

Towards the identification of MARCOS models based on intuitionistic fuzzy score functions

Bartłomiej Kizielewicz^{*}, Bartosz Paradowski, Jakub Więckowski, Wojciech Sałabun Research Team on Intelligent Decision Support Systems, Department of Artificial Intelligence Methods and Applied Mathematics,

Faculty of Computer Science and Information Technology

West Pomeranian University of Technology in Szczecin

ul. Żołnierska 49, 71-210 Szczecin, Poland

Email: {bartlomiej-kizielewicz, jakub-wieckowski, bartosz-paradowski, wojciech.salabun}@zut.edu.pl

Abstract—We encounter uncertainty in many areas. In decision-making, it is an aspect that allows for better modeling of real-world problems. However, many methods rely on crisp numbers in their calculations. It makes it necessary to use techniques that perform this conversion. In this paper, we address the problem of score functions assessment regarding their effectiveness and usefulness in the decision-making field. The selected methods were used to convert the intuitionistic fuzzy set matrix into crisp data, then used in the multi-criteria assessment. Managing the theoretical problem showed that the used techniques provide high similarity values. Moreover, they proved to be helpful when dealing with intuitionistic fuzzy sets in the decision-making area.

I. INTRODUCTION

Many multi-criteria decision-making problems are considered in areas where data are represented using crisp numbers [1]. However, uncertainty problems are difficult to represent using this approach. Therefore, many tools based on classical arithmetic methods have been developed to model uncertainty in decision problems [2]. Such tools allow us to model real-world problems more accurately and reflect uncertain knowledge flexibly. Uncertainty modeling tools are often used in multi-criteria decision-making problems due to their high reliability [3].

Several popular tools can be used to represent uncertain knowledge. Among the classical approaches are fuzzy sets (FS), based on the idea related to partial membership [4]. Over the years, fuzzy sets have seen many developments: Hesitant Fuzzy Sets (HFS) [5], Fermatean Fuzzy Sets (FFS) [6], Picture Fuzzy Sets (PFS) [7], or Intuitionistic Fuzzy Sets (IFS) [8]. Indeed, the main advantage of the generalization of fuzzy sets is a new approach to uncertainty modeling that considers new degrees of membership, which gives the expert the ability to adapt to the characteristics of the problem [9].

One of the most popular tools based on the idea of fuzzy sets is Intuitionistic Fuzzy Sets. This tool introduces the possibility of determining the degree of membership and nonmembership, thanks to which it is helpful in many areas such as decision-making and medical diagnosis [10], [11]. The wide use of Intuitionistic Fuzzy Sets has led to the development of this approach. A new similarity measure between intuitionistic fuzzy sets was proposed by Gohain et al. [12]. Szmidt et al. proposed a new proposal for attribute selection in models expressed by intuitionistic fuzzy sets [13]. Thao proposed new divergence measures of intuitionistic fuzzy sets from Archimedean t-conorm operators [14].

Using an extension of multi-criteria decision-making methods with fuzzy logic makes it possible to change the problem environment from crisp to uncertain. However, most Multi-Criteria Decision-Making (MCDM) related approaches operate in an environment based on crisp numbers [15]. To convert fuzzy data to crisp data, one can use point functions, whose idea in multi-criteria decision making was originally proposed by Chen and Tan [16]. However, the existence of multiple scoring functions means that their use within the same problem may be characterized by obtaining different results [17]. It creates a research gap that needs to be filled and determines which score function to select so that the results are satisfactory.

In this paper, we used five different score functions to convert Intuitionistic Fuzzy Sets to crisp values and assess the obtained decision matrix with the Measurement Alternatives and Ranking according to COmpromise Solution (MARCOS) method. The simulated data was used as the inputs to show the performance of the presented approach in the theoretical problem. Obtained results were then compared with selected correlation coefficients to point out the similarity of the used paths. The purpose of the study is to indicate the influence of the used score function regarding the differences obtained in multi-criteria ranking.

The rest of the paper is organized as follows. Section 2 presents the preliminaries of the IFS, the scores functions, the MARCOS method and selected similarity coefficients. In Section 3, the study case is shown, where the theoretical problem of the functioning of the different scores function is raised. Section 4 includes the description of the results obtained from the examined research. Finally, in Section 5, the summary is presented, and the conclusions are drawn.

II. PRELIMINARIES

A. Intuitionistic Fuzzy Sets

Definition II.1. An IFS A in a universe X is defined as an object of the following form:

where $\mu : X \to [0,1]$ and $\nu : X \to [0,1]$ such that $0 \leq \mu_j + \nu_j \leq 1$ for all $x_j \in X$. The values of μ_j and ν_j represent the degrees of membership and non-membership of $x_j \in X$ in A respectively [17].

For every $A \in IFS(X)$ (the class of IFSs in the universe X), the value of

$$\pi_j = 1 - \mu_j - \nu_j \tag{2}$$

represents the degree of hesitation (or uncertainty) associated with the membership of element $x_j \in X$ in IFS A, where $0 \leq \pi_i \leq 1$.

B. Score Functions

The purpose of the score function is to convert the uncertain data representation to a crisp value. Different approaches to performing such an action obtain diverse values as a final output. Selected score functions and the formulas for their calculations are presented below [17], [18], [19].

$$S_{\rm I}\left(X_{ij}\right) = \mu_{ij} - v_{ij} \tag{3}$$

$$S_{\rm II}\left(X_{ij}\right) = \mu_{ij} - v_{ij} \cdot \pi_{ij} \tag{4}$$

$$S_{\text{III}}(X_{ij}) = \mu_{ij} - \left(\frac{v_{ij} + \pi_{ij}}{2}\right) \tag{5}$$

$$S_{\rm IV}\left(X_{ij}\right) = \left(\frac{\mu_{ij} + v_{ij}}{2}\right) - \pi_{ij} \tag{6}$$

$$S_{\rm V}(X_{ij}) = \gamma \cdot \mu_{ij} + (1 - \gamma) \cdot (1 - v_{ij}), \quad \gamma \in [0, 1]$$
 (7)

where $S_I(X_{ij}), S_{II}(X_{ij}), S_V(X_{ij}) \in [-1, 1], S_{III}(X_{ij}) \in [-0.5, 1]$, and $S_{IV}(X_{ij}) \in [-1, 0.5]$.

