

Towards Objectification of Multi-Criteria Assessments: a Comparative Study on MCDA Methods

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Abstract—Objective evaluation in problems considering many, often conflicting criteria is challenging for the decision-maker. This paper presents an approach based on MCDA methods to objectify evaluations in the camera selection problem. The proposed approach includes three MCDA methods, TOPSIS, VIKOR, COMET, and two criterion weighting techniques. Two ranking similarity coefficients were used to compare the resulting rankings of the alternatives: WS and r_w . The performed research confirmed the importance of the appropriate selection of multi-criteria decision-making methods for the solved problem and the relevance of comparative analysis in method selection and construction of objective rankings of alternatives.

I. INTRODUCTION

DEALING with complex, real-world decision-making problems involves recognizing conflicting goals, making decisions with multiple criteria, and aiming for compromise solutions [1], [2]. In response to these requirements, many solutions dedicated to selected areas and general-purpose methods have been developed. Most research has focused on developing and improving new MCDA methods. They differ in many aspects, such as different techniques for determining the weights of criteria in the calculations, the complexity of the algorithms, the way preferences and evaluation criteria are represented, the type of data aggregation and the possibility of considering uncertain data [3].

Despite the existence of many MCDA methods, it is important to be aware that no method is perfect and can be considered suitable for applying to every decision situation or solving every decision problem [4], [5]. In such a condition, it becomes a significant research problem to select a decision support method suitable for the problem under consideration since only a properly selected method can provide a proper solution that reflects the decision maker's preferences [6]. The assessment of alternatives performed using MCDA methods requires considering the decision maker's preferences, which means that the final recommendation may change depending on those preferences [7].

Although there is observed a dynamic development of new MCDA methods and improved existing algorithms, relatively little attention is paid to their proper selection for a given decision problem. Applying the inappropriate method to a particular decision situation can reduce the quality of the recommendation, as different MCDA methods produce inconsistent results. Furthermore, the complexity, unrepeatability, or the fact that decision situations may occur simultaneously over a short time makes their analysis challenging. Consequently, it becomes necessary to apply formal procedures and guidelines for selecting MCDA methods in case of a partial lack of knowledge of the decision situation [8], [9].

Common real-life decision problems in which MCDA methods are applied to solve are issues like the mobile devices selection problem. Among them, there can be considered the mobile phone selection problem, the mobile handset selection problem, laptop selection problem, camera selection problem, where criteria can be features and functionalities such as the size of the in-build camera, battery talk time, brand, colour, camera size and resolution [10].

There are many methods of multi-criteria decision making belonging to different streams. Among them, the two main streams, i.e. the American school and the European school, stand out the most. In addition, there is also an approach that combines elements of both groups and the approach based on a set of rules. Examples of multi-criteria decision-making methods and their assignment to different streams (American, European, mixed or rule-based) are presented using a Table I.

This paper aims to present the study case of an objective camera selection multi-criteria problem. The authors' main objective was to perform a comparative analysis of the results obtained using three selected MCDA methods. Due to the goal of obtaining objective results in an automated process, the authors decided to choose two objective criteria weighting methods. It was assumed that due to the differences in the algorithms included in the MCDA methods, which cause

TABLE I
MULTI-CRITERIA DECISION-MAKING METHODS WITH STREAM AFFILIATIONS AND REFERENCES.

Stream	Acronym	Method Name	References
European	ELECTRE	ELimination Et Choix Traduisant la REalité (ELimination Et Choice Translating REality)	[11]
	PROMETHEE	Preference Ranking Organization METHOD for Enrichment of Evaluations	[12]
	TACTIC	Treatment of the Alternatives according To the Importance of Criteria	[13]
American	AHP	Analytic hierarchy process	[14]
	TOPSIS	Technique for the Order of Prioritisation by Similarity to Ideal Solution	[15]
	VIKOR	ViseKriterijumska Optimizacija I Kompromisno Resenje	[16]
	SMART	Simple Mutli-Attribute Rating Technique	[17]
Mixed	IDRA	Intercriteria Decision Rule Approach	[18]
	EVAMIX	Evaluation of Mixed Data	[19]
	PACMAN	Passive and Active Compensability Multicriteria ANalysis	[20]
Rule based	DRSA	Dominance-based rough set approach	[21]
	COMET	Characteristic Objects METHod	[22], [23]

various methods to provide different solutions to the same problems, benchmarking with several methods is an important stage in evaluating a multi-criteria problem. Because MCDA methods are intended to be used in many different fields, the need for a customized approach that considers the particular nature of the problem being analyzed occurs [24]. Using the correlation coefficients of the rankings in the next step allows an objective assessment of the convergence of the rankings and identification of methods that give consistent and outlier results in a specific problem.

