

Proposal of an efficient rank-ordering method based on subjectivity

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Abstract—An effective method to evaluate order relations based on subjectivity is proposed. This method is a kind of relative evaluation method and adopts “quicksort,” which is to be the fastest sorting algorithm, to reduce the number of comparisons and the time of evaluation, so that the method has light burden on subjects. Therefore it is expected that the evaluation criteria of subjects do not fluctuate during evaluation.

We test the validity and effectiveness of the method through an experiment. In the experiment, subjects evaluate order relations of five landscape photographs about preference using the proposal method and pairwise comparison method. The experimental result shows that there is a strong correlation between order relations obtained by both of methods.

We conclude that the proposal method is effective to evaluate order relation about preference of photograph. Additionally, it is suggested that the same thing applies to other kinds of subjective criteria.

I. INTRODUCTION

THIS paper discusses a new method to evaluate order relations based on individual subjectivity with a small number of comparisons and verify its effectiveness.

These days, there is necessity to evaluate order relations based on individual subjectivity such as comparing product designs, the usability of web sites or the quality of pictures. Additionally, as artificial intelligence develops, a larger number of teacher data of subjective evaluation is needed. In such situation, the order relation of stimuli has to be evaluated by many subjects in order to maintain generality, although it is difficult when the number of stimuli is large. Therefore methods to evaluate order relations are desirable to have little burden on subjects and a fewer number of comparisons.

To reflect individual subjectivity in order relations precisely, the evaluation criteria of subjects cannot fluctuate during the evaluation.

Subjective evaluation can be classified into absolute evaluation and relative evaluation. In absolute evaluation, stimuli are individually evaluated according to numeric rating scale.

When the number of stimuli is N , the number of evaluations is also N . Therefore even if the number of stimuli increases, the number of evaluations does not increase rapidly, but it gets difficult to keep evaluation criteria constant when the evaluation takes long. In relative evaluation, a stimulus is relatively evaluated with others. Pairwise comparison is frequently used relative evaluation method, which can reflect a tiny difference on the result. On the other hand, as the number of stimuli increases, the number of comparisons increases rapidly [1]. In this paper, pairwise comparison is assumed to be a method that compares all pairs of stimuli.

In this paper, we propose a type of relative rating method and test it through experiment. Using this method, order relation is evaluated with a small number of comparisons. This method is applied to order Japanese verbal expressions about pain and shown to have practicability [2]. Reference [2] reports that the method reduces burden on subjects and significant time without losing accuracy.

We extend this method to a subjective case other than pain.

II. EXISTING METHODS

In this section, well-known subjective evaluation methods are presented.

A. Scoring method

Scoring method is one of absolute evaluation methods. Every stimulus is evaluated according to discrete numeric rating scale one by one. Thus the number of evaluations is N when the number of stimuli is N . A disadvantage of this method is difficulty in maintaining constant evaluation criteria of subjects when the number of stimuli is large.

B. Whole ranking method

In whole ranking method, all stimuli are ordered at once, so that subjects need to compare multiple stimuli at one

evaluation. This might make evaluation criteria differ between subjects.

C. Pairwise comparison

Pairwise comparison [1] is one of the simplest method and frequently used for subjective evaluation such as preference test. In general, all pairs of stimuli are compared in this method. Thus precise results of order relations can be obtained. Nevertheless, the total number of comparisons P_N is large, $P_N = N(N - 1)/2$.

III. PROPOSAL METHOD, QUICK-PAIRWISE METHOD

We propose a type of relative rating method and name it “quick-pairwise method.” In this method, “quicksort [3],” which is to be the fastest sorting algorithm, is adopted to reduce the number of comparisons, so that it has a merit of simplicity of pairwise comparison with keeping the number of comparisons small.

A. Procedure

The procedure of quick-pairwise method follows below.

- 1) Choose a pivot from stimuli at random.
- 2) Compare the rest of stimuli with the pivot and divide them into a superior group, an equivalent group and an inferior group. If a stimulus is unable to compare, except it from the evaluation. If the pivot is excepted, choose a pivot again.
- 3) Choose a pivot from each of the superior group and the inferior group.
- 4) Divide the stimuli into the three groups in each groups.
- 5) Repeat this procedure until the numbers of stimuli in all groups are three or less.
- 6) Order stimuli in all groups

B. An evaluation example

How to order ten stimuli (A to J) by quick-pairwise method is shown below. In this example, stimuli are put in order of preference.

