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A hybrid CP/MP approach to supply chain modelling, optimization and analysis

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***Abstract*—The paper presents a concept and implementation of a novel hybrid approach to the modelling, optimization and analysis of the supply chain problems. Two environments, mathematical programming (MP) and constraint programming (CP), in which constraints are treated in different ways and different methods are implemented, were combined to use the strengths of both.**

This integration and hybridization, complemented with an adequate transformation of the problem, facilitates a significant reduction of the combinatorial problem. The whole process takes place at the implementation layer, which makes it possible to use the structure of the problem being solved, implementation environments and the very data. The superiority of the proposed approach over the classical scheme is proved by considerably shorter search time and example-illustrated wide-ranging possibility of expanding the decision and/or optimization models through the introduction of new logical constraints, frequently encountered in practice. The proposed approach is particularly important for the decision models with an objective function and many discrete decision variables added up in multiple constraints.

The presented approach will be compared with classical mathematical programming on the same data sets.

I. INTRODUCTION

The supply chain is commonly seen as a collection of various types of companies (raw materials, production, trade, logistics, transport, etc.) working together to improve the flow of products, information and finance. As the words in the term indicate, the supply chain is a combination of its individual links in the process of supplying products (material/products and services) to the market.

Huang [1] studied the shared information of supply chain production. This consists in the information shared between each network node determined by the model, which enables production, distribution and transport planning dependent on the purpose. The shared information process is vital for effective supply chain production, distribution and transport planning. In terms of centralized planning, the information flows from each node of the network where the decisions are made. Shared information includes the following groups of parameters: resources, inventory, production, transport, demand, etc. Minimization of total costs is the main purpose of the models presented in the literature, while maximization of revenues or sales is considered to a smaller scale [12].

The vast majority of the works reviewed [2]–[7], [9],[10],[12] have formulated their models as linear programming (LP), integer programming (IP) and mixed integer linear programming (MILP) problems and solved them using the Operations Research methods. Nonlinear programming, multi-objective programming, fuzzy programming with stochastic programming are used much less frequently [12] [25].

Problems related to the design, integration and management of the supply chain affect many aspects of production, distribution, warehouse management, supply chain structure, transport modes etc. Those problems are usually closely related to each other, some may influence one another to a greater or lesser extent. Because of the interconnectedness and a very large number of different constraints: resource, time, technological, and financial, the constraint–based environments are suitable for producing “natural” solutions for highly combinatorial problems. In the literature, references to modeling and optimizing supply chain problems using constraint–based environments are relatively few in number [11], [12].

This paper deals with a problem of supply chain modelling, optimization and analysis. An important contribution of the presented hybrid CP/MP approach is to propose a hybrid implementation platform that supports the modelling, optimization and analysis of decision problems in the supply chain. In this platform two environments, mathematical programming (MP) and constraint programming (CP), in which constraints are treated in different ways and different methods are implemented, were combined to use the strengths of both in the presented platform.

The rest of the paper is organized as follows: Section II describes our motivation and analyses the state of the art in this domain. Section III gives the concept of the novel hybrid CP/MP approach and implementation platform. The optimization model as an illustrative example is described in Section IV. Computational examples and tests of the implemented model are presented in Section V. The discussion on possible extensions of the proposed approach and conclusions is included in Section VI.

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II. MOTIVATION

We strongly believe that the constraint-based environment [13], [14], [16], [19] offers a very good framework for representing the knowledge and information needed for the decision support. The central issue for a constraint-based environment is a constraint satisfaction problem (CSP) [13]. Constraint satisfaction problem is the mathematical problem defined as a set of elements whose state must satisfy a number of constraints. Constraint satisfaction problems (CSPs) on finite domains are typically solved using a form of search. The most widely used techniques include variants of backtracking, constraint propagation, and local search. Constraint propagation embeds any reasoning that consists in explicitly forbidding values or combinations of values for some variables of a problem because a given subset of its constraints cannot be satisfied otherwise [16]. CSPs are frequently used in constraint programming. Constraint programming is the use of constraints as a programming language to encode and solve problems. Constraint logic programming (CLP) is a form of constraint programming (CP), in which logic programming is extended to include concepts from constraint satisfaction. A constraint logic program is a logic program that contains constraints in the body of clauses. Constraints can also be present in the goal. These environments are declarative. The declarative approach and the use of logic programming provide incomparably greater possibilities for decision problems modelling than the pervasive approach based on mathematical programming. Unfortunately, discrete optimization is not a strong suit of these environments.

Based on [8], [15], [16] and previous work [14], [17], [18], we observed some advantages and disadvantages of these environments. An integrated approach of constraint programming (CP) and mathematical programming (MP) can help to solve optimization problems that are intractable with either of the two methods alone [20]–[23]. Although mathematical programming and constraint programming have different roots, the links between the two environments have grown stronger in recent years.

Both MP and finite domain CP/CLP involve variables and constraints. However, the types of the variables and constraints that are used, and the way the constraints are solved, are different in the two approaches [23].

In both MILP and CP/CLP, there is a group of constraints that can be solved with ease and a group of constraints that are difficult to solve. The easily solved constraints in MILP are linear equations and inequalities over rational numbers.