For the solution of a described problem, a model-based approach including three MCDA methods, TOPSIS, VIKOR and COMET, has been applied, taking into account two techniques for determining the criteria weights: Mean Weighting, which gives equal weights and Entropy Weighting.

The rest of the paper is organized as follows. Section II provides the preliminaries and main fundamentals of the TOPSIS, VIKOR and COMET methods. In Section III, the study case, including evaluating alternatives and their types, is presented. Section IV shows the results of the performed assessment of alternatives. There is also presented the influence of the methods used in the authors' approach to the outcomes. Section V contains the summary of the conducted survey and conclusions.

II. PRELIMINARIES

A. The TOPSIS Method

The TOPSIS method compares the relative distances between the evaluated alternatives and the positive ideal solution (PIS) and the anti-ideal solution (negative ideal solution - NIS). The goal is to rank the alternatives such that the best alternative is as close as possible to the PIS and as far as possible from the NIS [25]. The TOPSIS method includes the five stages given below [26].

Step 1. Decision matrix is normalized.

In this approach, the greatest and the least values in the considered set are used. The formulas are described as follows (1) and (2):

$$r_{ij} = \frac{x_{ij} - \min_j(x_{ij})}{\max_j(x_{ij}) - X_{\min}} \quad (1)$$

$$r_{ij} = \frac{\max_j(x_{ij}) - x_{ij}}{\max_j(x_{ij}) - \min_j(x_{ij})} \quad (2)$$

Step 2. Weighted values of the normalized decision matrix v_{ij} are determined according to the Equation (3).

$$v_{ij} = w_i r_{ij} \quad (3)$$

Step 3. Calculate the positive ideal solution (PIS) values and negative anti-ideal solution (NIS) vectors. The PIS represented by the vector (4) expresses the maximum values for each criterion, and the NIS is represented by the vector (5) minimum values. It is unnecessary to divide the criteria into cost and profit criteria in this step because the cost criteria were transformed to profit criteria in the normalization step.

$$v_j^+ = \{v_1^+, v_2^+, \dots, v_n^+\} = \left\{ \max_j(v_{ij}) \right\} \quad (4)$$

$$v_j^- = \{v_1^-, v_2^-, \dots, v_n^-\} = \left\{ \min_j(v_{ij}) \right\} \quad (5)$$

Step 4. Calculate distance from PIS according to the Equation (6) and NIS, using the Equation (7) for each of the alternatives considered [6].

$$D_i^+ = \sqrt{\sum_{j=1}^n (v_{ij} - v_j^+)^2} \quad (6)$$

$$D_i^- = \sqrt{\sum_{j=1}^n (v_{ij} - v_j^-)^2} \quad (7)$$

Step 5. Calculate the outcome for each of the respected alternatives according to Equation (8). This score takes values between 0 and 1. Thus, the closer the value of a given alternative is to 1, the better is the alternative.

$$C_i = \frac{D_i^-}{D_i^- + D_i^+} \quad (8)$$

B. The VIKOR Method

The VIKOR method (VlseKriterijumska Optimizacija I Kompromisno Resenje), similarly to TOPSIS, takes distance measurement into account, but in this approach, the goal is to identify the alternative closest to the ideal solution. Therefore, the solution sought is a compromise solution [6]. The five steps of the VIKOR method are described below [27], [28], [29].

Step 1. Determinate the best f_j^* and the worst f_j^- value for the function of a particular criterion. For profit criteria, the Equation is used (9).

$$f_j^* = \max_i f_{ij}, \quad f_j^- = \min_i f_{ij} \quad (9)$$

whereas in the case of the cost criteria, the following Equation is used (10).

$$f_j^* = \min_i f_{ij}, \quad f_j^- = \max_i f_{ij} \quad (10)$$

Step 2. Calculate S_i and R_i with using Equations (11) and (12).

$$S_i = \sum_{j=1}^n w_j (f_j^* - f_{ij}) / (f_j^* - f_j^-) \quad (11)$$

$$R_i = \max_j [w_j (f_j^* - f_{ij}) / (f_j^* - f_j^-)] \quad (12)$$

Step 3. Calculate Q_i with using Equation (13).

$$Q_i = v(S_i - S^*) / (S^- - S^*) + (1 - v)(R_i - R^*) / (R^- - R^*) \quad (13)$$

where

$$S^* = \min_i S_i, \quad S^- = \max_i S_i$$

$$R^* = \min_i R_i, \quad R^- = \max_i R_i$$

v means the weight adopted for the strategy of "most criteria".

Step 4. Ranked alternatives S , R and Q are ordered in ascending order. Three ranked lists are the outcome.

Step 5. A compromise solution is proposed considering the conditions of good advantage and acceptable stability within the three vectors obtained in the previous step [29]. The best alternative is the one with the lowest value and the leading position in the ranking Q [30].