Firstly, choose a stimulus as a pivot. In this case, stimulus A is the pivot. Then, compare and divide the rest of stimuli whether it is preferred, not preferred or preferred equivalently to the pivot. At this process, except stimuli that are uncomparable for the subject. The result of the comparisons is shown in table I.

Next, choose a pivot in each of the preferred group and the not preferred group. In this case, B in the preferred group and C in the not preferred group are chosen as the pivots. Then, compare and divide the rest of the stimuli in each group likewise. The results of the comparisons are shown in table II and table III.

Because there is no group that has more than three stimuli now, compare each stimuli in all groups left. In this case, G is preferred to F and I is preferred to H. Table IV shows the whole result of this evaluation.

TABLE I
THE RESULT WHEN A IS THE PIVOT

preferred	equivalent	not preferred
B F	E	C D
G J		H I

TABLE II
THE RESULT WHEN B IS THE PIVOT

preferred	equivalent	not preferred
	J	G F

TABLE III
THE RESULT WHEN C IS THE PIVOT

preferred	equivalent	not preferred
D		H I

TABLE IV
THE ORDER RELATION OF THE STIMULI

1B	1J	3G	4F	5A	5E	7D	8C	9I	10H
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C. The number of comparisons

Let Q_N be the average number of comparisons when N is the number of stimuli. When a pivot is randomly chosen from the stimuli, recurrence formula about Q_N is

$$\begin{aligned}
 Q_N &= N - 1 + \frac{1}{N} \sum_{k=0}^{N-1} (Q_k + Q_{N-1-k}) \\
 &= N - 1 + \frac{2}{N} \sum_{k=0}^{N-1} Q_k
 \end{aligned}
 \tag{1}$$

$$Q_0 = 0$$

and using harmonic series H_N , Q_N is

$$\begin{aligned}
 Q_N &= 2(N + 1)H_N - 4N \\
 H_N &= 1 + \frac{1}{2} + \dots + \frac{1}{N} \\
 &= \sum_{k=1}^N \frac{1}{k}.
 \end{aligned}
 \tag{2}$$

Compared with the number of comparisons in pairwise comparison method, the ratio of Q_N to P_N is approximately 54% ($N = 10$), and 29% ($N = 30$).

Even if a subject makes the most inefficient judges, $Q_N = P_N$. Therefore the number of comparisons is smaller in quick-pairwise method than in pairwise comparison method.

IV. EXPERIMENT

We tested the effectiveness of quick-pairwise method through the following experiment.

A. Subjects

A hundred subjects who were aged 17 to 22 participated in this experiment.



Fig. 1. Photographs for the stimuli

B. Stimuli

Five landscape photographs, taken by the same photographer, were used as stimuli. They are shown in Fig. 1.

C. Process

Because pairwise comparison method can evaluate the order relation of stimuli precisely, we compared quick-pairwise method with pairwise comparison method. The subjects were presented with five photographs of stimuli. They put the stimuli in order of their preferences in both ways of quick-pairwise and pairwise comparison method using a web site which was made for this experiment. Firstly, they used quick-pairwise method to order, then after five minutes, pairwise comparison method was used.

A week later, 57 out of all subjects did the same experiment again. This second experiment was held to test our hypothesis that there might be fluctuations in the evaluation criteria of subjects that would cause difference between order relations obtained by two kinds of method.

To compare the order relations obtained from the two kinds of evaluation, let the order of stimulus be the number of superior stimuli plus one. Correlation coefficients are calculated between order relations for each subjects.

D. Web site for the experiment

We made a web site shown in Fig. 2 for this experiment. Subjects compare photographs by clicking their preferred photograph or a button of “=” After clicking one of them, the pair of displayed photographs updates. Until the evaluation ends, subjects only repeat clicking.

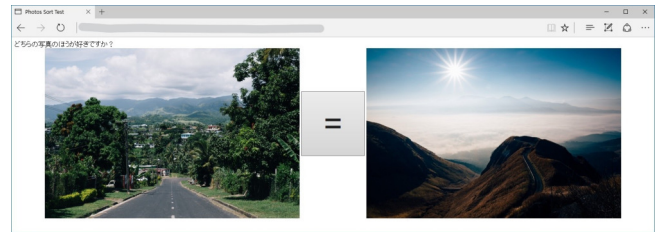


Fig. 2. Web site for the experiment

E. Result

We report the result of the experiment. Table V shows the average of the correlation coefficients of the obtained order relations. Table VI shows the correlation coefficients of the order relations evaluated by subjects who participated in the second experiment and had less than 0.75 correlation coefficient at the first experiment.