C. MARCOS method

The Measurement Alternatives and Ranking according to COmpromise Solution (MARCOS) method was introduced by Željko Stević in 2020 [20] as new multi-criteria decision making method, which was presented on study case of sustainable supplier selection in healthcare industries. This method provides new approach to solve decision problems, as it considers an anti-ideal and ideal solution at the initial steps of consideration of the problem. Moreover it proposes new way to determine utility functions and their further aggregation, while maintaining stability in the problems requiring large set of alternatives and criteria.

Step 1. The initial step requires to define set of n criteria and m alternatives to create decision matrix.

Step 2. Next, the extended initial matrix X should be formed by defining ideal (AI) and anti-ideal(AAI) solution.

$$X = \begin{array}{c} AII \\ A_1 \\ A_2 \\ \dots \\ A_m \\ AI \end{array} \begin{bmatrix} x_{aa1} & x_{aa2} & \dots & x_{aan} \\ x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \dots & \dots & \dots & \dots \\ x_{m1} & x_{22} & \dots & x_{mn} \\ x_{ai1} & x_{ai2} & \dots & x_{ain} \end{bmatrix}$$
(8)

The anti-ideal solution (AAI) which is the worst alternative is defined by equation (9), whereas the ideal solution (AI) is the best alternative in the problem at hand defined by equation (10).

$$AAI = \min_{i} x_{ij}$$
 if $j \in B$ and $\max_{i} x_{ij}$ if $j \in C$ (9)

$$AI = \max_{i} x_{ij}$$
 if $j \in B$ and $\min_{i} x_{ij}$ if $j \in C$ (10)

where B is a benefit group of criteria and C is a group of cost criteria.

Step 3. After defining anti-ideal and ideal solutions, the extended initial matrix X needs to be normalized, by applying equations (11) and (12) creating normalized matrix N.

$$n_{ij} = \frac{x_{ai}}{x_{ij}} \quad \text{if } j \in C \tag{11}$$

$$n_{ij} = \frac{x_{ij}}{x_{ai}} \quad \text{if } j \in B \tag{12}$$

Step 4. The weight for each criterion needs to be defined to present its importance in accordance to others. The weighted matrix V needs to be calculated by multiplying the normalized matrix N with the weight vector through equation (13).

$$v_{ij} = n_{ij} \times w_j \tag{13}$$

Step 5. Next, the utility degree K of alternatives in relation to the anti-ideal and ideal solutions needs to by calculated by using equations (14) and (15)

$$K_{i}^{-} = \frac{\sum_{i=1}^{n} v_{ij}}{\sum_{i=1}^{n} v_{aai}}$$
(14)

$$K_i^+ = \frac{\sum_{i=1}^n v_{ij}}{\sum_{i=1}^n v_{ai}}$$
(15)

Step 6. The utility function f of alternatives, which is the compromise of the observed alternative in relation to the ideal and anti-ideal solution, needs to be determined. Its done using equation (16).

$$f(K_i) = \frac{K_i^+ + K_i^-}{1 + \frac{1 - f(K_i^+)}{f(K_i^+)} + \frac{1 - f(K_i^-)}{f(K_i^-)}}$$
(16)

where $f(K_i^-)$ represents the utility function in relation to the anti-ideal solution and $f(K_i^+)$ represents the utility function in relation to the ideal solution.

Utility functions in relation to the ideal and anti-ideal solution are determined by applying equations (17) and (18).

$$f(K_i^-) = \frac{K_i^+}{K_i^+ + K_i^-}$$
(17)

$$f(K_i^+) = \frac{K_i^-}{K_i^+ + K_i^-}$$
(18)

Step 7. Finally, rank alternatives accordingly to the values of the utility functions. The higher the value the better is an alternative.

D. Rank similarity coefficients

In order to compare the performance of the score functions, it would be useful to compare the rankings obtained after evaluating the values calculated using these functions. For this purpose, one can use rank similarity coefficients, which are often used in the literature to compare the resulting rankings. In the case of our study, we decided to use weighted Spearman's correlation coefficient, which allows comparing rankings considering alternatives rated the best as more significant, and the WS ranking similarity coefficient, which the main assumption that the positions of top of the rankings has a more significant influence on similarity. The formulas for calculation of both coefficients are presented below in equation (19) for weighted Spearman's correlation and equation (20) for WS rank similarity coefficient.

$$r_w = 1 - \frac{6 \cdot \sum_{i=1}^n (x_i - y_i)^2 \left((N - x_i + 1) + (N - y_i + 1) \right)}{n \cdot (n^3 + n^2 - n - 1)}$$
(19)

$$WS = 1 - \sum_{i=1}^{n} \left(2^{-x_i} \frac{|x_i - y_i|}{\max\{|x_i - 1|, |x_i - N|\}} \right)$$
(20)

III. STUDY CASE

The use of fuzzy sets in multi-criteria problems is a popular approach to solving problems where uncertainty arises. It allows greater flexibility in modeling input data, thus ensuring that the actual values that determine the parameters can be represented. However, in many cases, the criteria are not considered in a binary way, or the corresponding values are not known precisely. Fuzzy sets are one of the possible ways to represent uncertainty [21]. In the following, we focus our attention on the problem of using Intuitionistic Fuzzy Sets and different score functions to point out differences and similarities in the results obtained by using these tools.

A randomly generated decision matrix of 6 alternatives and 4 criteria was used in the study. Each matrix element is represented in the form of an IFS, where the first value indicates the value of decisiveness, while the second determines the degree of indecisiveness. Then, based on the score functions described above, conversions of the uncertain matrix to a matrices represented in the form of sharp numbers were performed. The generated matrix is shown in Table I. The purpose of this operation is the need to indicate how a given score function affects the process of converting the data to a crisp form. Furthermore, it is crucial to determine whether the obtained matrices influence the obtained result through a multi-criteria analysis.

IV. RESULTS

A. Small example

Each type of previously presented score function was used to calculate crisp values for the matrix, which were shown in Tables respective to the used function. Table II presents values obtained by use of score function S_I . In the case of this score function, the spread of values in the range [-1, 1]is around 1.69, which might mean that this specific score function differentiates well between alternative values.

TABLE II CRISP SMALL DECISION MATRIX CALCULATED WITH S_I score function.