C. The COMET Method

The main advantage of the Characteristic Objects METHOD (COMET) is its resistance to the rank reversal paradox [31]. COMET method considers fuzzy sets theory. The important steps of this method are the determination and comparison of characteristic objects and the creation of a rule base. Then, each alternative is evaluated in a defuzzification process [25]. The five stages that the COMET method involves are provided below [32], [33], [34].

Step 1. Definition of the space of the problem. The expert determines the dimensionality of the problem with the selection r criteria, C_1, C_2, \dots, C_r . Then a set of fuzzy numbers is selected for each criterion C_i , e.g., $\{\tilde{C}_{i1}, \tilde{C}_{i2}, \dots, \tilde{C}_{ic_i}\}$ according to the Equation (14).

$$\begin{aligned} C_1 &= \{\tilde{C}_{11}, \tilde{C}_{12}, \dots, \tilde{C}_{1c_1}\} \\ C_2 &= \{\tilde{C}_{21}, \tilde{C}_{22}, \dots, \tilde{C}_{2c_2}\} \\ &\vdots \\ C_r &= \{\tilde{C}_{r1}, \tilde{C}_{r2}, \dots, \tilde{C}_{rc_r}\} \end{aligned} \quad (14)$$

where C_1, C_2, \dots, C_r are the ordinals of the fuzzy numbers for all criteria.

Step 2. The generation of characteristic objects (COs) with the usage of the Cartesian product of the fuzzy numbers' cores of all the criteria according to the Equation (15).

$$CO = \langle C(C_1) \times C(C_2) \times \dots \times C(C_r) \rangle \quad (15)$$

As a result, an ordered set of all COs is obtained (16).

$$\begin{aligned} CO_1 &= \langle C(\tilde{C}_{11}), C(\tilde{C}_{21}), \dots, C(\tilde{C}_{r1}) \rangle \\ CO_2 &= \langle C(\tilde{C}_{11}), C(\tilde{C}_{21}), \dots, C(\tilde{C}_{r2}) \rangle \end{aligned} \quad (16)$$

$$CO_t = \langle C(\tilde{C}_{1c_1}), C(\tilde{C}_{2c_2}), \dots, C(\tilde{C}_{rc_r}) \rangle$$

where t is the count of COs and is equal to Equation (17).

$$t = \prod_{i=1}^r c_i \quad (17)$$

Step 3. Assessment of characteristic objects by identifying the Matrix of Expert Judgment MEJ by comparing pairwise objects COs by the expert. The MEJ matrix is presented as Equation (18).

$$MEJ = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \dots & \alpha_{1t} \\ \alpha_{21} & \alpha_{22} & \dots & \alpha_{2t} \\ \dots & \dots & \dots & \dots \\ \alpha_{t1} & \alpha_{t2} & \dots & \alpha_{tt} \end{pmatrix} \quad (18)$$

where α_{ij} is the outcome of comparing CO_i and CO_j by the expert. The more preferred characteristic object receives a value of 1, and the less preferred object receives a value of 0. If the preferences are equal, both objects get a value of half. This step depends totally on the expert's knowledge and can be represented as (19).

$$\alpha_{ij} = \begin{cases} 0.0, & f_{exp}(CO_i) < f_{exp}(CO_j) \\ 0.5, & f_{exp}(CO_i) = f_{exp}(CO_j) \\ 1.0, & f_{exp}(CO_i) > f_{exp}(CO_j) \end{cases} \quad (19)$$

where the expert function f_{exp} denotes the empirical preferences of the expert.

After the MEJ matrix is provided, a vertical vector of the Summed Judgments SJ is obtained as shown by Equation 20.

$$SJ_i = \sum_{j=1}^t \alpha_{ij} \quad (20)$$

Finally, preference values are determined for each characteristic object. As a result, a vertical vector P is obtained, where the i -th row contains the approximate value of preference for CO_i .

Step 4. Each CO and its preference value is converted to a fuzzy rule by using the following Equation (21)

$$IF \ C(\tilde{C}_{1i}) \ AND \ C(\tilde{C}_{2i}) \ AND \ \dots \ THEN \ P_i \quad (21)$$

In this procedure, a complete fuzzy rule base is prepared.

Step 5. Inference and getting the final ranking. Each alternative is represented as a set of values, e.g.

$A_i = \{\alpha_{i1}, \alpha_{i2}, \alpha_{ri}\}$. This set refers to the criteria C_1, C_2, \dots, C_r . Mamdani's fuzzy inference technique is used to determine the preference of the i -th decision variant. The constant rule base guarantees that the results obtained are unequivocal, which makes COMET completely resistant to the rank reversal paradox [35].

