M4	S3	S2		20.00
....								
Z_20	F2	P10	C1	M2	S3	S2	56.00	31.00

Z_10	F2	P10	C1	M5	S3	S1		25.00
Z_20	F2	P10	C2	M2	S2	S1	4.00	4.00
Z_10	F2	P10	C2	M5	S3	S1	5.00	5.00

i	s	d	$Xb_{i,s,d}$	i	s	d	$Xb_{i,s,d}$
F1	C1	S2	7.00	F2	C2	S2	3.00
F1	C1	S3	8.00	F2	C2	S3	4.00
F2	C1	S3	8.00				

s	j	d	$Yb_{s,j,d}$	s	j	d	$Yb_{s,j,d}$
C1	M1	S1	13.00	C1	M5	S1	12.00
C1	M2	S1	1.00	C1	M5	S2	3.00
C1	M2	S2	10.00	C2	M1	S1	2.00
C1	M3	S1	9.00	C2	M2	S1	2.00
C1	M4	S1	2.00	C2	M3	S1	9.00
C1	M4	S2	7.00	C2	M5	S1	9.00

APPENDIX A3

TABLE V
ANALYSIS OF THE IMPACT PARAMETER V_S FOR FC (EXAMPLE P2)

$V=V_1=V_2=V_3$	Fc^{opt}	Distributor capacity (Vs)utilization		
		V_1	V_2	V_3
200	22 058	199	176	70
300	21 142	300	145	0
400	21 137	435	10	0
450	20 439	445	0	0
500	20 439	445	0	0
550	20 439	445	0	0

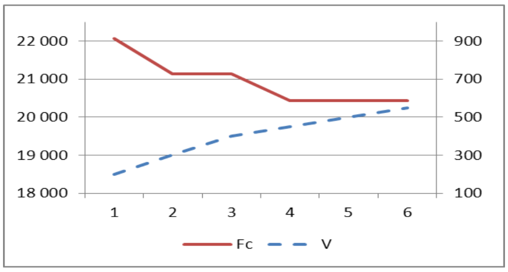

Fig. 4 The value of the objective function depending on the parameter V (Example P2)

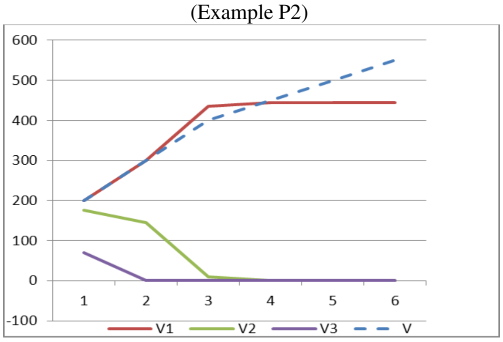

Fig. 5 Capacity utilization (Vs) for distributor s=1, s=2,s=3 (Example P2)

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