A_i	C_1	C_2	C_3	C_4
A_1	-0.039081	0.137740	-0.351916	0.691538
A_2	-0.558417	-0.455956	-0.649449	0.010564
A_3	-0.405613	-0.006527	0.361583	0.244181
A_4	-0.211142	-0.122171	-0.596428	0.159583
A_5	0.479454	-0.860203	0.830694	-0.110298
A_6	0.375293	0.328710	-0.797545	-0.822696

In Table III values calculated through execution of score function S_{II} are presented. This function is defined as the degree of membership minus the product of the non-membership and hesitation degrees, and even though it provides values from the same range as S_I , it can be seen that there are less negative values. Moreover, it is clear that in this example, the spread of calculated values is significantly smaller, as in this case, it's around 0.99.

TABLE III CRISP SMALL DECISION MATRIX CALCULATED WITH S_{II} score function.

A_i	C_1	C_2	C_3	C_4
A_1	0.041174	0.510925	0.243452	0.726737
A_2	0.205952	0.075299	0.021391	0.089091
A_3	-0.041371	0.060761	0.568287	0.587950
A_4	-0.141754	0.392034	0.001438	0.324662
A_5	0.506242	-0.056156	0.849442	0.389249
A_6	0.383334	0.368876	-0.004356	-0.014798

The values obtained through the equation of score function S_{III} are shown in Table IV. This specific function operates in the range [-0.5, 1] and is similar to the previous one but subtracts the arithmetic mean of the non-membership and hesitation degrees. As a result, provided values spread around 1.24, which translates into a high differentiation of the individual IFS values from the initial decision matrix.

 TABLE I

 Small decision matrix represented by intuitionistic fuzzy sets.

A_i	$C_1 \ (\mu, u)$	$C_2 \ (\mu, u)$	$C_3\ (\mu, u)$	$C_4 \ (\mu, u)$
A_1	(0.17125, 0.21033)	(0.53664,0.39890)	(0.28872, 0.64063)	(0.73657,0.04503)
A_2	(0.21496, 0.77338)	(0.18588, 0.64183)	(0.11440,0.76385)	(0.20609, 0.19553)
A_3	(0.13443, 0.54004)	(0.17854, 0.18506)	(0.60514,0.24356)	(0.60220, 0.35801)
A_4	(0.03524, 0.24638)	(0.41634, 0.53851)	(0.11939, 0.71582)	(0.40974, 0.25016)
A_5	(0.52621, 0.04675)	(0.02438, 0.88458)	(0.85215, 0.02146)	(0.41781, 0.52811)
A_6	(0.39471,0.01942)	(0.41035,0.08164)	(0.06241, 0.85995)	(0.05100,0.87369)

TABLE IV CRISP SMALL DECISION MATRIX CALCULATED WITH S_{III} score function.

A_i	C_1	C_2	C_3	C_4
A_1	-0.243131	0.304959	-0.066927	0.604858
A_2	-0.177553	-0.221182	-0.328406	-0.190864
A_3	-0.298357	-0.232197	0.407707	0.403293
A_4	-0.447136	0.124515	-0.320909	0.114611
A_5	0.289310	-0.463433	0.778231	0.126715
A_6	0.092065	0.115525	-0.406387	-0.423503

The function S_{IV} is defined as the arithmetic mean of the membership and non-membership degrees minus the hesitation degree, which operates in the range [-1, 0.5]. The spread of the values obtained is around 1.06, which is slightly less than the previous function, but still shows that the IFS values are significantly differentiated from each other.

TABLE V CRISP SMALL DECISION MATRIX CALCULATED WITH S_{IV} score function.

A_i	C_1	C_2	C_3	C_4
A_1	-0.427641	0.403307	0.394019	0.172409
A_2	0.482520	0.241570	0.317362	-0.397573
A_3	0.011706	-0.454602	0.273041	0.440315
A_4	-0.577559	0.432286	0.252824	-0.010152
A_5	-0.140560	0.363438	0.310422	0.418877
A_6	-0.378809	-0.262015	0.383544	0.387039

Values for last score function, namely S_V are presented in Table VI. This function represents a mixed result of positive and negative outcome expectations and operates in the same range as S_I and S_{II} . In this case, no negative values were received even though the range in which operates this function includes negative values. The spread of values received from this function is around 0.85, which is the lowest of presented score functions, considering its range.

TABLE VI CRISP SMALL DECISION MATRIX CALCULATED WITH S_V score function.

A_i	C_1	C_2	C_3	C_4
A_1	0.480460	0.568870	0.324042	0.845769
A_2	0.220791	0.272022	0.175275	0.505282
A_3	0.297193	0.496736	0.680791	0.622091
A_4	0.394429	0.438915	0.201786	0.579791
A_5	0.739727	0.069898	0.915347	0.444851
A_6	0.687646	0.664355	0.101228	0.088652

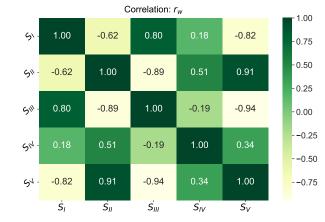
Table VII presents preference values calculated by execution of MARCOS method. The values of preference for respective alternatives show the differences between considered score functions. The function S_I has irregular distribution, where only one value is significantly higher than the rest. But in the case of this function, the difference between the highest and lowest value is almost 0.3, which shows that the values do not have a high spread. On the contrary, the score function S_{II} provides a higher spread of 0.426, which might be preferable as it better distinguishes the differences between alternatives.

Score function S_{III} provides the smallest spread of values of all functions, namely 0.047. In such a case, it may be perceived as the difference between alternatives is insignificant, which is rarely preferable in case of decision problems. Evaluated values from score function S_{IV} yielded values that spread around 0.23, which is not the highest of presented score functions but might be useful in some cases. The last score function S_V provided the highest values in this Table, which might be visually better perceived by some decision-makers, as the differences between the alternatives are more readily apparent. The spread is around 0.33, which is the secondhighest. Considering those values, functions S_{II} and S_V are the most representative and might be preferred by numerous decision-makers.

TABLE VII PREFERENCES FOR SMALL DECISION MATRIX COMPUTED WITH MARCOS METHOD FOR S_I - S_V score functions.

A_i	S_{I}	S_{II}	S_{III}	$S_{\rm IV}$	$S_{ m V}$
A_1	-0.064287	0.584944	-0.017914	0.174813	0.700419
A_2	0.233428	0.173407	0.034649	0.177619	0.366605
A_3	0.005450	0.374305	0.010045	0.080901	0.643830
A_4	0.091397	0.231148	0.022694	0.051248	0.514291
A_5	0.054616	0.599214	-0.011511	0.278166	0.644428
A_6	0.025817	0.359104	0.008765	0.056061	0.525352

Figure 1 presents alternatives ranked by preference obtained through considered score functions. On the graph, the differences in evaluation are clearly visible as, for example, the score function S_{III} and S_I ranked alternative A_1 as the worst. In contrast, score function S_V ranked this alternative as the worst. On the other hand, almost all functions placed alternative A_6 fourth.