D. Entropy Weighting Method

In the entropy weighting method, the criteria weight is calculated using a measure of uncertainty in the information [36].

Step 1. Normalization of input data using sum normalization method (22).

$$p_{ij} = \frac{x_{ij}}{\sum_{i=1}^m x_{ij}} \quad i = 1, \dots, m; \quad j = 1, \dots, n \quad (22)$$

Step 2. Calculation of the entropy value of j th criterion using Equation (23).

$$E_j = -\frac{\sum_{i=1}^m p_{ij} \ln(p_{ij})}{\ln(m)} \quad j = 1, \dots, n \quad (23)$$

Step 3. Calculation of the objective weight of j th criterion according to the Equation (24).

$$w_j = \frac{1 - E_j}{\sum_{i=1}^n (1 - E_j)} \quad j = 1, \dots, n \quad (24)$$

E. Mean Weighting Method

Criteria weights are calculated according to the Equation (25), where n is the number of criteria [37].

$$w_j = 1/n \quad (25)$$

F. Weighted Spearman's Rank Correlation Coefficient

The symmetrical r_w correlation coefficient is calculated by the Equation (26). The sample size is N and x_i and y_i are the positions in rankings which are compared [38].

$$r_w = 1 - \frac{6 \sum_{i=1}^N (x_i - y_i)^2 ((N - x_i + 1) + (N - y_i + 1))}{N^4 + N^3 - N^2 - N} \quad (26)$$

G. The WS similarity coefficient

The asymmetrical WS similarity coefficient is calculated according to Equation (27), where N is size of sample and x_i and y_i are the positions in the compared rankings x and y . For this coefficient, changes in the positions at the top of the ranking influence most significantly its value [39].

$$WS = 1 - \sum_{i=1}^N 2^{-x_i} \frac{|x_i - y_i|}{\max(|x_i - 1|, |x_i - N|)} \quad (27)$$

III. STUDY CASE

This work aimed to study the effect of three different MCDA methods TOPSIS, VIKOR and COMET, on the evaluation results of 20 different camera models. Data on the evaluation criteria values of the selected camera models were obtained from various websites. The selected quantitative criteria represent camera parameters considered by customers during purchase decisions. In modelling decision problems, a very significant issue is determining the importance of decision criteria. There are methods in the literature to obtain the values of criteria weights [38]. In this study, two objective criteria weighting methods were applied: Mean Weighting, which gives equal weights and Entropy Weighting. The selected criteria according to which the alternatives were evaluated are included in Table II. In the next steps of the study, a comparative analysis between the MCDA methods used was performed for each of the criteria weighting methods used. Finally, two ranking correlation coefficients were used to determine the convergence of the obtained rankings: symmetrical r_w and asymmetrical WS .

TABLE II
SELECTED CRITERIA USED IN EVALUATION OF CAMERA MODELS

C_i	Name	Type	Unit
C_1	Thickness	Cost	Millimeters [mm]
C_2	Width	Cost	Millimeters [mm]
C_3	Height	Cost	Millimeters [mm]
C_4	Weight	Cost	Gram [g]
C_5	Resolution	Profit	Megapixel [Mpx]
C_6	4K	Profit	Frames per second [FPS]
C_7	FullHD	Profit	Frames per second [FPS]
C_8	HD	Profit	Frames per second [FPS]
C_9	Viewing angle	Profit	Radian [°]
C_{10}	Battery life	Profit	Minutes [min]
C_{11}	Price	Cost	Polish zloty [PLN]

IV. RESULTS AND DISCUSSION

The values of each criterion for the alternatives evaluated are given in the decision matrix, displayed in Table III. The decision matrix, normalized by using the Minimum-Maximum normalization method for each weighting technique and MCDA method, is presented in Table IV. For the TOPSIS and COMET methods, the best alternative is the alternative that scored the highest preference value. Therefore, the lower the preference value, the lower the alternative is ranked. For the VIKOR method, the opposite is true. In its case,

TABLE III
THE PERFORMANCE TABLE OF THE ALTERNATIVES $A_1 - A_{20}$.

