Proceedings of the Federated Conference on Computer Science and Information Systems pp. 35–38
DOI: 10.15439/2018F160
ISSN 2300-5963 ACSIS, Vol. 15

Ranking Rough Sets in Pawlak Approximation Spaces

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***Abstract*—Applying the cardinality of finite sets, interval numbers can be assigned to rough sets represented by nested sets. Borrowing two different comparison methods from Multiple Attribute Decision Making analysis, rough sets are compared and ranked on the model of interval numbers. Some special cases are investigated. Illustrative examples are presented relying on both methods. The calculated results are compared and interpreted.**

***Index Terms*—Rough sets, interval arithmetic, Possibility Degree Method, Midpoints Comparison Method.**

I. Introduction

ROUGH set theory (RST) was proposed by Pawlak in the early 1980's [1]. Information system in the Pawlak's sense can be viewed to some extent as a Multiple Attribute Decision Making (MADM) scheme (see, e.g., [2]).

In RST, rough sets represented by nested sets can be considered as an *interval set structure* to represent nonnumeric uncertainty on the model of interval numbers [3]. In our approach, however, by the cardinality of finite sets, *interval numbers* are assigned to rough sets represented by nested sets. Then, borrowing Possibility Degree Method and Midpoints Comparison Methods from MADM, rough sets can be compared and ranked numerically based on these interval numbers.

Section II presents some elementary notations for reasons of clarity. Section III and IV state fundamental knowledge about rough sets and interval arithmetic, respectively. Section V shows two comparison methods of interval numbers, namely, Possibility Degree Method and Midpoints Comparison Method. Then, it deals with the comparison and ranking of rough sets applying these two methods. It also contains simplified illustrative examples.

II. Basic Notations

Let U be a nonempty set, and $\mathcal{P}(U)$ denote the power set of U. Set operations union, intersection, difference, and complementation are denoted by $\cup$, $\cap$, $\setminus$, and ^{c}, respectively. Let $S \in P(U)$, and $\mathcal{S} \subseteq \mathcal{P}(U)$ be a nonempty family of sets. $|S|$ denotes the cardinality of S. $\cup\mathcal{S}$ and $\cap\mathcal{S}$ are defined by:

$$\cup\mathcal{S} = \{u \mid \exists S \in \mathcal{S}(u \in S)\}, \ \cap\mathcal{S} = \{u \mid \forall S \in \mathcal{S}(u \in S)\}.$$

If $\mathcal{S}$ is empty, the conventions $\cup\emptyset = \emptyset$ and $\cap\emptyset = X$ are used.

The shorthand expression "iff" is used for "if and only if".

From now on, throughout the paper let U be a finite nonempty set of objects called the *universe*.

III. Rough Sets

Notions of rough set theory can be represented in many forms. For our purposes, their constructive granule based definitions [4] are formulated as follows.

Let E be an equivalence relation on U. The partition of U generated by E is denoted by U/E. The subset $[u]_E \in \mathcal{P}(U)$ is an equivalence class from U/E containing $u \in U$. The members of U/E are called *elementary sets* or simply *base sets*. Any union of base sets is referred to as *definable set*. By definition, $\emptyset$ is definable for any equivalence relation on U. Their collection is denoted by $\mathcal{D}_{U/E}(\subseteq \mathcal{P}(U))$.

The principal notions of RST are defined by:

$$\mathsf{l} : \mathcal{P}(U) \to \mathcal{D}_{U/E}, \ S \mapsto \cup\{[u]_E \in U/E \mid [u]_E \subseteq S\},$$
$$\mathsf{u} : \mathcal{P}(U) \to \mathcal{D}_{U/E}, \ S \mapsto \cup\{[u]_E \in U/E \mid [u]_E \cap S \neq \emptyset\}.$$

Values $\mathsf{l}(S)$ and $\mathsf{u}(S)$ are commonly called the *lower* and *upper approximations* of S. With the above notations, the ordered quintuple $PAS = \langle U, U/E, \mathcal{D}_{U/E}, \mathsf{l}, \mathsf{u}\rangle$ is called a finite Pawlak approximation space.