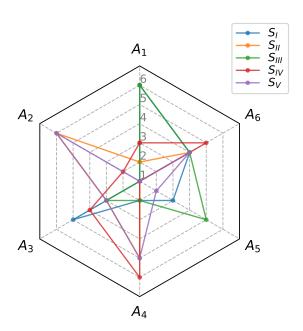
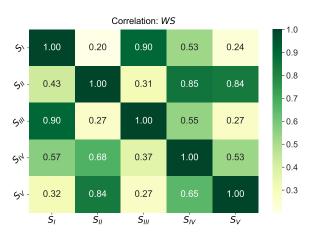


Fig. 1. Radar chart of MARCOS rankings.

Fig. 2. Weighted Spearman's correlation heatmap of MARCOS rankings for small decision matrix.

Additionally, the rankings were compared using the WS rank similarity coefficient, which as well is presented as a correlation matrix in the form of a heatmap as Figure 3. This coefficient shows which pair of compared rankings are not symmetrical, meaning that rankings are not identical neither the change in position is between exactly the same alternatives. As it can be seen once again, the pair S_I and S_{III} and pair S_{II} and S_V are characterized by a high degree of similarity. Moreover, comparing S_{II} and S_{IV} where S_{II} is treated as yields high similarity.



To better visualize differences between presented score functions, rankings obtained by execution of the MARCOS method were compared using similarity coefficients. The first coefficient, namely Weighted Spearman's correlation coefficient, is presented in Figure 2 as a correlation matrix in the form of a heatmap. This coefficient shows high similarities of rankings, which resulted through execution of score functions S_{II} and S_V . The previous examination showed that those two functions behave rather similarly, resulting in crisp values of IFS. The next pair of functions that are quite similar is S_{III} and S_I .

Fig. 3. WS correlation heatmap of MARCOS rankings for small decision matrix.

B. Big example

The next example that was taken into consideration consists of twenty alternatives and six criteria, which created the decision matrix presented in Table VIII. This approach provides a view of how specific score functions behave in larger multicriteria decision problems.

Similarly to the smaller example, for a decision matrix consisting of IFS, crisp matrix was calculated using score

 TABLE VIII

 BIG DECISION MATRIX REPRESENTED BY INTUITIONISTIC FUZZY SETS.

A_i	C_1	C_2	C_3	C_4	C_5	C_6
712	(μ, u)	(μ, u)				
A_1	(0.09095,0.71850)	(0.13774,0.49164)	(0.20716,0.00628)	(0.06600,0.90771)	(0.93253,0.01122)	(0.17497,0.50594)
A_2	(0.19634, 0.76889)	(0.05385,0.74241)	(0.07135, 0.15414)	(0.44855, 0.19833)	(0.15521,0.30392)	(0.41031,0.09120)
A_3	(0.02148,0.09223)	(0.37886,0.44763)	(0.20687, 0.35686)	(0.11724, 0.55262)	(0.15676, 0.80864)	(0.28854,0.00747)
A_4	(0.80399,0.04354)	(0.10795, 0.17483)	(0.80161,0.11037)	(0.49469,0.30475)	(0.18160,0.29511)	(0.02199, 0.37757)
A_5	(0.59997,0.06593)	(0.52386, 0.23834)	(0.25322,0.03477)	(0.86408, 0.08472)	(0.14660, 0.03157)	(0.35019,0.03213)
A_6	(0.17915,0.01657)	(0.35069,0.17044)	(0.23634, 0.71446)	(0.71924, 0.26769)	(0.16976, 0.73265)	(0.13227, 0.62576)
A_7	(0.39394,0.41539)	(0.59632,0.02533)	(0.28756, 0.53541)	(0.03969, 0.82749)	(0.44033,0.44300)	(0.18513, 0.47813)
A_8	(0.09366, 0.55802)	(0.78447,0.18155)	(0.15418, 0.51586)	(0.51218,0.19691)	(0.54950,0.29227)	(0.78470,0.19383)
A_9	(0.20754, 0.11127)	(0.02822, 0.77145)	(0.11259, 0.43723)	(0.10478, 0.83100)	(0.43970,0.01513)	(0.36435, 0.54747)
A_{10}	(0.53145,0.38788)	(0.51920, 0.27552)	(0.47281, 0.39902)	(0.29417, 0.18583)	(0.44656, 0.32535)	(0.56993, 0.28041)
A_{11}	(0.41454, 0.55076)	(0.60336,0.17182)	(0.25771,0.20216)	(0.43993,0.40186)	(0.34019,0.46780)	(0.28601, 0.21097)
A_{12}	(0.00178, 0.27447)	(0.21928, 0.08916)	(0.22282, 0.15183)	(0.52120,0.09153)	(0.10013, 0.27936)	(0.44117, 0.20162)
A_{13}	(0.57693,0.40495)	(0.07804, 0.58253)	(0.54650,0.08093)	(0.67845, 0.08846)	(0.19737, 0.36442)	(0.72426,0.00283)
A_{14}	(0.03320,0.27491)	(0.17390,0.71133)	(0.78701,0.18097)	(0.15871, 0.82272)	(0.68930,0.04541)	(0.16032, 0.47469)
A_{15}	(0.14737, 0.32855)	(0.62481, 0.01155)	(0.34158, 0.62958)	(0.61442,0.01856)	(0.85899,0.08471)	(0.29505, 0.22883)
A_{16}	(0.46437, 0.49039)	(0.46334, 0.19572)	(0.20792, 0.70879)	(0.08940, 0.36645)	(0.00819, 0.58278)	(0.04862, 0.11211)
A_{17}	(0.17621, 0.05731)	(0.39896, 0.25935)	(0.54071,0.45444)	(0.11264, 0.77514)	(0.70301,0.02073)	(0.40974, 0.52313)
A_{18}	(0.03117,0.46166)	(0.48374,0.35310)	(0.09871, 0.32470)	(0.15036,0.08845)	(0.25587, 0.41844)	(0.03895, 0.57066)
A_{19}	(0.37151,0.06187)	(0.29695, 0.33688)	(0.03437, 0.25546)	(0.63877,0.03488)	(0.87320,0.08054)	(0.37052,0.44601)
A_{20}	(0.26295, 0.58775)	(0.32911,0.29911)	(0.08203,0.25447)	(0.74980,0.20059)	(0.26421,0.04034)	(0.46111,0.50190)

functions. The resultant matrix with crisp values is presented in Table IX.