Integrity constraints are difficult to solve using mathematical programming methods and often the real problems of MILP make them NP-hard.

In CP/CLP, domain constraints with integers are easy to solve. The system of such constraints can be solved over integer variables in polynomial time. The inequalities between more than two variables, general linear constraints and symbolic constraints are difficult to solve, which makes real problems in CP/CLP NP-hard. This type of constraints reduces the strength of constraint propagation. As a result, CP/CLP is incapable of finding even the first feasible solution. This is the greatest weakness of this approach.

As mentioned earlier, the vast majority of decision-making models for the problems of production, logistics, supply chain are formulated in the form of mathematical programming (MIP, MILP, IP).

Due to the structure of these models (adding together discrete decision variables in the constraints and the objective function) and a large number of discrete decision variables (integer and binary), they can only be applied to small problems. Another weakness is that only linear constraints can be used. In practice, the issues related to the production, distribution and supply chain constraints are often logical, nonlinear, etc. For these reasons the problem was formulated in a new way

The motivation and contribution behind this work was to create a hybrid method for supply chain decision problems modelling and optimization instead of using mathematical programming or constraint programming separately. It follows from the above that what is difficult to solve in one environment can be easy to solve in the other. Furthermore, such a hybrid CP/MP approach allows the use of all layers of the problem to solve it (Fig. 1). And finally, the transformation of the problem to a form that can fully exploit the strengths of the constraint propagation.

The hybrid method is not inferior to its component elements applied separately. This is due to the fact that the number of decision variables and the search area are reduced. The extent of the reduction directly affects the effectiveness of the method.

III.THE CONCEPT OF THE CP/MP HYBRID APPROACH

Due to the structure of the decision models for supply chain problems (summing of discrete decision variables in the constraints and the objective function) and a large number of discrete decision variables (integer and/or binary) they can only be applied to small problems. Another disadvantage is that only linear constraints can be used. In practice, the issues related to the production, distribution and su25]5ply chain constraints are often logical, nonlinear, etc. For these reasons the problem was formulated in a new way.

In our approach to modeling and optimization these problems we proposed the implementation platform, where:

- knowledge related to the supply chain can be expressed as linear and logical constraints (implementing all types of constraints of the previous MILP models [17], and introducing new types of constraints (logical, nonlinear, symbolic etc.));
- the decision models solved using the proposed platform can be formulated as a pure model of MILP or of CP/CLP, or it can also be a hybrid model with logical and other types of constraints;
- the problem is modelled in CP/CLP, which is far more flexible than MILP;

- the possibility to transform the problem of using the flexibility of declarative environment (CP/CLP) is introduced;
- the novel method of constraint propagation is proposed (obtained by transforming the decision model to explore its structure and properties);
- constrained domains of decision variables, new constraints and values for some variables are transferred from CP-based environment into MP-based environment;
- the efficiency of finding solutions to larger size problems is increased.

The concept of the proposed implementation platform is presented in Fig. 1. In the first stage, a formal model is implemented in the form of predicates in CLP and the data in the form of facts. In the next step constraint propagation is performed. Constraint propagation is one of the basic methods of CLP. As a result, the variable domains are narrowed, and in some cases, the values of variables are set, or even the solution can be found. In order to increase the efficiency of the constraint propagation transformation of the problem and its representation can be made. The transformation uses the structure and properties of the problem. The most common effect is a change in the representation of the problem by reducing the number of decision variables, and the introduction of additional constraints and variables, changing the nature of the variables, etc.

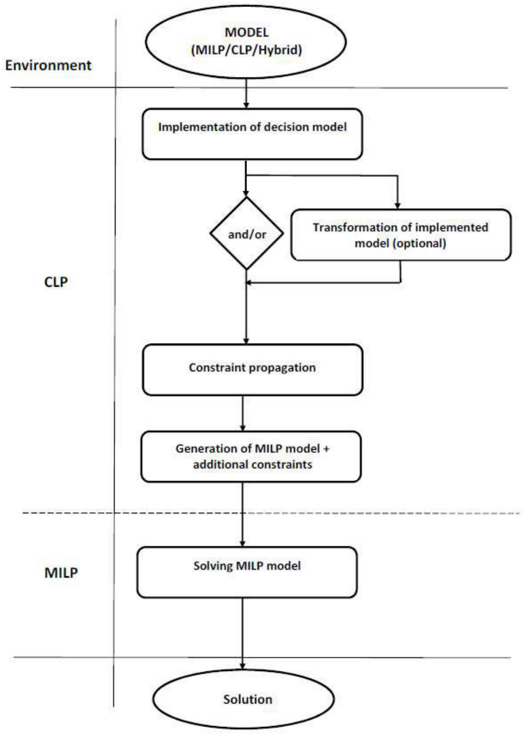

Fig. 1 Concept scheme of the CP/MP hybrid approach

The next step is the generation of the MILP model using predicates in CLP. All the information obtained in previous stages are used during the generation of the model. The final step is to solve the model by the solver MILP.

The implementation details of the CP/MP hybrid approach have been discussed in [24]. The motivation was to offer the most effective tools for model–specific constraints and solution efficiency.

