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Application of survival function in robust scheduling ofproductionjobs

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Abstract—**Scheduling production jobs in the real production system requires considering a number of factors which may prove to exert a negative effect on the production processes. Hence the need for the identification and compensation of potential disruptions as early as at the production planning stage. The aim of this paper is to employ the survival and the hazard function to anticipate potential disruptions of the schedule so that they could be absorbed to produce a robust job schedule.**

I. Introduction

SCHEDULING of production jobs has received much attention from academic researchers, which has led to numerous works published in the field. Authors have proposed various solutions aimed at creating effective production schedules [1]–[5]. Many current solutions, however, are of a purely theoretical character and are frequently unfeasible in existing production systems [6]–[7].

Practice shows that each production process involves a variety of factors impede the performance [7]–[8]. It is for that reason that we can observe the trend referred to as robust scheduling, which describes job scheduling under uncertainty [7].

This paper describes the development of a robust job schedule based on empirical data regarding the selected technological machine failure. In order to determine the selected reliability parameters, we have employed the Life-time Data Analysis, also referred to as the Survival Analysis [9]. Moreover, we proposed new service buffer input method.

II. Scheduling under Machine Failure Constraint

Robust scheduling represents a process that produces a schedule that is able to absorb disruptions, *i.e.* can account for changing parameters of the production process [7–8]. This type of scheduling is composed of the predictive phase (pertaining to the planning stage) and the reactive phase (pertaining to the production stage) [10]–[11].

Researchers indicate several sources of uncertainty in the production process, including: job processing times, preparation and completion times, works transport availability and times, machine availability, workers and tools availability, raw material/semi-finished product shortage or delay [7], [12]–[14]. Although they are largely of random character, the knowledge of the character of uncertainty factors is of crucial importance in robust scheduling [8], [15].

An increasing number of studies into robust scheduling of production jobs regard resource availability as the major source of disruption in the production process. In practice, this is strictly connected with the failure of machines processing production jobs [10], [16]–[17]. Various solutions are proposed in this area of research.

In his study, M. T. Jensen [16] adopts a deterministic approach and regards machine failure as the times of failure occurrence, and subsequently tests the developed schedules for various numbers of machines and jobs. A. Davenport *et al.* [17], S. Gürel *et al.* [18] and V. J. Leon *et al.* [19] include in their works a typical probability distribution and apply the obtained data in developing robust schedules. Many authors suggest employing the times pertaining to the field of Preventive Maintenance (PM) [20]. Deepu [21], Hong Gao [22], W. M. Kempa [23] or B. Skołud [24] employ MTTF, MTBR, MTTR and MTTFF factors in prediction of potential failure, with a view to developing robust schedules. These authors make use of the redundancy-based techniques, which are widely applied in the research in the field. An extensive body of literature [16]–[17], [21]–[22] emphasises the need for acquisition and analysis of historical data of machine failure as an invaluable source of knowledge in robust scheduling of production jobs.

III. Problem Formulation

Formulation of the job scheduling problem under uncertainty demands establishing the following [25]:

– set of processed jobs J, which is the set of n technological processes (jobs) to carry out:

$$J = \{J_1, J_2, ..., J_n\}, \tag{1}$$

– set of machines M, which is the set m of technological machines processing production jobs:

$$M = \{M_1, M_2, ..., M_m\}, \tag{2}$$

– $m \times n$ matrix of machine orders MO representing the rank of jobs on particular machines:

$$MO = [o_{ij}], \tag{3}$$

where: o_{ij} – ranking of jobs i on the machine j taking the value of: o_{ij} = 0 – when the operation i is not processed on the machine j, $o_{ij} = \{1,..., m\}$ – when the operation i is processed on the machine j;

– matrix of processing times PT containing data regarding processing times of particular technological operations:

$$PT=[p_{ij}], \tag{4}$$

where: p_{ij} – processing time of job i on the machine j, while for $o_{ij} = 0$, also $p_{ij} = 0$.

The general problem with scheduling in job-shop conditions consists in ordering jobs from the set of jobs J between the machines from the set of machines M, accounting for the technology described in the matrix MO, so that the resulting schedule corresponds to the furthest extent with the defined objective criterion.

In order to produce the robust schedule, which will absorb potential disruptions in the stock of machines, it is crucial determine for each m of the uncertain machines:

– the set of failure times of machines FT_m containing data on machine failure times:

$$FT_m=\{f_{m1}, f_{m2}, \ldots, f_{mi}\}, \tag{5}$$

where: f_{mi} – is a factor determining the probable machine failure times;

– the set of time buffers TB_m, which contains data on the machine servicing time buffers that must be included in the development of the robust schedule:

$$TB_m=\{t_{m1}, t_{m2}, \ldots, t_{mi}\}. \tag{6}$$

To determine the specified values of the sets which are crucial to developing the robust schedule of production jobs we have employed selected Survival Analysis techniques. The applied techniques enable the determination of the survival model of a given object or phenomenon, and produce data that may be used in the prediction of survival patterns [10]. It was resolved that the analysis of the character of the technological machine failure occurrence will be conducted by means of the survival and hazard functions in the robust schedule.