TABLE X CRISP BIG DECISION MATRIX CALCULATED WITH S_I score function.

TABLE IX Preferences for BIG decision matrix computed with MARCOS method for S_I - S_V score functions.

A_i	S_{I}	S_{II}	$S_{\rm III}$	$S_{\rm IV}$	$S_{ m V}$
A_1	-0.036549	0.190882	-0.085154	-0.041990	0.451390
A_2	-0.030858	0.146254	-0.109542	0.059565	0.465973
A_3	-0.028761	0.126735	-0.129638	0.081650	0.459555
A_4	0.033628	0.355654	0.068392	0.049577	0.658271
A_5	0.067533	0.500764	0.122542	0.119319	0.763359
A_6	-0.019921	0.272210	-0.038713	-0.046104	0.489817
A_7	-0.019389	0.278088	-0.007072	-0.101019	0.487457
A_8	0.029940	0.462223	0.143382	-0.127860	0.643773
A_9	-0.050486	0.130363	-0.126984	0.004657	0.411298
A_{10}	0.030223	0.483942	0.140541	-0.105493	0.646673
A_{11}	0.013713	0.371025	0.056929	-0.055130	0.590749
A_{12}	0.014285	0.170247	-0.078569	0.216041	0.596157
A_{13}	0.036599	0.465783	0.134680	-0.048441	0.671756
A_{14}	-0.020070	0.282331	-0.011157	-0.077203	0.501384
A_{15}	0.045216	0.480237	0.126290	-0.027890	0.697439
A_{16}	-0.030772	0.129525	-0.105932	0.042165	0.452381
A_{17}	0.005362	0.395817	0.049615	-0.063564	0.574704
A_{18}	-0.033528	0.026981	-0.153698	0.105713	0.449290
A_{19}	0.034387	0.394112	0.073114	0.025449	0.672488
A_{20}	0.005835	0.327543	0.020393	-0.003544	0.576575

The first function, namely S_I yielded results presented in Table X. In this case, the standard deviation is 0.411, which is pretty high considering the range of this function, and it tells us that this specific function provided differentiated results. Moreover, the spread of those values is 1.76, which once again, as in the smaller numerical example, shows that this function makes use of a significant part of the range it operates in.

A_i	C_1	C_2	C_3	C_4	C_5	C_6
A_1	-0.6276	-0.3539	0.2009	-0.8417	0.9213	-0.3310
A_2	-0.5725	-0.6886	-0.0828	0.2502	-0.1487	0.3191
A_3	-0.0708	-0.0688	-0.1500	-0.4354	-0.6519	0.2811
A_4	0.7605	-0.0669	0.6912	0.1899	-0.1135	-0.3556
A_5	0.5340	0.2855	0.2184	0.7794	0.1150	0.3181
A_6	0.1626	0.1802	-0.4781	0.4516	-0.5629	-0.4935
A_7	-0.0215	0.5710	-0.2478	-0.7878	-0.0027	-0.2930
A_8	-0.4644	0.6029	-0.3617	0.3153	0.2572	0.5909
A_9	0.0963	-0.7432	-0.3246	-0.7262	0.4246	-0.1831
A_{10}	0.1436	0.2437	0.0738	0.1083	0.1212	0.2895
A_{11}	-0.1362	0.4315	0.0555	0.0381	-0.1276	0.0750
A_{12}	-0.2727	0.1301	0.0710	0.4297	-0.1792	0.2395
A_{13}	0.1720	-0.5045	0.4656	0.5900	-0.1670	0.7214
A_{14}	-0.2417	-0.5374	0.6060	-0.6640	0.6439	-0.3144
A_{15}	-0.1812	0.6133	-0.2880	0.5959	0.7743	0.0662
A_{16}	-0.0260	0.2676	-0.5009	-0.2770	-0.5746	-0.0635
A_{17}	0.1189	0.1396	0.0863	-0.6625	0.6823	-0.1134
A_{18}	-0.4305	0.1306	-0.2260	0.0619	-0.1626	-0.5317
A_{19}	0.3096	-0.0399	-0.2211	0.6039	0.7927	-0.0755
A_{20}	-0.3248	0.0300	-0.1724	0.5492	0.2239	-0.0408

The results obtained using the S_{II} function are presented in the Table XI. In this case, values are characterized by a standard deviation of 0.29 and a spread of 1.16. Because this function operates in the same interval as S_I , namely [-1,1], they can be easily compared. And just as in the small numerical example, here too, the function S_{II} achieves smaller values of spread and standard deviation.

TABLE XI CRISP BIG DECISION MATRIX CALCULATED WITH S_{II} score function.

A_i	C_1	C_2	C_3	C_4	C_5	C_6
A_0	-0.0460	-0.0445	0.2022	0.0421	0.9319	0.0135
A_1	0.1696	-0.0974	-0.0480	0.3785	-0.0092	0.3648
A_2	-0.0603	0.3012	0.0512	-0.0652	0.1288	0.2833
A_3	0.7974	-0.0174	0.7919	0.4336	0.0272	-0.2047
A_4	0.5779	0.4672	0.2285	0.8597	0.1207	0.3303
A_5	0.1658	0.2691	0.2012	0.7157	0.0983	-0.0191
A_6	0.3147	0.5867	0.1928	-0.0702	0.3887	0.0241
A_7	-0.1007	0.7783	-0.0160	0.4549	0.5033	0.7805
A_8	0.1317	-0.1263	-0.0842	0.0514	0.4315	0.3161
A_9	0.5002	0.4626	0.4217	0.1975	0.3724	0.5280
A_{10}	0.3954	0.5647	0.1485	0.3763	0.2504	0.1799
A_{11}	-0.1969	0.1576	0.1279	0.4858	-0.0732	0.3691
A_{12}	0.5696	-0.1197	0.5163	0.6578	0.0377	0.7235
A_{13}	-0.1570	0.0923	0.7812	0.1434	0.6773	-0.0129
A_{14}	-0.0248	0.6206	0.3234	0.6076	0.8542	0.1861
A_{15}	0.4422	0.3966	0.1489	-0.1100	-0.2302	-0.0455
A_{16}	0.1323	0.3103	0.5385	0.0256	0.6973	0.3746
A_{17}	-0.2030	0.4261	-0.0885	0.0830	0.1196	-0.1838
A_{18}	0.3365	0.1736	-0.1470	0.6274	0.8695	0.2887
A_{19}	0.1752	0.2179	-0.0868	0.7399	0.2362	0.4425

The score function S_{III} yielded values presented in Table XII. This function operates in a different range than the two previous. Considering the operative range of this function, the standard deviation value of 0.35 and spread of 1.39 are definitely high values. Results similar to those obtained in the small numerical example show that this function is stable and, at the same time, uses a large part of the interval in which it operates, providing relatively different values for the different alternatives.