IV. EXAMPLES OF SUPPLY CHAIN OPTIMIZATION

The proposed approach was used and tested on two supply chain optimization models.

First model was formulated as a mixed linear integer programming (MILP) problem [18] under constraints (2) .. (23) in order to test the proposed approach (Fig. 1) against the classical integer programming approach [17]. Then the hybrid model (1) .. (26) was implemented and solved. Indices, parameters and decision variables used in the models together with their descriptions are summarized in Table I. The simplified structure of the supply chain network for this model, composed of producers, distributors and customers is presented in Fig. 2.

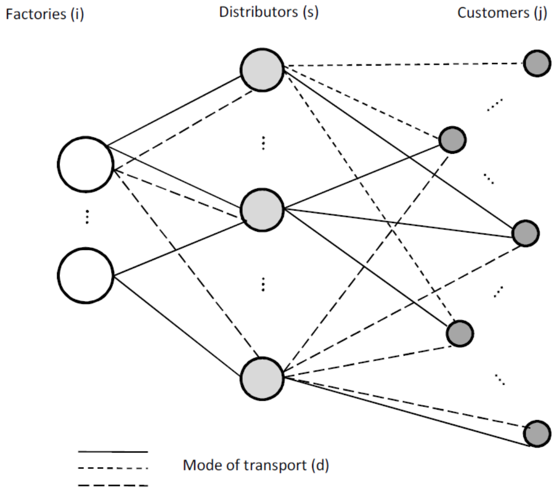

Fig. 2 The simplified structure of the supply chain network

Both models are the cost models that take into account three other types of parameters, i.e., the spatial parameters (area/volume occupied by the product, distributor capacity and capacity of transport unit), time (duration of delivery and service by distributor, etc.) and the transport mode.

The main assumptions made in the construction of these models were as follows:

- the shared information process in the supply chain consists of resources (capacity, versatility, costs), inventory (capacity, versatility, costs, time), production (capacity, versatility, costs), product (volume), transport (cost, mode, time), demand, etc;
- part of the supply chain has a structure as in Fig. 2.;
- transport is multimodal (several modes of transport, a limited number of means of transport for each mode);
- the environmental aspects of use of transport modes are taken into account;
- different products are combined in one batch of transport;
- the cost of supplies is presented in the form of a function (in this approach, linear function of fixed and variable costs);

- the models have linear or linear and logical constraints.

TABLE I.
INDICES, PARAMETERS AND DECISION VARIABLES

Symbol	Description
Indices	
k	product type (k=1..O)
j	delivery point/customer/city (j=1..M)
i	manufacturer/factory (i=1..N)
s	distributor /distribution center (s=1..E)
d	mode of transport (d=1..L)
N	number of manufacturers/factories
M	number of delivery points/customers
E	number of distributors
O	number of product types
L	number of mode of transport
Input parameters	
F_s	the fixed cost of distributor/distribution center *s*
P_k	the area/volume occupied by product *k*
V_s	distributor *s* maximum capacity/volume
$W_{i,k}$	production capacity at factory *i* for product *k*
$C_{i,k}$	the cost of product *k* at factory *i*
$R_{s,k}$	if distributor *s* can deliver product *k* then $R_{s,k}=1$, otherwise $R_{s,k}=0$
$Tp_{s,k}$	the time needed for distributor *s* to prepare the shipment of product *k*
$Tc_{j,k}$	the cut-off time of delivery to the delivery point/customer *j* of product *k*
$Z_{j,k}$	customer demand/order *j* for product *k*
Zt_d	the number of transport units using mode of transport *d*
Pt_d	the capacity of transport unit using mode of transport *d*
$Tf_{i,s,d}$	the time of delivery from manufacturer *i* to distributor *s* using mode of transport *d*
$K1_{i,s,k,d}$	the variable cost of delivery of product *k* from manufacturer *i* to distributor *s* using mode of transport *d*
$R1_{i,s,d}$	if manufacturer *i* can deliver to distributor *s* using mode of transport *d* then $R1_{i,s,d}=1$, otherwise $R1_{i,s,d}=0$
$A_{i,s,d}$	the fixed cost of delivery from manufacturer *i* to distributor *s* using mode of transport *d*
$Koa_{s,j,d}$	the total cost of delivery from distributor *s* to customer *j* using mode of transport *d*
$Tm_{s,j,d}$	the time of delivery from distributor *s* to customer *j* using mode of transport *d*
$K2_{s,j,k,d}$	the variable cost of delivery of product *k* from distributor *s* to customer *j* using mode of transport *d*
$R2_{s,j,d}$	if distributor *s* can deliver to customer *j* using mode of transport *d* then $R2_{s,j,d}=1$, otherwise $R2_{s,j,d}=0$
$G_{s,j,d}$	the fixed cost of delivery from distributor *s* to customer *j* using mode of transport *d*
$Kog_{s,j,d}$	the total cost of delivery from distributor *s* to customer *j* using mode of transport *d*
Od_d	the environmental cost of using mode of transport *d*
Decision variables	
$X_{i,s,k,d}$	delivery quantity of product *k* from manufacturer *i* to distributor *s* using mode of transport *d*
$Xa_{i,s,d}$	if delivery is from manufacturer *i* to distributor *s* using mode of transport *d* then $Xa_{i,s,d}=1$, otherwise $Xa_{i,s,d}=0$
$Xb_{i,s,d}$	the number of courses from manufacturer *i* to distributor *s* using mode of transport *d*
$Y_{s,j,k,d}$	delivery quantity of product *k* from distributor *s* to customer *j* using mode of transport *d*
$Ya_{s,j,d}$	if delivery is from distributor *s* to customer *j* using mode of transport *d* then $Ya_{s,j,d}=1$, otherwise $Ya_{s,j,d}=0$
$Yb_{s,j,d}$	the number of courses from distributor *s* to customer *j* using mode of transport *d*
Tc_s	if distributor *s* participates in deliveries, then $Tc_s=1$, otherwise $Tc_s=0$
CW	Arbitrarily large constant