Alternatives	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}
SONY FDR-X3000	29.40	83.00	47.00	114	12.0	30	120	240	170	90	1717.75
DJI Pocket 2 Creator Combo	30.00	38.10	124.70	117	16.0	60	60	60	93	70	2389.00
GÖTZE & JENSEN S-Line SC501	29.28	59.27	41.13	58	16.0	30	60	120	170	78	239.99
GOPRO HERO9	33.60	71.00	55.00	159	23.6	60	240	240	132	140	2099.00
Xblitz Move 4K+	21.00	59.00	41.00	66	16.0	24	60	120	170	70	439.00
DJI Osmo Action	35.00	65.00	42.00	134	12.0	60	240	240	145	60	1087.00
Insta360 ONE R-1-Inch Edition	47.00	79.00	54.00	158	19.0	60	120	120	360	72	2499.00
GOPRO HERO7	28.30	62.30	44.90	116	12.0	30	60	60	130	90	999.99
DJI Osmo Pocket	36.90	28.60	121.60	130	12.0	60	120	120	80	80	1099.00
GOXTREME Enduro	32.00	59.00	41.00	60	16.0	30	120	120	170	60	302.96
GOPRO HERO8	28.40	66.30	48.60	126	12.0	60	240	240	132	135	1629.00
Insta360 One X2	29.80	46.00	113.00	47	18.0	50	50	50	360	72	2099.00
SJCAM A20	20.20	64.00	80.00	70	8.0	24	60	120	166	480	699.99
LAMAX X9.1	33.00	60.00	44.00	59	12.0	30	60	120	170	90	388.00
MANTA MM9259	29.00	59.00	41.00	55	16.0	30	60	120	170	120	299.00
SJCAM SJ4000 WiFi	29.00	59.00	41.00	182	12.0	30	30	60	94	140	249.00
LAMAX Action X3.1 Atlas	29.80	59.20	41.00	65	16.0	30	60	120	160	90	219.99
SJCAM SJ10 Pro	28.80	62.50	41.00	70	12.0	60	120	120	170	138	1399.99
GOXTREME Pioneer	24.00	40.00	59.00	60	12.0	10	30	30	140	78	269.99
TRACER eXplore SJ 4561	30.00	60.00	45.00	201	16.0	30	30	30	170	90	199.99

TABLE IV
NORMALIZED DECISION MATRIX

A_i	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}
A_1	0.3745	0.0000	0.6231	0.4328	0.5085	0.5000	0.5000	1.0000	0.4722	0.1875	0.3126
A_2	0.3617	0.5410	0.0000	0.4179	0.6780	1.0000	0.2500	0.2500	0.2583	0.1458	0.0440
A_3	0.3770	0.2859	0.6702	0.7114	0.6780	0.5000	0.2500	0.5000	0.4722	0.1625	0.9040
A_4	0.2851	0.1446	0.5589	0.2090	1.0000	1.0000	1.0000	1.0000	0.3667	0.2917	0.1601
A_5	0.5532	0.2892	0.6712	0.6716	0.6780	0.4000	0.2500	0.5000	0.4722	0.1458	0.8243
A_6	0.2553	0.2169	0.6632	0.3333	0.5085	1.0000	1.0000	1.0000	0.4028	0.1250	0.5650
A_7	0.0000	0.0482	0.5670	0.2139	0.8051	1.0000	0.5000	0.5000	1.0000	0.1500	0.0000
A_8	0.3979	0.2494	0.6399	0.4229	0.5085	0.5000	0.2500	0.2500	0.3611	0.1875	0.5998
A_9	0.2149	0.6554	0.0249	0.3532	0.5085	1.0000	0.5000	0.5000	0.2222	0.1667	0.5602
A_{10}	0.3191	0.2892	0.6712	0.7015	0.6780	0.5000	0.5000	0.5000	0.4722	0.1250	0.8788
A_{11}	0.3957	0.2012	0.6103	0.3731	0.5085	1.0000	1.0000	1.0000	0.3667	0.2812	0.3481
A_{12}	0.3660	0.4458	0.0938	0.7662	0.7627	0.8333	0.2083	0.2083	1.0000	0.1500	0.1601
A_{13}	0.5702	0.2289	0.3585	0.6517	0.3390	0.4000	0.2500	0.5000	0.4611	1.0000	0.7199
A_{14}	0.2979	0.2771	0.6472	0.7065	0.5085	0.5000	0.2500	0.5000	0.4722	0.1875	0.8447
A_{15}	0.3830	0.2892	0.6712	0.7264	0.6780	0.5000	0.2500	0.5000	0.4722	0.2500	0.8804
A_{16}	0.3830	0.2892	0.6712	0.0945	0.5085	0.5000	0.1250	0.2500	0.2611	0.2917	0.9004
A_{17}	0.3660	0.2867	0.6712	0.6766	0.6780	0.5000	0.2500	0.5000	0.4444	0.1875	0.9120
A_{18}	0.3872	0.2470	0.6712	0.6517	0.5085	1.0000	0.5000	0.5000	0.4722	0.2875	0.4398
A_{19}	0.4894	0.5181	0.5269	0.7015	0.5085	0.1667	0.1250	0.1250	0.3889	0.1625	0.8920
A_{20}	0.3617	0.2771	0.6391	0.0000	0.6780	0.5000	0.1250	0.1250	0.4722	0.1875	0.9200