TABLE XII CRISP BIG DECISION MATRIX CALCULATED WITH S_{III} score function.

A_i	C_1	C_2	C_3	C_4	C_5	C_6
0	-0.3636	-0.2934	-0.1893	-0.4010	0.8988	-0.2375
1	-0.2055	-0.4192	-0.3930	0.1728	-0.2672	0.1155
2	-0.4678	0.0683	-0.1897	-0.3241	-0.2649	-0.0672
3	0.7060	-0.3381	0.7024	0.2420	-0.2276	-0.4670
4	0.3999	0.2858	-0.1202	0.7961	-0.2801	0.0253
5	-0.2313	0.0260	-0.1455	0.5789	-0.2454	-0.3016
6	0.0909	0.3945	-0.0687	-0.4405	0.1605	-0.2223
7	-0.3595	0.6767	-0.2687	0.2683	0.3243	0.6770
8	-0.1887	-0.4577	-0.3311	-0.3428	0.1595	0.0465
9	0.2972	0.2788	0.2092	-0.0587	0.1698	0.3549
10	0.1218	0.4050	-0.1134	0.1599	0.0103	-0.0710
11	-0.4973	-0.1711	-0.1658	0.2818	-0.3498	0.1618
12	0.3654	-0.3829	0.3197	0.5177	-0.2039	0.5864
13	-0.4502	-0.2391	0.6805	-0.2619	0.5339	-0.2595
14	-0.2789	0.4372	0.0124	0.4216	0.7885	-0.0574
15	0.1966	0.1950	-0.1881	-0.3659	-0.4877	-0.4271
16	-0.2357	0.0984	0.3111	-0.3310	0.5545	0.1146
17	-0.4532	0.2256	-0.3519	-0.2745	-0.1162	-0.4416
18	0.0573	-0.0546	-0.4484	0.4582	0.8098	0.0558
19	-0.1056	-0.0063	-0.3770	0.6247	-0.1037	0.1917

Table XIII presents results obtained using function S_{IV} . The calculated spread of values, being around 1.32, similar to the functions S_I and S_{III} shows significant use of the range in which this function operates. Moreover, the standard deviation value of about 0.36 is close to the value obtained by the function S_{III} , which might indicate that those functions might yield similar results.

TABLE XIII CRISP BIG DECISION MATRIX CALCULATED WITH S_{IV} score function.

A_i	C_1	C_2	C_3	C_4	C_5	C_6
0	0.2142	-0.0559	-0.6798	0.4606	0.4156	0.0214
1	0.4478	0.1944	-0.6618	-0.0297	-0.3113	-0.2477
2	-0.8294	0.2397	-0.1544	0.0048	0.4481	-0.5560
3	0.2713	-0.5758	0.3680	0.1992	-0.2849	-0.4007
4	-0.0012	0.1433	-0.5680	0.4232	-0.7327	-0.4265
5	-0.7064	-0.2183	0.4262	0.4804	0.3536	0.1370
6	0.2140	-0.0675	0.2345	0.3008	0.3250	-0.0051
7	-0.0225	0.4490	0.0051	0.0636	0.2627	0.4678
8	-0.5218	0.1995	-0.1753	0.4037	-0.3178	0.3677
9	0.3790	0.1921	0.3077	-0.2800	0.1579	0.2755
10	0.4480	0.1628	-0.3102	0.2627	0.2120	-0.2545
11	-0.5856	-0.5373	-0.4380	-0.0809	-0.4308	-0.0358
12	0.4728	-0.0092	-0.0589	0.1504	-0.1573	0.0906
13	-0.5378	0.3278	0.4520	0.4721	0.1021	-0.0475
14	-0.2861	-0.0455	0.4567	-0.0505	0.4155	-0.2142
15	0.4321	-0.0114	0.3751	-0.3162	-0.1135	-0.7589
16	-0.6497	-0.0125	0.4927	0.3317	0.0856	0.3993
17	-0.2608	0.2553	-0.3649	-0.6418	0.0115	-0.0856
18	-0.3499	-0.0493	-0.5653	0.0105	0.4306	0.2248
19	0.2761	-0.0577	-0.4952	0.4256	-0.5432	0.4445

Table XIV presents values calculated using function S_V . The standard deviation of calculated values is 0.21, whereas the spread value is 0.88. This function operates in the same range as functions S_I and S_{II} , which makes it the worst in diversifying values in comparison to those two. Even though a small standard deviation and spread characterize those values, this function might be useful when such values are expected.

TABLE XIV CRISP BIG DECISION MATRIX CALCULATED WITH S_V score function.

A_i	C_1	C_2	C_3	C_4	C_5	C_6
0	0.1862	0.3231	0.6004	0.0791	0.9607	0.3345
1	0.2137	0.1557	0.4586	0.6251	0.4256	0.6596
2	0.4646	0.4656	0.4250	0.2823	0.1741	0.6405
3	0.8802	0.4666	0.8456	0.5950	0.4432	0.3222
4	0.7670	0.6428	0.6092	0.8897	0.5575	0.6590
5	0.5813	0.5901	0.2609	0.7258	0.2186	0.2533
6	0.4893	0.7855	0.3761	0.1061	0.4987	0.3535
7	0.2678	0.8015	0.3192	0.6576	0.6286	0.7954
8	0.5481	0.1284	0.3377	0.1369	0.7123	0.4084
9	0.5718	0.6218	0.5369	0.5542	0.5606	0.6448
10	0.4319	0.7158	0.5278	0.5190	0.4362	0.5375
11	0.3637	0.5651	0.5355	0.7148	0.4104	0.6198
12	0.5860	0.2478	0.7328	0.7950	0.4165	0.8607
13	0.3791	0.2313	0.8030	0.1680	0.8219	0.3428
14	0.4094	0.8066	0.3560	0.7979	0.8871	0.5331
15	0.4870	0.6338	0.2496	0.3615	0.2127	0.4683
16	0.5595	0.5698	0.5431	0.1687	0.8411	0.4433
17	0.2848	0.5653	0.3870	0.5310	0.4187	0.2341
18	0.6548	0.4800	0.3895	0.8019	0.8963	0.4623
19	0.3376	0.5150	0.4138	0.7746	0.6119	0.4796

The rankings obtained using the MARCOS method are grouped in the barplot shown in Figure 4. As can be seen, the obtained rankings differ significantly from each other, highlighting how important it is to choose an appropriate score function. Additionally, it can be seen that on the podium of the ranking, the functions S_{II} , S_{III} , and S_V behave similarly. Still, in the further positions, significant discrepancies appear.