A. Objective function

The objective function (1) defines the aggregate costs of the entire chain and consists of five elements. The first element comprises the fixed costs associated with the operation of the distributor involved in the delivery (e.g. distribution centre, warehouse, etc.). The second element corresponds to environmental costs of using various means of transport. Those costs are dependent on the number of courses of the given means of transport, and on the other hand, on the environmental levy, which in turn may depend on the use of fossil fuels and carbon-dioxide emissions [26],[27].

The third component determines the cost of the delivery from the manufacturer to the distributor. Another component is responsible for the costs of the delivery from the distributor to the end user (the store, the individual client, etc.). The last component of the objective function determines the cost of manufacturing the product by the given manufacturer.

Formulating the objective function in this manner allows comprehensive cost optimization of various aspects of supply chain management. Each subset of the objective function with the same constrains provides a subset of the optimization area and makes it much easier to search for a solution.

$$\begin{gathered}\sum_{s=1}^{E} F_s \cdot Tc_s + \sum_{d=1}^{L} Od_d \Big(\sum_{i=1}^{N}\sum_{s=1}^{E} Xb_{i,s,d} + \sum_{s=1}^{E}\sum_{j=1}^{M} Yb_{j,s,d}\Big) + \\ \sum_{i=1}^{N}\sum_{s=1}^{E}\sum_{d=1}^{L} Koa_{i,s,d} + \sum_{s=1}^{E}\sum_{j=1}^{M}\sum_{d=1}^{L} Kog_{s,j,d} + \sum_{i=1}^{N}\sum_{k=1}^{O}\Big(C_{ik} \cdot \sum_{s=1}^{E}\sum_{d=1}^{L} X_{i,s,k,d}\Big)\end{gathered} \tag{1}$$

B. Constraints

The model was based on constraints (2) .. (26) Constraint (2) specifies that all deliveries of product *k* produced by the manufacturer *i* and delivered to all distributors *s* using mode of transport *d* do not exceed the manufacturer's production capacity.

Constraint (3) covers all customer *j* demands for product *k* $(Z_{j,k})$ through the implementation of delivery by distributors *s* (the values of decision variables $Y_{i,s,k,d}$). The flow balance of each distributor *s* corresponds to constraint (4). The possibility of delivery is dependent on the distributor's technical capabilities – constraint (5). Time constraint (6) ensures the terms of delivery are met. Constraints (7a), (7b), (8) guarantee deliveries with available transport taken into account. Constraints (9), (10), (11) set values of decision variables based on binary variables Tc_s, $Xa_{i,s,d}$, $Ya_{s,j,d}$. Dependencies (12) and (13) represent the relationship based on which total costs are calculated. In general, these may be any linear functions. The remaining constraints (14)..(23) arise from the nature of the model (MILP).

Constraint (24) allows the distribution of exclusively one of the two selected products in the distribution center *s*. Similarly, constraint (25) allows the production of exclusively one of the two selected products in the factory *i*.

Constraint (26) allows the transport of exclusively one of the two selected products in the same route and transport unit.

Those constraints result from technological, marketing, sales or safety reasons. Therefore, some products cannot be distributed and/or produced and/or transported together. The constraint can be re-used for different pairs of product *k* and for some of or all distribution centers *s* and factories *i.* A logical constraint like this cannot be easily implemented in a mathematical programming model. Only declarative application environments based on constraint satisfaction problem (CSP) make it possible to easy implement constraints such as (24), (25), (26).

The addition of constraints of that type changes the model class. It is a hybrid model (1)..(26).