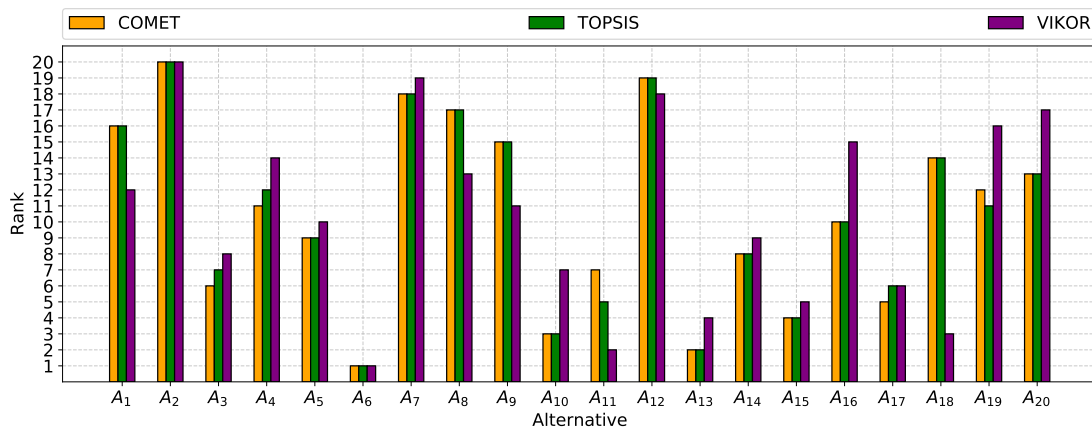


Fig. 1. Comparison of rankings received with using Entropy Weighting for TOPSIS, VIKOR and COMET

TABLE V
PREFERENCE VALUES AND RANKINGS OBTAINED WITH USING ENTROPY WEIGHTING FOR TOPSIS, VIKOR AND COMET

A_i	$COMET_{pref}$	$TOPSIS_{pref}$	$VIKOR_{pref}$	$COMET_{rank}$	$TOPSIS_{rank}$	$VIKOR_{rank}$
A_1	0.3902	0.4287	0.3557	16	16	12
A_2	0.0848	0.2060	0.9500	20	20	20
A_3	0.5901	0.5424	0.2118	6	7	8
A_4	0.4709	0.4853	0.4203	11	12	14
A_5	0.5385	0.5176	0.2609	9	9	10
A_6	0.6677	0.5885	0.1240	1	1	1
A_7	0.1787	0.3136	0.8273	18	18	19
A_8	0.3827	0.4192	0.3778	17	17	13
A_9	0.4278	0.4459	0.3531	15	15	11
A_{10}	0.6377	0.5703	0.2029	3	3	7
A_{11}	0.5796	0.5463	0.1397	7	5	2
A_{12}	0.1631	0.2806	0.6954	19	19	18
A_{13}	0.6489	0.5877	0.1506	2	2	4
A_{14}	0.5619	0.5302	0.2232	8	8	9
A_{15}	0.5988	0.5494	0.1856	4	4	5
A_{16}	0.4969	0.4978	0.4415	10	10	15
A_{17}	0.5964	0.5459	0.1976	5	6	6
A_{18}	0.4580	0.4682	0.1433	14	14	3
A_{19}	0.4687	0.4856	0.4451	12	11	16
A_{20}	0.4641	0.4850	0.4674	13	13	17

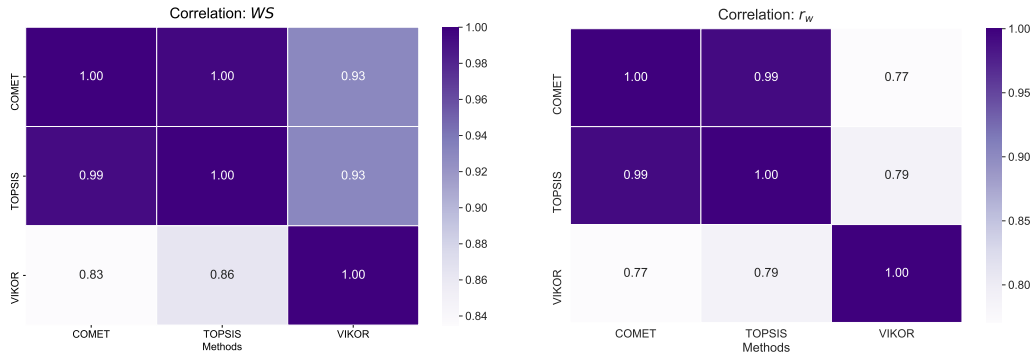


Fig. 2. WS and r_w correlation heat maps for TOPSIS, VIKOR and COMET with using Entropy Weighting