Having given an approximation pair, to identify and characterize the features of set approximations in RST, the following fundamental notions are defined:

- *boundary* of S is $\mathsf{bnd}(S) = \mathsf{u}(S) \setminus \mathsf{l}(S)$;
- S is *exact* (*crisp*), if $\mathsf{l}(S) = \mathsf{u}(S)$, i.e., $\mathsf{bnd}(S) = \emptyset$;
- S is *rough* (*inexact*), if it is not exact, i.e., $\mathsf{bnd}(S) \neq \emptyset$.

In RST the notions of exactness and definability coincide.

For any set S, an approximation pair divides the universe U into three mutual disjoint regions:

- $POS(S) = \mathsf{l}(S)$ — *positive region* of S;
- $NEG(S) = U \setminus \mathsf{u}(S) = \mathsf{u}^c(S)$ — *negative region* of S;
- $BN(S) = \mathsf{bnd}(S)$ — *borderline region* of S.

There are (at least) four equivalent definitions of rough sets, see, e.g., [5], [6]. In the following, the nested pair of sets $\langle \mathsf{l}(S), \mathsf{u}(S)\rangle$ will be used to represent rough sets. It is a family of inexact sets in such a way that for any $T \in \langle \mathsf{l}(S), \mathsf{u}(S)\rangle$, $\mathsf{l}(S) = \mathsf{l}(T)$, $\mathsf{u}(S) = \mathsf{u}(T)$ and $\mathsf{l}(S) \subseteq T \subseteq \mathsf{u}(S)$ hold.

Proposition III.1 ([7], Proposition 3.2) *Let $S_1 \subseteq S_2$. The pair $\langle S_1, S_2\rangle$ is a rough set of the form $\langle \mathsf{l}(S), \mathsf{u}(S)\rangle$ for a set S ($S_1 \subseteq S \subseteq S_2$) if and only if S_1 and S_2 are definable and $S_2 \setminus S_1$ does not contain any singleton base set.*

IV. Basics of Interval Arithmetic

An *interval number* or *interval* [2], [8] is a closed real interval of the form $a = [a^l, a^u] = \{x \in \mathbb{R} \mid a^l \leq x \leq a^u\}$. If $a^l = a^u$, $[a^l, a^u]$ contains a single real number $a = a^l = a^u$.

Two intervals $a = [a^l, a^u]$ and $b = [b^l, b^u]$ are said to be equal, in notation $a = b$, if $a^l = b^l$ and $a^u = b^u$.

The most common special terms for an interval a are:

- $m(a) = \frac{1}{2}(a^l + a^u)$ is the *midpoint* or *center* of a;
- $w(a) = a^u - a^l$ is the *width* or *diameter* of a.

Binary operations $+, -, \cdot, /$, addition, subtraction, multiplication, and division, respectively, can be defined on the set of intervals. Their endpoint formulae are the following [8]:

$$\begin{aligned}
a + b &= [a^l + b^l, a^u + b^u];\\
a - b &= a + (-b) = [a^l - b^u, a^u - b^l], -b = [-b^u, -b^l];\\
a \cdot b &= [\min\{a^l b^l, a^l b^u, a^u b^l, a^u b^u\},\\
&\qquad\qquad \max\{a^l b^l, a^l b^u, a^u b^l, a^u b^u\}];\\
a/b &= a \cdot (1/b), \text{ where } 1/b = [1/b^u, 1/b^l]\ (0 \notin b).
\end{aligned}$$

For nonnegative intervals a, b ($0 \leq a^l, b^l$), multiplication and division formulae are simplified to:

- $a \cdot b = [a^l b^l, a^u b^u]$;
- $a/b = [a^l/b^u, a^u/b^l]$, provided in addition that $0 < b^l$.

V. Comparing and Ranking Rough Sets

A. Possibility Degree Method

Many different equivalent methods have been proposed to compare two interval numbers [2], [9].

Definition V.1 ([2], Definition 4.5) *Let $a = [a^l, a^u]$, $b = [b^l, b^u]$ be two nonnegative intervals with $w(a) > 0$ or $w(b) > 0$. The* possibility degree *of $a \geq b$ is defined by*

$$p(a \geq b) = \max\left\{1 - \max\left\{\frac{b^u - a^l}{w(a) + w(b)}, 0\right\}, 0\right\}.$$

It is also said that $p(a \geq b)$ is the possibility degree of a over b.