$$\sum_{s=1}^{E}\sum_{d=1}^{L} X_{i,s,k,d} \cdot R_{s,k} \le W_{i,k} \text{ for } i=1..N, k=1..O \tag{2}$$

$$\sum_{s=1}^{E}\sum_{d=1}^{L} (Y_{s,j,k,d} \cdot R_{s,k}) \ge Z_{j,k} \text{ for } j=1..M, k=1..O \tag{3}$$

$$\sum_{i=1}^{N}\sum_{d=1}^{L} X_{i,s,k,d} = \sum_{j=1}^{M}\sum_{d=1}^{L} Y_{s,j,k,d} \text{ for } s=1..E, k=1..O \tag{4}$$

$$\sum_{k=1}^{O} (P_k \cdot \sum_{i=1}^{N}\sum_{d=1}^{L} X_{i,s,k,d}) \le Tc_s \cdot V_s \text{ for } s=1..E \tag{5}$$

$$\begin{gathered} Xa_{i,s,d} \cdot Tf_{i,s,d} + Xa_{i,s,d} \cdot Tp_{s,k} + Ya_{s,j,d} \cdot Tm_{s,j,d} \le Tc_{j,k} \\ \text{for } i=1..N,\ s=1..E, j=1..M,\ k=1..O, d=1..L \end{gathered} \tag{6}$$

$$\begin{gathered} R1_{i,s,d} \cdot Xb_{i,s,d} \cdot Pt_d \ge X_{i,s,k,d} \cdot P_k \\ \text{for } \ i=1..N, s=1..E, k=1..O, d=1..L \end{gathered} \tag{7a}$$

$$\begin{gathered} R2_{s,j,d} \cdot Yb_{s,j,d} \cdot Pt_d \ge Y_{s,j,k,d} \cdot P_k \\ \text{for } \ s=1..E,\ j=1..M,\ k=1..O, d=1..L \end{gathered} \tag{7b}$$

$$\sum_{i=1}^{N}\sum_{s=1}^{E} Xb_{i,s,d} + \sum_{j=1}^{M}\sum_{s=1}^{E} Yb_{j,s,d} \le Zt_d \ \text{ for } d=1..L \tag{8}$$

$$\sum_{i=1}^{N}\sum_{d=1}^{L} Xb_{i,s,d} \le CW \cdot Tc_s \ \text{ for } s=1..E \tag{9}$$

$$Xb_{i,s,d} \le CW \cdot Xa_{i,s,d} \ \text{ for } i=1..N, s=1..E, d=1..L \tag{10}$$

$$Yb_{s,j,d} \le CW \cdot Ya_{s,j,d} \ \text{ for } s=1..E, j=1..M, d=1..L \tag{11}$$

$$\begin{gathered} Koa_{i,s,d} = A_{i,s,d} \cdot Xb_{i,s,d} + \sum_{k=1}^{O} K1_{i,s,k,d} \cdot X_{i,s,k,d} \\ \text{for } \ i=1..N, s=1..E, d=1..L \end{gathered} \tag{12}$$

$$\begin{gathered} Kog_{s,j,d} = G_{s,j,d} \cdot Yb_{j,s,d} + \sum_{k=1}^{O} K2_{s,j,k,d} \cdot Y_{s,j,k,d} \\ \text{for } s=1..E, j=1..M, d=1..L \end{gathered} \tag{13}$$

$$X_{i,s,k,d} \ge 0 \text{ for } i=1..N, s=1..E, k=1..0,\ d=1..L \tag{14}$$

$$Xb_{i,s,d} \ge 0 \text{ for } i=1..N,\ s=1..E,\ d=1..L, \tag{15}$$

$$Yb_{s,j,d} \ge 0 \text{ for } \ s=1..E,\ j=2..M, \ \ d=1..L, \tag{16}$$

$$X_{i,s,k,d} \in C \ \text{ for } i=1..N, s=1..E,\ k=1..0, d=1..L, \tag{17}$$

$$Xb_{i,s,d} \in C \ \text{ for } i=1..N,\ s=1..E,\ d=1..L \tag{18}$$

$$Y_{s,j,k,d} \in C \text{ for } s=1..E, j=1..M,\ k=1..0, d=1..L \tag{19}$$

$$Yb_{s,j,d} \in C \ \text{ for } \ s=1..E,\ j=1..M, \ \ d=1..L, \tag{20}$$

$$Xa_{i,s,d} \in \{0,1\} \text{ for } i=1..N,\ s=1..E,\ d=1..L, \tag{21}$$

$$Ya_{s,j,d} \in \{0,1\} \text{ for } \ s=1..E,\ j=2..M, \ \ d=1..L, \tag{22}$$

$$Tc_s \in \{0,1\} \text{ for } s=1..E \tag{23}$$

$$\text{ExclusionD}(k_1,k_2,s) \text{ for } k_1,k_2 \in 1..O,\ s \in 1..E,\ k_1 \neq k_2 \tag{24}$$

$$\text{ExclusionP}(k_1,k_2,i) \text{ for } k_1,k_2 \in 1..O,\ i \in 1..N,\ k_1 \neq k_2 \tag{25}$$

$$\text{ExclusionT}(k_1,k_2) \text{ for } k_1 \in 1..O,\ k_2 \in 1..O,\ k_1 \neq k_2 \tag{26}$$