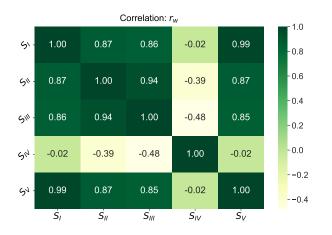


Fig. 5. Weighted Spearman's correlation heatmap of MARCOS rankings for big decision matrix.

The correlations of the rankings obtained from the big decision matrix data are shown in Figures 5 and 6 using heatmaps. The former, describing values for the weighted Spearman's correlation coefficient, shows high correlation values for all scoring functions, excluding the S_{IV} function. When it was used, the rankings calculated using the MARCOS method were significantly different.

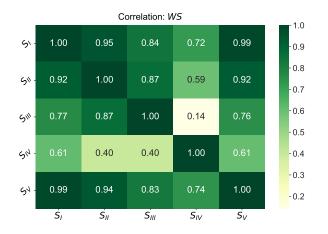


Fig. 6. WS correlation heatmap of MARCOS rankings for big decision matrix

In contrast, the similarity of the rankings calculated with WS coefficient, shown in Figure 6 also indicated that the S_{IV} function showed the least consistent results with the other techniques used. The strongest similarity of rankings could be observed for the pair of methods S_I and S_V , which is 0.99. In contrast, the lowest consistency of rankings is 0.14 for the pair of methods S_{III} and S_{IV} . It indicates a significant discrepancy, which confirms the importance of the influence of the used scoring function on the obtained results.

C. WS comparison

To generalize the results and examine the similarities between the scoring functions used, 1000 simulations were performed for randomly generated decision matrices. Each of the generated matrices was subjected to the techniques described earlier, and the resulting crisp matrices were used in a multi-criteria analysis using the MARCOS method. The figures and tables below show the values calculated for the similarities of the obtained rankings. The WS rank similarity coefficient determined their consistency.

Visualizations for selected scoring functions are presented below, together with tables describing selected statistics of the obtained data. Figure 7 shows the distribution of ranking similarity values for the simulations performed. The rankings obtained using the $S_{\rm II}$ function were compared with the other methods. It is worth noting that for the functions $S_{\rm III}$ and $S_{\rm V}$, the similarity of the rankings was high and concentrated in a narrow area. It shows a high consistency in how IFS conversions to crisp values are performed, which translates into high reproducibility in evaluating alternatives.

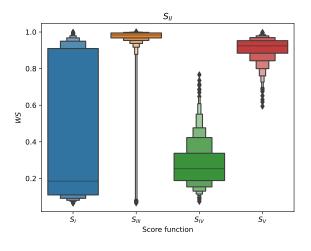


Fig. 7. Distribution of rankings similarity values using the $S_{\rm II}$ score function.

Table XV contains the statistics calculated from the simulations, including a comparison of the performance of the function $S_{\rm I}$ with the others. The variance and standard deviation were most negligible for the functions $S_{\rm III}$ and $S_{\rm V}$, as confirmed by the data shown in Figure 7. On the other hand, the most significant standard deviation (0.384707) was seen when comparing the results obtained using the $S_{\rm I}$ function.

TABLE XV Statistics for results obtained using the $S_{\rm II}$ score function.

S_i	Standard deviation	Variance	Mean
S_{I}	0.384707	0.147999	0.446851
$S_{\rm III}$	0.098412	0.009685	0.967222
S_{IV}	0.118155	0.013961	0.276578
$S_{\rm V}$	0.060845	0.003702	0.910355

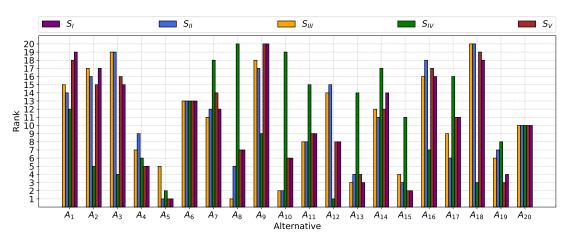


Fig. 4. MARCOS rankings for big decision matrix for S_I - S_V score functions.

Figure 8 shows the similarity distribution obtained for the comparison of results using the $S_{\rm III}$ function together with the other functions. As in the previous case, the highest similarity of rankings was observed for the functions $S_{\rm II}$ and $S_{\rm V}$. In addition, the lowest consistency of results was noted when comparing with the ranking obtained using $S_{\rm IV}$.

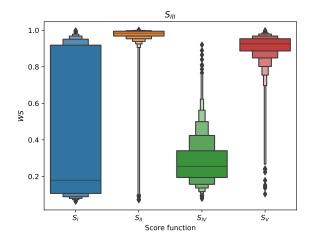
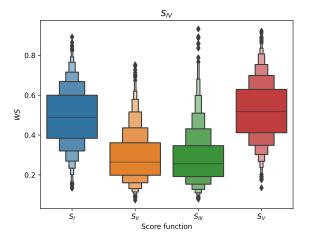


Fig. 8. Distribution of rankings similarity values using the $S_{\rm III}$ score function.

TABLE XVI STATISTICS FOR RESULTS OBTAINED USING THE $S_{\rm HI}$ score function.

S_i	Standard deviation	Variance	Mean
S_{II}	0.391059	0.152927	0.452026
S_{II}	0.098297	0.009662	0.968083
S_{IV}	0.125697	0.015800	0.281541
S_{V}	0.093363	0.008717	0.906154

A visualization of the similarity distribution of the rankings obtained using the scoring function S_{IV} compared to the other functions is shown in Figure 9. It can be seen that none of the techniques used gives a strong rankings correlation. Instead, it causes the results obtained to vary, making it essential to bear in mind that the choice of scoring function directly impacts the results obtained.



data obtained when comparing the rankings of the functions $S_{\rm III}$ with the others. The highest average ranking similarity value is 0.968083 for the method pair $S_{\rm III}$ and $S_{\rm II}$. It demonstrates the high consistency of the results and shows that the two functions can be used interchangeably without much effect on the rankings in most cases.