Theorem V.2 ([2], Theorem 4.1) *Let $a = [a^l, a^u], b = [b^l, b^u]$ and $c = [c^l, c^u]$ be three nonnegative intervals. For their possibility degrees, the following properties hold:*

1) $0 \leq p(a \geq b) \leq 1$.
2) $p(a \geq b) + p(b \geq a) = 1$. *Especially,* $p(a \geq a) = \frac{1}{2}$.
3) $p(a \geq b) = 1$ *iff* $b^u \leq a^l$.
4) $p(a \geq b) = 0$ *iff* $a^u \leq b^l$.
5) $p(a \geq b) \geq \frac{1}{2}$ *iff* $a^u + a^l \geq b^u + b^l$.
Especially, $p(a \geq b) = \frac{1}{2}$ *iff* $a^u + a^l = b^u + b^l$.
6) If $p(a \geq b) \geq \frac{1}{2}$ *and* $p(b \geq c) \geq \frac{1}{2}$, *then* $p(a \geq c) \geq \frac{1}{2}$.

It is said that

- a *superior to* b in the degree $p(a \geq b)$, in notation $a \succ b$, if $p(a \geq b) > p(b \geq a)$;
- a is *indifferent to* b, in notation $a \sim b$, if $p(a \geq b) = p(b \geq a) = \frac{1}{2}$;
- a is *inferior to* b in the degree $p(b \geq a)$, in notation $a \prec b$, if $p(b \geq a) > p(a \geq b)$.

Let $\{S_1, \ldots, S_n\} \subseteq \mathcal{P}(U)$ be a family of sets. Let us form the rough sets relating to them by their nested pair representations: $RS_i = \langle \mathsf{l}(S_i), \mathsf{u}(S_i)\rangle$ $(i = 1, 2, \ldots, n)$.

The cardinality of finite sets, as some sort of "size" of them, plays a key role in the rough set theory. Applying it, interval numbers can be assigned to the above rough sets:

$$RS_i \mapsto [RS_i] = [|\mathsf{l}(S_i)|, |\mathsf{u}(S_i)|]\ (i = 1, 2, \ldots, n).$$

To avoid heavy notations, the following simplified notations are introduced: $|\mathsf{l}(S_i)|$, $|\mathsf{u}(S_i)|$, $|\mathsf{bnd}(S_i)|$ are denoted by S_i^l, S_i^u, S_i^{bnd}, respectively.

By applying the method described by Xu in [2], ranking of rough sets can be carried out in the following steps:

Step 1. Provided that $w([RS_i]) > 0$ $(i = 1, \ldots, n)$, comparing each rough set with all rough sets as $(i, j = 1, 2, \ldots, n)$:

$$\begin{aligned}
p_{ij} &= p([RS_i] \geq [RS_j])\\
&= \max\left\{1 - \max\left\{\frac{\mathsf{S}_j^u - \mathsf{S}_i^l}{w([RS_i]) + w([RS_j])}, 0\right\}, 0\right\};
\end{aligned}$$

arranging the numbers p_{ij}'s in a possibility degree matrix:

$$P = \begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1n}\\ p_{21} & p_{22} & \cdots & p_{2n}\\ & & \vdots & \\ p_{n1} & p_{n2} & \cdots & p_{nn} \end{pmatrix}.$$

Of course, $p_{ij} \geq 0$, $p_{ij} + p_{ji} = 1$, $p_{ii} = \frac{1}{2}$ for $i, j = 1, \ldots, n$.

Step 2. Summing the numbers line by line:

$$p_i = \sum_{j=1}^{n} p_{ij}\ \ (i = 1, 2, \ldots, n).$$

Step 3. Ranking rough sets RS_i in descending (increasing) order in accordance with the values p_i's $(i = 1, 2, \ldots, n)$. The ith rough set is ranked higher (lower) than the jth rough set, if $p_i > p_j$ $(p_i < p_j)$.