C. Model transformation

Due to the nature of the decision problem (adding up decision variables and constraints involving a lot of variables), the constraint propagation efficiency decreases dramatically. Constraint propagation is one of the most important methods in CLP affecting the efficiency and effectiveness of the CLP and novel hybrid implementation platform (Fig. 1). For that reason, research into more efficient and more effective methods of constraint propagation was conducted. The results included different representation of the problem and the manner of its implementation. The classical problem modeling in the CLP environment consists in building a set of predicates with parameters. Each CLP predicate has a corresponding multi-dimensional vector representation. While modeling both problems, (1) .. (23) and (1) .. (26), quantities *i, s, k, d* and decision variable $X_{i,s,k,d}$ were vector parameters. The process of finding the solution may consist in using the constraints propagation methods, labeling of variables and the backtracking mechanism [13]. The quality of constraints propagation and the number of backtrackings are affected to a high extent by the number of parameters that must be specified/labeled in the given predicate/vector. In both models presented above, the classical problem representation included five parameters: *i, s, k, d* and $X_{i,s,k,d}$. Considering the domain size of each parameter, the process is complex and time-consuming. Our idea was to transform the problem by changing its representation without changing the very problem. All permissible routes were first generated based on the fixed data and a set of orders, then the specific values of parameters *i, s, k, d* were assigned to each of the routes. In this way, only decision variables $X_{i,s,k,d}$ (deliveries) had to be specified. This transformation allows only one parameter search instead of five. This is possible due to the flexibility and features of the CLP environment.

This transformation fundamentally improved the efficiency of the constraint propagation and reduced the number of backtracks. A route model is a name adopted for the models that underwent the transformation (MILP-R).

D. Decision-making support

The proposed models in this platform can support decision-making in the following areas:

- the optimization of total cost of the supply chain (Table II);
- the selection of the transport fleet number, capacity and modes for specific total costs;
- the sizing of distributor warehouses and the study of their impact on the overall costs (Table III, Fig. 3.);
- the selection of transport routes for optimal total cost (Fig. 4.).

Detailed studies of these topics are being conducted and will be described in our future articles.

V. NUMERICAL EXPERIMENTS

In order to verify and evaluate the proposed approach, many numerical experiments were performed. All the examples relate to the supply chain with seven manufacturers *($i=1..7$)*, three distributors *($s=1..3$)*, ten customers *($j=1..10$)*, three modes of transport *($d=1..3$)*, and twenty types of products *($k=1..20$)*. Experiments began with nine examples of P1 .. P9 for the optimization MILP model (1) .. (23). The examples differ in terms of capacity available to the distributors s (V_s), the number of transport units using the mode of transport d (Zt_d) and the number of orders (*No*). The first series of experiments was designed to show the benefits and advantages of the hybrid approach. For this purpose the model (1) .. (23) was implemented in both the hybrid and integer programming environments. The experiments that follow were conducted to optimize examples P1..P9 for the optimization HP model (1) .. (26) in the hybrid approach.

Numeric data of input parameters for examples P1.. P9 are shown in Appendix A1. The results in the form of the objective function and the computation time are shown in Table II.

TABLE II
THE RESULTS OF NUMERICAL EXAMPLES FOR BOTH APPROACHES

P(No)	**Hybrid Approach**		**Integer Programming**		**Hybrid Approach**	
	MILP-R		**MILP**		**HM**	
	Fc	***T***	***Fc***	***T***	***Fc***	***T***
P1(100)	10791	416	15459	900**	10891	402
P2(90)	9263	323	9636	900**	9377	452
P3(80)	8388	522	8854	900**	8522	438
P4(60)	6330	345	6330	900**	6444	383
P5(40)	4473	203	4473	743	4708	223
P6(30)	3488	83	3488	503	3664	181
P7(20)	2877	23	2877	383	2894	31
P8(15)	2266	7	2266	503	2282	13
P9(10)	1756	2	1756	355	1756	2
Fc	the value of the objective function					
T	time of finding solution (in seconds)					
*	the feasible value of the objective function after the time T					
**	calculation was stopped after 900 s					
MILP	MILP model implementation in the IP environment.					
MILPT	MILP model after transformation-implementation in the hybrid implementation platform.					
HM	Hybrid model after transformation-implementation in the hybrid implementation platform.					

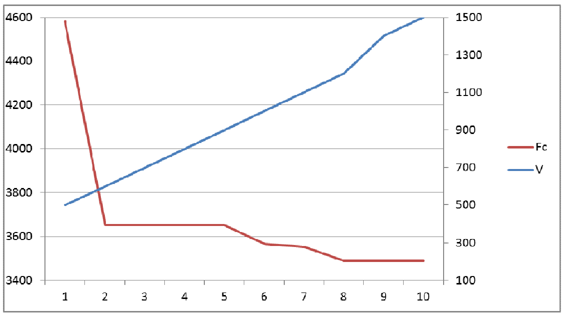

Fig. 3 The value of the objective function depending on the parameter V (Example P6)

For each example the solution for the MILP-R implementation was found faster than that for the MILP implementation. Moreover, for examples P1 .. P4, the traditional approach based on integer programming gives only feasible solution (calculation was stopped after 900 s) despite using highly efficient MILP solvers. It is obvious that the solution of the hybrid model (HM) was, due to its nature, only possible using the hybrid platform. Also, the proposed environment brought the expected results. The results were obtained in only a slightly longer period of time than that necessary for MILP-R (examples P1 .. P9).