TABLE VI
PREFERENCE VALUES AND RANKINGS OBTAINED WITH USING MEAN WEIGHTING (EQUAL WEIGHTS) FOR TOPSIS, VIKOR AND COMET

A_i	$COMET_{pref}$	$TOPSIS_{pref}$	$VIKOR_{pref}$	$COMET_{rank}$	$TOPSIS_{rank}$	$VIKOR_{rank}$
A_1	0.4516	0.4646	0.7887	13	13	14
A_2	0.3565	0.4042	1.0000	20	20	20
A_3	0.5323	0.5233	0.4939	8	8	6
A_4	0.5797	0.5516	0.0356	2	2	2
A_5	0.5358	0.5249	0.5375	7	7	9
A_6	0.5731	0.5487	0.5186	3	3	8
A_7	0.4353	0.4590	0.8249	15	15	16
A_8	0.4025	0.4248	0.7055	17	18	13
A_9	0.4223	0.4462	0.8539	16	16	17
A_{10}	0.5421	0.5315	0.5875	6	5	10
A_{11}	0.5815	0.5565	0.0192	1	1	1
A_{12}	0.4814	0.4878	0.6455	12	12	11
A_{13}	0.5159	0.5111	0.6457	10	10	12
A_{14}	0.4882	0.4913	0.5150	11	11	7
A_{15}	0.5424	0.5313	0.2023	5	6	4
A_{16}	0.3868	0.4232	0.9327	19	19	19
A_{17}	0.5268	0.5195	0.4292	9	9	5
A_{18}	0.5459	0.5351	0.0790	4	4	3
A_{19}	0.4393	0.4630	0.8160	14	14	15
A_{20}	0.3923	0.4281	0.9205	18	17	18

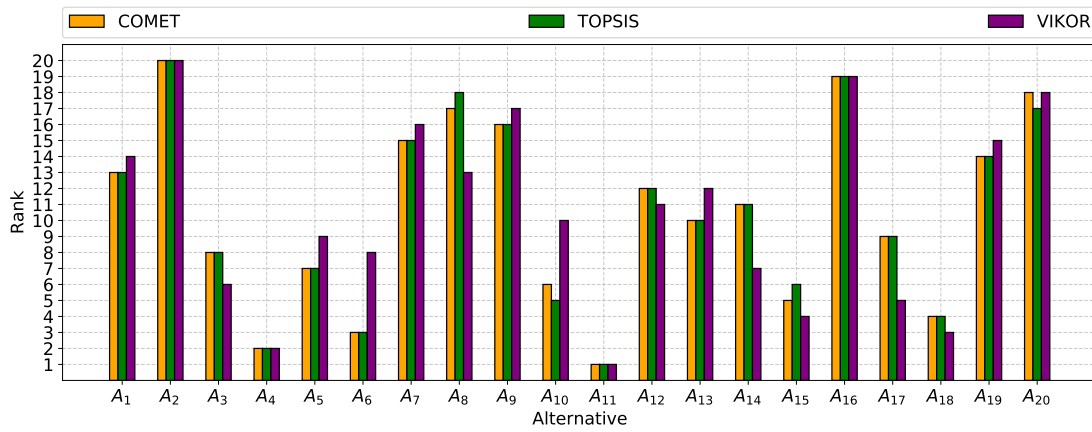


Fig. 3. Comparison of rankings received with using Mean Weighting (Equal Weights) for TOPSIS, VIKOR and COMET

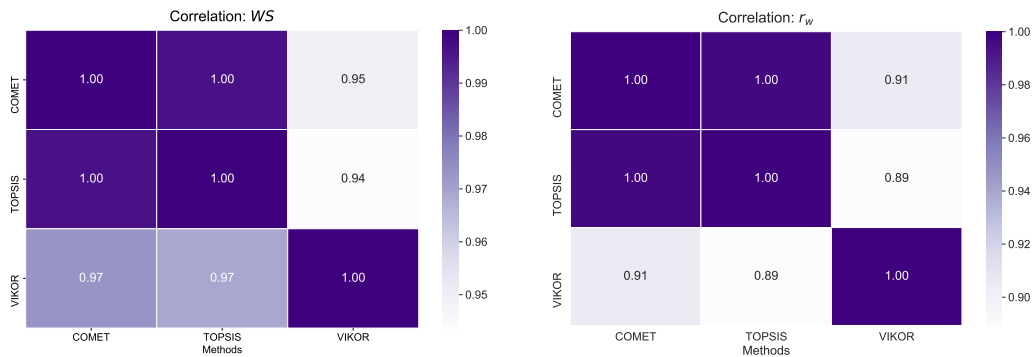


Fig. 4. WS and r_w correlation heat maps for TOPSIS, VIKOR and COMET with using Mean Weighting (Equal Weights)

the best alternative is the alternative for which the lowest preference value was calculated. Therefore, as the preference value increases, the alternative decreases in ranking.