B. Possibility Degree Method — A Special Case

The sets $S_1, S_2 \in \mathcal{P}(U)$ form an orthopair, if $S_1 \cap S_2 = \emptyset$. An orthopair is a reasonable means to represent bipolar information. Bipolarity arises in a natural way in RST as positive and negative regions. According to the Dubois and Prade typology [10], [11], orthopair models usually belong under the "Type II: Symmetric bivariate unipolarity". This bipolarity type well fits the nature of bipolarity representation in RST [12].

Let $\langle S_1, S_2\rangle$ be an orthopair. S_1 and S_2 are called the *positive* and *negative* reference set, respectively. Here, the positive and negative adjectives claim nothing else, only the sets S_1 and S_2 are well separated.

Let us form the rough sets relating to S_1, S_2 by their nested pair rough set representations:

$$RS_1 = \langle \mathsf{l}(S_1), \mathsf{u}(S_1)\rangle \text{ and } RS_2 = \langle \mathsf{l}(S_2), \mathsf{u}(S_2)\rangle.$$

By the above Steps 1–3, the following entities can be obtained with which the constituents of an orthopair can be ranked:

$$\begin{aligned}
p_1 &= p_{11} + p_{12} = p([RS_1] \geq [RS_1]) + p([RS_1] \geq [RS_2]),\\
p_2 &= p_{21} + p_{22} = p([RS_2] \geq [RS_1]) + p([RS_2] \geq [RS_2]).
\end{aligned}$$

Several interpretations of the obtained results can be stated:

- $p_1 > p_2$ ($p_1 < p_2$) means that the positive (negative) reference set is ranked higher than the negative (positive) reference set.
- $p([RS_1] \geq [RS_2]) = 1$ iff $\mathsf{S}_2^u \leq \mathsf{S}_1^l$.
It means that the positive reference set is certainly superior to the negative reference iff the number of elements of U which can *possibly be classified as belonging to the negative reference set* is less than or equal to the number of elements of U which can *certainly be classified as belonging to the positive reference set*.
- $p([RS_1] \geq [RS_2]) = 0$ iff $p([RS_2] \geq [RS_1]) = 1$ iff $\mathsf{S}_1^u \leq \mathsf{S}_2^l$.
It means that the negative reference set is certainly superior to the positive reference set iff the number of elements of U which can *possibly be classified as belonging to the positive reference set* is less than or equal to the number of elements of U which can *certainly be classified as belonging to the negative reference set*.
- $p([RS_1] \geq [RS_2]) = \frac{1}{2}$ iff $\mathsf{S}_1^u + \mathsf{S}_1^l = S_2^u + \mathsf{S}_2^l$ iff $\mathsf{S}_1^u - \mathsf{S}_2^u = \mathsf{S}_2^l - \mathsf{S}_1^l$. Let $\mathsf{S}_1^u - \mathsf{S}_2^u = \mathsf{S}_2^l - \mathsf{S}_1^l = K$.
$K = 0$ means that the possibility degree of the positive reference set over the negative reference set is equal to $\frac{1}{2}$, iff the number of elements of U which can *possibly* be classified as belonging to the positive and negative reference sets, respectively, are equal, and, at the same time, the number of elements of U which can *certainly* be classified as belonging to the positive and negative reference sets, respectively, are also equal.
Similar interpretations can be made for $K > 0$ and $K < 0$.

C. Possibility Degree Method — Illustrative Examples

These examples deal with studying the symptoms of thyroid dysfunctions. Although the problem emerged in Csajbók et al. [13], a substantially different solution is presented here.

Thyroid dysfunction diagnosis via clinical symptoms is an important problem [14]. We deal with only hypothyroidism and hyperthyroidism thyroid disorders [15]. The thyroid gland produces thyroid hormone. Hyperthyroidism occurs when the thyroid gland is "overactive", i.e., releases too much hormone, whereas hypothyroidism takes place when the thyroid gland is "underactive", i.e., does not produce enough hormone.

Let us consider a data table given in Table I, taken from [13]. It contains clinical symptoms which may indicate that someone, a patient, develops hypothyroidism or hyperthyroidism, perhaps neither of them. There are, of course, more symptoms of hypothyroidism and hyperthyroidism, but the example has been simplified here for illustrative purposes.