IV. SURVIVAL AND HAZARD FUNCTIONS

Let T be a non-negative random variable with the probability density function $f(t)$, $t > 0$ and the cumulative distribution function

$$F(t)=P(T<t). \tag{7}$$

Bellow we assume, that the random variable T represents the waiting time until the failure (death of plant). In the literature the variable T denotes the survival time [26]. The value $F(t)$ determines the probability that the failure (breakdown) occurs by duration. The survival function

$$S(t)=P(T\geq t)=1-F(t)=\int_t^{\infty} f(s)\,ds \tag{8}$$

presents the probability of correct work of a machine just before duration t (the probability of surviving to duration t), generally the probability that the failure (breakdown) does not occur by duration t. The survival characteristic of a machine may be presented by a hazard function

$$h(t)=\lim_{dt\to 0}\frac{P(t\leq T<t+dt \mid T\geq t)}{dt}= \tag{9}$$

$$=\lim_{dt\to 0}\frac{\int_t^{t+dt} f(s)\,ds}{dt P(T\geq t)}=\frac{f(t)}{S(t)}$$

The value of this function represents an instantaneous rate of occurrence of failure [9]. From (8) the formula (9) we may rewrite as

$$h(t)=-\frac{d}{dt}\ln S(t) \tag{10}$$

By solving the expression (10) we obtain a formula for the survival function

$$S(t)=\exp(-H(t)) \tag{11}$$

where $H(t)=\int_0^t h(s)\,ds$ is called the cumulative hazard function. The cumulative hazard function represents the sum of risks occurring from the duration 0 to t [9].

V. NUMERICAL EXAMPLE

The techniques for developing robust schedules presented in the preceding sections will be now presented in practice, to analyse the machine failure and servicing times at one of the representatives of the automotive industry. The data obtained from the analysis was afterwards employed in the scheduling of production jobs.

A. The Survival and Hazard Function

Let $\{(t_i, d_i)\}_{1\leq k\leq n}$ be a sequence of described failures, where t_i is a time after which the failure occurred but d_i – number of this events. We assume that times $\{t_i\}_{1\leq k\leq n}$ are ordered, $0 < t_1 < \ldots < t_k$. Fig. 1 presents the empirical cumulative distribution function

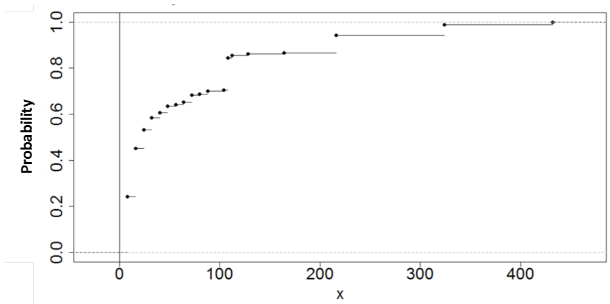

Fig. 1 The cumulative distribution function of failure

The survival function (8) is usually obtained with the Kaplan-Meier method. The estimate of the survival function is given by the following formula

$$\hat{S}(t)=\begin{cases} 1, & t<t_1, \\ \prod_{t_i \leq t} \frac{r_i - d_i}{r_i}, & t_1 \text{¿} t \end{cases} \quad (12)$$

where r_i represents the number of individuals at risk at time t_i, $1 < i < k$ (number of individuals who die at time t_i or later) and is calculated as $r_i = \sum_{j=i}^{k} d_j$. Fig. 2 presents the survival function with 95% confidence intervals. From (11) the estimate of cumulative hazard function may be obtained as

$$\hat{H}(t) = -\ln(\hat{S}(t)) \quad (13)$$

The full black curve with jumps in Fig. 3 represents the values of estimate of the cumulative hazard function $\hat{H}(t)$. Another method of estimating the cumulative hazard function is the Nelson-Aalen estimator

$$\bar{H}(t) = \sum_{t_i \leq t} \frac{d_i}{r_i} \quad (14)$$

which is represented in Fig. 3 by the red broken curve with jumps.

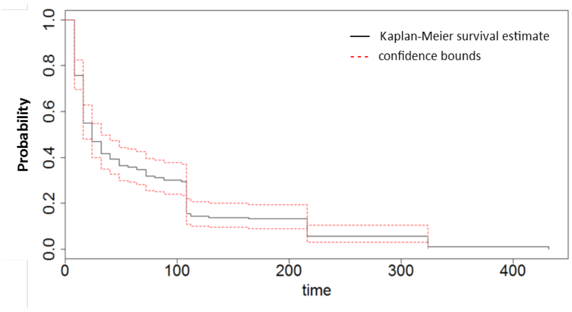

Fig. 2 The survival function – Kaplan-Meier estimate

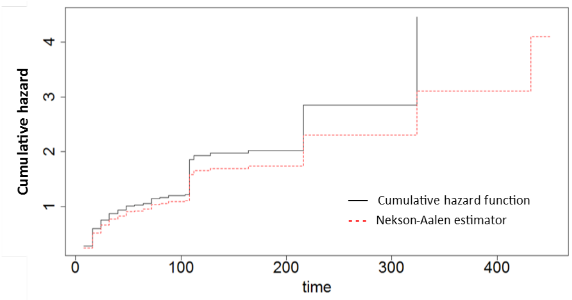

Fig. 3 The cumulative hazard function and Nelson-Aalen estimate

The data obtained from both the survival and hazard functions may be used in the robust schedule development. Decreasing survival translates to a longer life of the object, and consequently higher probability of machine failure. In terms of hazard, the abrupt jumps denote numerous instances of failures in given periods. High values of intensity function denote high risk of machine failure [10].