A. Results for Entropy Weighting

Preference values and rankings obtained for each MCDA method with applying entropy weights are contained in Table V. Comparison of rankings is visualized in Figure 1. In the rankings obtained using entropy weights, only two alternatives ranked equally in the rankings created by the MCDA methods applied. These include A_6 , which is the leader in all three rankings, and A_2 , which is always last. Analysis of the obtained rankings allows us to conclude that the VIKOR method has the most significant impact on the differences in rankings. Ranking received using this method demonstrates the most divergent values (range of differences including four positions for A_1 , A_8 , A_9 , A_{10} , A_{20} , five positions for A_{11} , A_{16} , and eleven positions for A_{18}). The rankings obtained using TOPSIS and COMET methods show very high convergence. As many as 15 alternatives are in identical positions. For the remaining alternatives, the differences are minimal (one position for A_3 , A_4 , A_{17} , A_{19} and two positions for A_{11}).

The similarity between the rankings obtained using each MCDA method was then examined. Results of investigation of rankings' similarity are displayed in Figure 2. Two ranking similarity coefficients were used to investigate the correlation: WS and r_w . The highest value of asymmetrical WS coefficient was noticed for COMET and TOPSIS (1.00) and TOPSIS and COMET (0.99). The lower value was received for COMET and VIKOR and TOPSIS and VIKOR (0.93). The lowest correlation was observed for VIKOR and TOPSIS (0.96) and VIKOR and COMET (0.83).

When investigating the similarity of rankings using the r_w coefficient, the highest correlation was found for COMET and TOPSIS (0.99), lower for TOPSIS and VIKOR (0.79), and lowest for VIKOR and COMET (0.77). Thus, the ranking similarity examination results confirm the outliers in the ranking achieved by the VIKOR method.

B. Results for Mean Weighting

Preference values for TOPSIS, VIKOR and COMET with applying equal weights are contained in Table VI. Comparison of rankings is illustrated in Figure 3. The same rankings for the three MCDA methods were obtained for only four alternatives when Mean Weighting was used. Among them are A_{11} , which

is the ranking leader, A_4 in second place, A_{16} in second-to-last place, and A_2 in the last place. Thus, another alternative is the leader for Mean Weighting than Entropy Weighting. The most significant differences between the obtained rankings were observed for the VIKOR method (range of differences including five positions for A_6 , A_8 , A_{10} , and four positions for A_{14} and A_{17}). On the other hand, for Mean Weighting, the rankings obtained with the TOPSIS and COMET methods were the most consistent. The rankings were identical for as many as 16 alternatives, while for four alternatives (A_8 , A_{10} , A_{15} , A_{20}), the differences included only one position.

Values of ranking similarity coefficients are displayed in Figure 4. In the ranking similarity study, the highest WS value was obtained for COMET and TOPSIS (1.00), followed by VIKOR and COMET and VIKOR and TOPSIS (0.97), and the lowest for COMET and VIKOR (0.95) and TOPSIS and VIKOR (0.94). The highest r_w value was received for COMET and TOPSIS (1.00), lower for VIKOR and COMET (0.91), and lowest for TOPSIS and VIKOR (0.89).

The results of the performed research demonstrate that the complexity of decision problems containing many different criteria makes it difficult to identify a universal method to obtain the best solution for various problems. Therefore, when there is a need to obtain an objective solution to multi-criteria decision problems, hybrid approaches, in which different algorithms are combined to solve the decision problem, seem to be suitable [40]. A well-known example is the hybrid DSS 3.0 system proposed by Budzinski and Becker. In this system, the values of criteria weights are determined by the AHP method, while the ELECTRE Tri method is used to create the ranking [41]. The described hybrid approach is worth attention and consideration in further research directions.

V. CONCLUSIONS

This paper aimed to investigate the effect of selected MCDA methods and objective weighting techniques on the objectivity of the resulting rankings. The case study in this work was the camera selection problem. The results obtained confirm that several conditions must be respected to obtain appropriate assessment results using MCDA methods. First, it is essential to select methods for the problem to be adequately solved. Second, benchmarking with other methods allowing for comparative analysis is required. Also, a proper selection of criteria weights that reflect the preferences of the decision-maker is recommended.

The study shows that the most comparable rankings were achieved using TOPSIS and COMET methods. Outlier results of the VIKOR method contribute to the disturbance of objectivity of received results. Due to the careful selection of several MCDA methods and the comparative analysis performed, it was possible to determine a set of methods providing convergent and objective results. Obtained results encourage continuing the research with other MCDA methods to extend the set of methods enabling objectivization of evaluations in the undertaken problem.

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