V. DISCUSSION

We will discuss the effectiveness of quick-pairwise method compared with pairwise comparison method.

The first point to be discussed is the validity of quick-pairwise method. The average of correlation coefficient between the order relations obtained at first evaluation with quick-pairwise and pairwise comparison method of table V is 0.854. It shows that there is a strong correlation between the order relations. This result suggests that quick-pairwise method is as valid as pairwise comparison method to evaluate order relations about preference.

Even though it is supposed that quick-pairwise method is valid, it should be noted that there still are differences between the order relations obtained by two kinds of method. One plausible explanation for the differences is that there are fluctuations in the evaluation criteria of subjects. To examine this explanation, we will discuss correlation coefficients between the first and second pairwise comparison.

Table V shows that the average of correlation coefficient between the first and second pairwise comparison is 0.725, which is less than the average of correlation coefficient between the first quick-pairwise and pairwise comparison, 0.874.

Let us focus on low correlation coefficients shown in table VI. Correlation coefficients between quick-pairwise and pairwise comparison of subject A are 0.384 at the first evaluation and 0.354 at the second. Those are too low to say that quick-pairwise method is strongly correlated with pairwise comparison method. On the other hand, his correlation coefficient between the first and the second pairwise comparison is 0.294, which is smaller than 0.354 and 0.384, that are correlation coefficients between quick-pairwise and pairwise comparison. The same thing applies to a half of the subjects in table VI. This result indicates that ordering stimuli even by pairwise comparison method, which can evaluate precisely, there are differences between the obtained order relations. Thus, it is quite possible that the differences between the order relations obtained by quick-pairwise and pairwise comparison method

TABLE V
THE ORDER RELATION OF STIMULI

number of subjects	average of correlation coefficient			
	1st quick-pairwise and pairwise	2nd quick-pairwise and pairwise	1st and 2nd quick-pairwise	1st and 2nd pairwise
100*	0.854	—	—	—
57 [†]	0.874	0.854	0.754	0.725
12 [‡]	0.634	0.812	0.580	0.612

* all the subjects

[†] subjects who participated in second evaluation

[‡] subjects who participated in second evaluation and had less than 0.75 correlation coefficient at first evaluation

TABLE VI
SUBJECTS WITH LOW CORRELATION COEFFICIENT AT THE FIRST EVALUATION

subject	correlation coefficient			
	1st quick-pairwise and pairwise	2nd quick-pairwise and pairwise	1st and 2nd quick-pairwise	1st and 2nd pairwise
A	0.384	0.354	0.853	0.294
B	0.512	0.645	-0.500	-0.333
C	0.530	0.900	0.700	0.707
D	0.600	0.671	0.700	-0.244
E	0.671	0.900	0.600	0.671
F	0.700	0.632	0.700	0.791
G	0.700	0.900	0.400	0.700
H	0.700	0.900	0.700	0.900
I	0.700	0.945	0.700	0.945
J	0.700	1.000	0.700	1.000
K	0.700	1.000	0.700	1.000
L	0.707	0.894	0.707	0.894

were not caused by the small number of comparisons of quick-pairwise method but by fluctuations in the evaluation criteria of subjects.

The next discussion deals with the effectiveness of quick-pairwise method as a method to evaluate order relation. It is possible that order relation is cycled using pairwise comparison method. For example, stimuli A, B and C may be judged that A is superior to B, and B is superior to C, but C is superior to A. In this case, order relation cannot be evaluated, and when such evaluation happens in more than four stimuli, it may be a noise of the whole order relation. In fact, it happened in 20% of the order relations obtained by pairwise comparison method at the first evaluation of the experiment. However, it cannot happen with quick-pairwise method. From the above-described point of view, it seems that quick-pairwise method is effective to evaluate order relations.

VI. CONCLUSION

We conclude that quick-pairwise method evaluates precise order relations about preference of photograph with a small number of comparisons and light burden on subjects. Besides, using quick-pairwise method, cycle of order relations does not happen by fluctuations in the evaluation criteria of subjects.

VII. VIEW OF THE FUTURE

In this paper, we tested the validity of quick-pairwise method about preference of photograph. More evaluations of order relation are needed to improve the credibility of the result and discussion.

Our future research will be evaluations of order relations of stimuli other than photograph. Furthermore, experiments about other subjective cases are required to prove the effectiveness of quick-pairwise method.

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