B. Applied Survival Analysis Results in Robust Scheduling

The presented analyses provided data that was subsequently employed in the robust schedule development for the following scenarios *m* × *n:* 3×2, 3×3, 3×4, 4×3, 4×5 and 4×6. The values of *MO* and *PT* were randomly generated. The obtained data was used to elaborate standard Schedule. The scheduling method applied was the dispatching rules, whereas the objective criterion was the maximum makespan (C_{max}) – LiSA software was used in the study. The data form enterprise obtained from the failure analysis was applied to produce the robust schedule. The application of the survival and hazard functions is presented in Fig. 4.

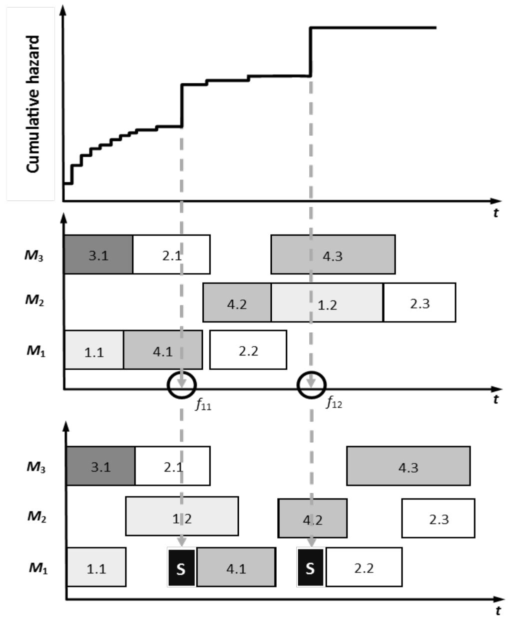

Fig. 4 The hazard function employed in the development of the robust schedule (S – machining servicing buffers)

It was established that the resulting plot of function determines the failure characteristics of the machine M_1. The set of failure times FT_1 was obtained from the results of analysis of the survival and hazard functions:

$$f_{11} = 108 \text{ [h]};\ f_{12} = 216 \text{ [h]} \quad (15)$$

and the values of the machine servicing time buffer TB_1:

$$t_{11} = t_{12} = 55 \text{ [min]} \approx 0.92 \text{ [h]} \quad (16)$$

The values of machine servicing time buffers were obtained from the empirical indicator MTTR.

In scheduling it was established that the machine M_1 worked for 100 h prior to commencement of production, hence the machine servicing time buffer f_{11} was set to occur after 8 h, and the buffer f_{22} occurred after 116 h (if necessary). The machines worked to 65% capacity, which provided the basis for the generation of the elements of matrix *MO*. The maximum job processing time was 16 h, therefore elements of matrix *PT* were also randomly generated from the range of $p_{ij} \in \langle 0; 16 \rangle$. The results of analysis are presented in Table I.

TABLE I.
RESULTS OF ROBUST SCHEDULING USING SURVIVAL AND HAZARD FUNCTIONS

Dispatching rules	Nominal schedule C_{max} [h]						Robust schedule C_{max} [h]					
	3×2	3×3	3×4	4×4	4×5	4×6	3×2	3×3	3×4	4×4	4×5	4×6
LPT	35	41	28	51	51	54	44	41	29	51	60	54
SPT	47	43	31	51	50	67	47	43	31	51	56	67
FCFS	47	43	31	51	53	54	47	43	31	51	62	54
LQUE	35	41	28	51	51	54	44	41	29	51	60	54
EDD	47	41	31	51	50	54	47	41	31	51	56	54

VI. CONCLUSION

The analysis of results obtained from the robust scheduling of production jobs indicates that the inclusion of the machine servicing time buffer M_1 did not exert a considerable effect on C_{max}. In the majority of the analysed scenarios the difference between the standard and the robust schedule was negligible (approx. 1 h), or practically non-existent. It was only in the case of 3×2 scheduling problem (scheduling with dispatching rules) and problem 4×5 that a substantial discrepancy of schedule makespans was observed (on average 8.14 h). The difference in question resulted from the fact that in these particular cases, the machine M_1 was heavily burdened with jobs, and simultaneously the values of processing times were considerably high.

Further investigations should concentrate on introducing a procedure limiting the machine servicing time buffers in job processing, with a view to obtaining lower values of scheduling assessment. That help to implement proposed method it the real production systems. Job scheduling under uncertainty requires further development and employing various inference and analysis engines.

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