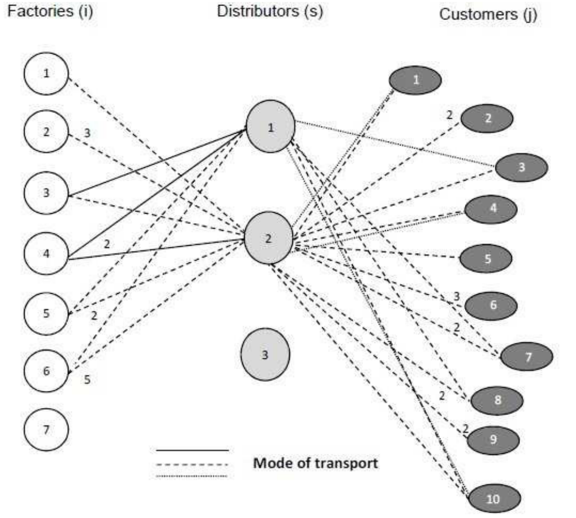

Fig. 4 Optimal transport network for the optimal solution ($Fc^{opt}=8388$) for P3 (no number means one)

TABLE III
ANALYSIS OF THE IMPACT PARAMETER V_S FOR FC (EXAMPLE P6)

$V=V_1= V_2= V_3$	**Fc^{opt}**	**Distributor capacity (Vs) utilization**		
		V_1	**V_2**	**V_3**
500	4581	470	336	350
600	3653	570	586	0
700	3653	570	586	0
800	3653	570	586	0
900	3653	256	900	0
1000	3567	188	968	0
1100	3554	130	1026	0
1200	3488	0	1156	0
1400	3488	0	1156	0

VI. CONCLUSION

The efficiency of the proposed approach is based on the reduction of the combinatorial problem. This means that using the hybrid approach practically for all models of this class, the same or better solutions are found faster (the optimal instead of the feasible solutions). Another element contributing to the high efficiency of the method is a possibility to determine the values or ranges of values for some of the decision variables (phase P3). All effective LINGO solvers can be used in the hybrid method.

It should be emphasized that with the used approach it is possible not only to solve optimization problems faster, but also to solve much larger problems than in the [17].

Therefore, the proposed solution is highly recommended for all types of decision problems in supply chain or for other problems with similar structure. This structure is characterized by the constraints of many discrete decision variables and their summation. Furthermore, this method can model and solve problems with logical constraints.

In addition to the undoubted effectiveness of the proposed hybrid approach, should underline the possibility of modeling decision problems. The proposed approach can be created a new class of decision problems - hybrid problems that are not only familiar with the constraints from mathematical programming models but also new types of constraints such as logical constraints.

Further work will focus on running the optimization models with non-linear and other logical constraints, multi-objective, uncertainty etc. in the hybrid optimization framework. It is planned also apply this method to various types of scheduling problems [14],[28],[29].

APPENDIX A1

TABLE IV

DATA FOR COMPUTATIONAL EXAMPLES P1,P2,P3,P4,P5,P6,P7,P8,P9

k	V_k	k	V_k
01	1	11	8
02	2	12	4
03	5	13	5
04	9	14	5
05	3	15	7
06	4	16	8
07	5	17	9
08	6	18	1
09	7	19	4
10	8	20	6

j
01
02
03
04
05
06
07
08
09
10

d	Pt_s	Zt_s	Od_s
S1	400	25	240
S2	200	40	180
S3	100	80	100

i
01
02
03
04
05
06
07

k	k
06	07
09	10

k	i	k
01	01	02
01	02	01

s	V_s	F_s
C1	4000	100
C2	4000	100
C3	4000	400

i	s	d	$K_{i,s,d}$	$T_{i,s,d}$
03	01	01	35	4
04	01	01	44	5
07	01	01	17	2
01	01	02	5	1
02	01	02	15	2
03	01	02	18	3
04	01	02	22	4
05	01	02	16	3
06	01	02	18	3
07	01	01	8	1
03	02	01	46	5
04	02	01	38	4
07	02	01	35	4
01	02	02	18	3
02	02	02	14	2
03	02	02	24	4
04	02	02	17	3
05	02	02	8	1
06	02	02	5	1
07	02	02	17	3
03	03	01	5	1
04	03	01	34	4
07	03	01	48	5
01	03	02	15	3
02	03	02	30	5
03	03	02	5	1
04	03	02	15	3
05	03	02	15	3
06	03	02	20	4
07	03	02	22	3

i	k	$W_{i,k}$	$C_{i,k}$
01	01	500	0
01	02	500	0
01	03	500	50
01	04	500	50
02	01	500	50
02	02	500	50
02	03	500	0
02	04	500	0
03	05	500	0
03	06	500	0
03	07	500	0
03	08	500	0
04	08	500	30
04	09	500	0
04	10	500	0
04	11	500	0
07	05	500	40
07	06	500	30
07	07	500	40
07	08	500	50
07	09	500	40
07	10	500	60
07	11	500	10
05	12	500	0
05	13	500	0
05	14	500	10
05	18	500	0
05	19	500	0
05	20	500	20
06	14	500	0
06	15	500	0
06	16	500	0
06	17	500	0
06	18	500	20
06	19	500	20
06	17	500	0
06	20	500	0