Clinical symptoms which are taken into account are the following: Weight change, Edema, Tachycardia, Increased sweating, Mood. Hypothyroidism and hyperthyroidism can accurately be diagnosed with laboratory tests. The last two columns in Table I are based on these results.

In the example, the universe U is a set of clinically observed patients: $U = \{P_1, P_2, P_3, P_4, P_5\}$. Let $S_1 = \{P_2, P_3\}$ and $S_2 = \{P_4, P_5\}$ be the sets of patients who demonstrably suffer from hypothyroidism and hyperthyroidism, respectively.

Example V.3 *If the column "Weight change" is chosen, the universe U can be partitioned into $\{P_1, P_5\}$, $\{P_2, P_3\}$, and $\{P_4\}$, reflecting the weight change being "no change", "gain", "loss", respectively. Then, based on this partition,*

$$\mathsf{l}(S_1) = \{P_2, P_3\},\ \mathsf{u}(S_1) = \{P_2, P_3\},\ \textit{i.e.,}\ [RS_1] = [2, 2];$$
$$\mathsf{l}(S_2) = \{P_4\},\ \mathsf{u}(S_2) = \{P_1, P_4, P_5\},\ \textit{i.e.,}\ [RS_2] = [1, 3].$$

Since $2+2 = 1+3$, $p([RS_1] \geq [RS_1]) = \frac{1}{2}$, by Theorem V.2, (5). That is $[RS_1]$ is indifferent to $[RS_2]$. It can be interpreted as follows: with respect to our knowledge represented in Table I and partitioning U by "Weight change", weight change does not contribute specifically to developing any of hypothyroidism and hyperthyroidism.

Example V.4 *If the columns "Edema" and "Mood" are chosen, the universe U can be partitioned into $\{P_5\}$ and $\{P_1, P_2, P_3, P_4\}$, reflecting the edema and mood being "Edema = yes", "Mood = nervousness" and "Edema = no", "Mood = no", respectively. Then, based on this new partition,*

$$\mathsf{l}(S_1) = \emptyset,\ \mathsf{u}(S_1) = \{P_1, P_2, P_3, P_4\},\ \textit{i.e.,}\ [RS_1] = [0, 4];$$
$$\mathsf{l}(S_2) = \{P_5\},\ \mathsf{u}(S_2) = \{P_1, P_2, P_3, P_4, P_5\},\ \textit{i.e.,}\ [RS_2] = [1, 5].$$

With a simple calculation, we have

$$p([RS_1] \geq [RS_2]) =$$
$$= \max\left\{1 - \max\left\{\frac{\mathsf{S}_2^u - \mathsf{S}_1^l}{w([RS_1]) + w([RS_2])}, 0\right\}, 0\right\} = \frac{3}{8},$$

and $p([RS_2] \geq [RS_1]) = 1 - p([RS_1] \geq [RS_2]) = \dfrac{5}{8}$.

These results can be interpreted as follows: with respect to our knowledge represented in Table I and partitioning U by "Edema" and "Mood", the overall contribution of the clinical symptoms edema and mood to the presence of

- *hypothyroidism has the possibility degree $\frac{3}{8}$,*
- *hyperthyroidism has the possibility degree $\frac{5}{8}$.*

D. Midpoints Comparison Method

In **Theorem V.2**, properties (3) and (4) mean that the possibility degree of a over b is equal to 0 or 1 iff they do not have a common area *regardless of the distance between a and b.*

To overcome this problem, Dymova et al. [16] proposed a method to measure the distance between intervals which, in addition, also indicates which interval is greater/lesser.

Let $a = [a^l, a^u]$, $b = [b^l, b^u]$ be two intervals and form their subtraction: $c = a - b = [c^l, c^u] = [a^l - b^u, a^u - b^l]$. Clearly, $c^l \leq 0$ and $c^u \geq 0$, if a and b overlap each other.

Then, the proposed distance measure between a and b is:

$$\Delta(a, b) = \frac{1}{2}\left(\left(a^l - b^u\right) + \left(a^u - b^l\right)\right) = m(a) - m(b).$$

That is, $\Delta(a, b)$ is simply the difference of the midpoints of a and b. This immediately implies that for intervals a and b with common midpoints, $\Delta(a, b) = 0$ holds.