s	j	d	$K_{s,j,d}$	$T_{s,j,d}$	s	j	d	$K_{s,j,d}$	$T_{s,j,d}$
01	01	02	10	1	02	05	02	16	2
01	02	02	29	3	02	06	02	5	1
01	03	02	34	3	02	07	02	35	4
01	04	02	44	4	02	08	02	36	4
01	05	02	31	3	02	09	02	28	3
01	06	02	35	3	02	10	02	41	5
01	07	02	17	2	02	01	03	26	3
01	08	02	45	5	02	02	03	16	2
01	09	02	57	6	02	03	03	36	3
01	10	02	17	2	02	04	03	6	3
01	01	03	5	1	02	05	03	5	2
01	02	03	19	3	02	06	03	25	1
01	03	03	24	3	02	07	03	26	3
01	04	03	34	4	02	08	03	26	3
01	05	03	21	3	02	09	03	18	2
01	06	03	25	3	02	10	03	31	4
01	07	03	7	2	03	01	02	33	4
01	08	03	35	5	03	02	02	59	6
01	09	03	40	6	03	03	02	5	1
01	10	03	7	2	03	04	02	34	4
02	01	02	36	4	03	05	02	30	4
02	02	02	26	3	03	06	02	45	5
02	03	02	46	4	03	07	02	48	5
02	04	02	38	4	03	08	02	69	7
03	05	03	20	3	03	09	02	10	2
03	06	03	30	4	03	10	02	52	6
03	07	03	32	4	03	01	03	23	3
03	08	03	50	6	03	02	03	40	4
03	09	03	8	1	03	03	03	5	1
03	10	03	40	6	03	04	03	24	3

s	k	$T_{s,k}$	s	k	$T_{s,k}$	s	k	$T_{s,k}$
01	01	1	02	01	1	03	01	2
01	02	1	02	02	1	03	02	2
01	03	1	02	03	1	03	03	2
01	04	1	02	04	1	03	04	2
01	05	1	02	05	1	03	05	2
01	06	1	02	06	1	03	06	2
01	07	1	02	07	1	03	07	2
01	08	1	02	08	1	03	08	2
01	09	1	02	09	1	03	09	2
01	10	1	02	10	1	03	10	2
01	11	1	02	11	1	03	11	2
01	12	1	02	12	1	03	12	2
01	13	1	02	13	1	03	13	2
01	14	1	02	14	1	03	14	2
01	15	1	02	15	1	03	15	2
01	16	1	02	16	1	03	16	2
01	17	1	02	17	1	03	17	2
01	18	1	02	18	1	03	18	2
01	19	1	02	19	1	03	19	2
01	20	1	02	20	1	03	20	2

Name	k	j	$T_{j,k}$	$Z_{j,k}$	Name	k	j	$T_{j,k}$	$Z_{j,k}$
z0101	01	01	25	8	z0105	05	01	30	10
z0116	16	01	50	7	z0106	06	01	20	10
z0201	01	02	10	10	z0219	19	02	20	10
z0202	02	02	40	8	z0220	20	02	10	10
z0301	01	03	20	10	z0302	02	03	10	10
z1019	19	10	15	8	z0303	03	03	10	8

z1020	20	10	35	8	z0419	19	04	30	10
z0901	01	09	30	10	z0420	20	04	25	10
z0401	01	04	40	10	z0501	01	05	15	10
z0505	05	05	60	8	z0502	02	05	10	10
z1013	13	10	15	8	z1015	15	10	10	8
z0911	11	09	20	10	z1016	16	10	20	10
z0912	12	09	25	10	z1017	17	10	20	10
z0806	06	08	50	8	z1018	18	10	30	10
z0807	07	08	60	10	z0917	17	09	30	10
z0705	05	07	60	10	z0918	18	09	30	10
z0706	06	07	20	8	z0919	19	09	40	10
z0604	04	06	30	10	z0920	20	09	40	8
z0605	05	06	35	10	z0809	09	08	55	10
z0606	06	06	50	8	z0810	10	08	30	8
z0103	03	01	10	8	z0811	11	08	30	10
z0209	09	02	20	8	z0812	12	08	20	10
z0309	09	03	30	10	z0708	08	07	30	8
z0410	10	04	40	10	z0709	09	07	60	10
z0514	14	05	30	8	z0710	10	07	30	10
z0614	14	06	20	10	z0711	11	07	10	10
z0719	19	07	10	8	z0609	09	06	10	8
z0720	20	07	30	10	z0610	10	06	30	10
z0818	18	08	25	8	z0611	11	06	30	10
z0819	19	08	25	10	z0612	12	06	30	10
z0102	02	01	15	10	z0107	07	01	30	10
z0104	04	01	15	10	z0108	08	01	45	10
z0203	03	02	50	10	z0109	09	01	30	10
z0204	04	02	20	10	z0110	10	01	10	10
z0304	04	03	10	8	z0210	10	02	20	10
z0305	05	03	30	10	z0211	11	02	30	10
z0406	06	04	40	8	z0217	17	02	20	10
z0407	07	04	50	10	z0218	18	02	10	10
z0512	12	05	30	8	z0308	08	03	30	10
z0513	13	05	20	10	z0312	12	03	30	10
z0615	15	06	10	10	z0315	15	03	20	10
….									

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