Remark V.5 *It may seem that the measure $\Delta(a, b)$ is too simple. For its discussion, see [16]. In addition, on the important role of midpoints in comparison of intervals, see [17].*

TABLE I
CLINICAL SYMPTOMS OF THYROID DYSFUNCTION AND DIAGNOSIS BASED ON TEST RESULTS

No.	Weight change	Edema	Tachy-cardia	Increased sweating	Mood	Hypothy-roidism	Hyperthy-roidism
P_1	no change	no	no	no	normal	no	no
P_2	gain	no	no	no	normal	yes	no
P_3	gain	no	yes	no	normal	yes	no
P_4	loss	no	yes	yes	normal	no	yes
P_5	no change	yes	no	yes	nervousness	no	yes

E. Comparing the Two Methods

In [16], *experimental observations* show that the sign of $\Delta(a, b)$ is positive (negative), if $a \succ b$ ($a \prec b$). In addition, $abs(\Delta(a, b))$ is close to the Hamilton distance d_H and Euclidean distance d_E of the intervals a and b, where

$$\begin{aligned} d_H &= \frac{1}{2}\left(abs\left(a^l - b^l\right) + abs\left(a^u - b^u\right)\right), \\ d_E &= \frac{1}{2}\sqrt{\left(a^l - b^l\right)^2 + \left(a^u - b^u\right)^2}. \end{aligned}$$

In regard to these experimental observations, let us compare our numerical results which were calculated with the help of the possibility degree method and midpoints comparison method.

S_1, S_2 are the sets of patients who demonstrably suffer from hypothyroidism and hyperthyroidism, respectively.

According to **Example V.3**, $[RS_1] = [2, 2]$; $[RS_2] = [1, 3]$, where RS_1, RS_2 are the rough sets concerning S_1, S_2 and based on the partition of U formed by "Weight change".

By applying the possibility degree method, $p([RS_1] \geq [RS_2]) = \frac{1}{2}$, i.e., $[RS_1]$ is indifferent to $[RS_2]$.

By applying the midpoints comparison method, the intervals $[RS_1]$, $[RS_2]$ are equal, i.e, $\Delta([RS_1],[RS_2]) = 0$, since their midpoints are equal. Of course, the sign rule does not work here.

The one interpretation is in accordance with the other.

According to **Example V.4** $[RS_1] = [0, 4]$; $RS_2] = [1, 5]$, where RS_1, RS_2 are the rough sets concerning S_1, S_2 and relying on the partition of U formed by "Edema" and "Mood".

By applying the possibility degree method, $p([RS_2] \geq [RS_1]) = \frac{5}{8}$, i.e., $[RS_1]$ is inferior to $[RS_2]$, $[RS_1] \prec [RS_2]$, in the degree $\frac{5}{8}$.

By applying the midpoints comparison method,

$$\Delta([RS_1], [RS_2]) = m([RS_1]) - m([RS_2]) = 2 - 3 = -1.$$

According to the sign rule of the midpoint comparison method, since the sign of $\Delta([RS_1], [RS_2])$ is negative, $[RS_1]$ is lesser than $[RS_2]$. This result coincides with the result $[RS_1] \prec [RS_2]$ obtained by the possibility degree method.

If the midpoints of two intervals are the same, there is no sense in comparing $abs(\Delta([RS_1], [RS_2]))$ with the Hamilton and Euclidean distances. This is the case in **Example V.3**.

In **Example V.4**, $abs(\Delta([RS_1], [RS_2])) = 1$. In this case, Hamilton and Euclidean distances can be calculated. For $[RS_1] = [0, 4]$, $[RS_2] = [1, 5]$, $d_H = 1$ and $d_E = \frac{\sqrt{2}}{2} \approx 0, 71$. The Hamilton distance is the same as $abs(\Delta([RS_1], [RS_2]))$, and Euclidean distance estimates it to some extent.

VI. CONCLUSION

The paper has presented two comparison and ranking methods for rough sets in Pawlak approximation spaces. Although the two methods are borrowed from Multiple Attribute Decision Making analysis, their application to rough sets is a new approach. Based on the presented calculations and interpretations, it seems that this approach deserves attention.

Acknowledgement The authors would like to thank the anonymous referees for their useful comments and suggestions.

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