In turn, Table XVI describes the statistical values for the

Fig. 9. Distribution of rankings similarity values using the $S_{\rm IV}$ score function.

The determined statistical features for the comparisons of the function S_{IV} with the others are listed in Table XVII. The average correlation value oscillates between a value of 0.284001 for feature $S_{\rm III}$ and 0.522578 for feature $S_{\rm V}.$ It shows that a low consistency of results is obtained regardless of the technique used. On the other hand, the standard deviation for the similarity of the rankings is similar across all functions. It shows that the quality of the correlation is also affected by the input data, which can improve or worsen the consistency of the rankings.

TABLE XVII Statistics for results obtained using the $S_{\rm IV}$ score function.

S_i	Standard deviation	Variance	Mean
S_{I}	0.145812	0.021261	0.491630
$S_{\rm II}$	0.124837	0.015584	0.289953
$S_{\rm III}$	0.133394	0.017794	0.284001
$S_{\rm V}$	0.147851	0.021860	0.522578

The similarity results for the other functions used in the study, i.e., $S_{\rm I}$ and $S_{\rm V}$, show that the first function gives a similar similarity of rankings to the other techniques. Still, it oscillates within a value of 0.4, indicating low consistency of the results. On the other hand, the second function shows a high similarity of performance together with the functions $S_{\rm II}$ and $S_{\rm III}$. It confirms the trend of possible interchangeable use of these functions in converting IFS to crisp values in multicriteria problems.

V. CONCLUSION

Decision-making appears in many parts of life, so developing this particular branch of technology is crucial. However, often in decision-making problems, the problem of uncertainty and fuzzy values arise, which makes standard methods inapplicable. For this reason, it is worth taking a closer look at the possibilities of defuzzification of such problems.

In the study carried out, five score functions that allow achieving crisp values from intuitionistic fuzzy sets were compared. Each of the functions allows obtaining completely different values, which ultimately will significantly influence the results of the rankings. The study showed that in the smaller problem, the functions S_I and S_{III} should be preferred in decision-making problems because of the high distinction of individual values between them. However, the more extensive problem and simulations for 1000 decision matrices showed that functions $S_{\rm II}$, $S_{\rm III}$ and $S_{\rm V}$ proved to be the most coherent techniques. Moreover, those functions presented high similarity in resulting rankings rendering them equally capable.

In future studies, it would be meaningful to address this issue regarding the reference ranking to compare the performance of the used score functions to indicate their reliability in practical problems. In addition, it would verify the usefulness and effectiveness of presented score functions in the decisionmaking process, which is obligatory to obtain credible results. In addition, future research would need to consider real decision-making tasks.

ACKNOWLEDGMENT

The work was supported by the National Science Centre, Decision number 2021/41/B/HS4/01296 (B.K. and W.S).

REFERENCES

- [1] X. Gandibleux, "Multiple criteria optimization: state of the art annotated bibliographic surveys," 2006.
- C. C. Aggarwal and S. Y. Philip, "A survey of uncertain data algorithms [2] and applications," IEEE Transactions on knowledge and data engineering, vol. 21, no. 5, pp. 609-623, 2008.
- [3] P. Ziemba, J. Jankowski, and J. Wątróbski, "Online comparison system with certain and uncertain criteria based on multi-criteria decision analysis method," in International Conference on Computational Collective Intelligence. Springer, 2017, pp. 579-589.
- [4] D. Dubois and H. Prade, "Membership functions," in Fuzzy Approaches for Soft Computing and Approximate Reasoning: Theories and Applications. Springer, 2021, pp. 5-20.
- [5] V. Torra, "Hesitant fuzzy sets," International journal of intelligent systems, vol. 25, no. 6, pp. 529-539, 2010.
- [6] T. Senapati and R. R. Yager, "Fermatean fuzzy sets," Journal of Ambient Intelligence and Humanized Computing, vol. 11, no. 2, pp. 663-674, 2020.
- [7] B. C. Cuong and V. Kreinovich, "Picture fuzzy sets," Journal of
- [7] D. C. Coong and Y. Richnovich, Trethe 1022y sets, *Journal of Computer Science and Cybernetics*, vol. 30, no. 4, pp. 409–420, 2014.
 [8] P. Ejegwa, S. Akowe, P. Otene, and J. Ikyule, "An overview on intuitionistic fuzzy sets," *Int. J. Sci. Technol. Res*, vol. 3, no. 3, pp. 145, 2014. 142-145, 2014.
- [9] M. J. Khan, M. I. Ali, P. Kumam, W. Kumam, M. Aslam, and J. C. R. Alcantud, "Improved generalized dissimilarity measure-based vikor method for pythagorean fuzzy sets," International Journal of Intelligent Systems, vol. 37, no. 3, pp. 1807-1845, 2022.
- [10] P. Thakur, B. Kizielewicz, N. Gandotra, A. Shekhovtsov, N. Saini, A. B. Saeid, and W. Sałabun, "A new entropy measurement for the analysis of uncertain data in mcda problems using intuitionistic fuzzy sets and copras method," Axioms, vol. 10, no. 4, p. 335, 2021.
- [11] S. Faizi, W. Sałabun, T. Rashid, S. Zafar, and J. Wątróbski, "Intuitionistic fuzzy sets in multi-criteria group decision making problems using the characteristic objects method," Symmetry, vol. 12, no. 9, p. 1382, 2020.
- [12] B. Gohain, R. Chutia, P. Dutta, and S. Gogoi, "Two new similarity measures for intuitionistic fuzzy sets and its various applications, International Journal of Intelligent Systems, 2022.
- [13] E. Szmidt, J. Kacprzyk, and P. Bujnowski, "Three term attribute description of atanassov's intuitionistic fuzzy sets as a basis of attribute selection," in 2021 IEEE International Conference on Fuzzy Systems (*FUZZ-IEEE*). IEEE, 2021, pp. 1–6. [14] N. X. Thao, "Some new entropies and divergence measures of intu-
- itionistic fuzzy sets based on archimedean t-conorm and application in supplier selection," Soft Computing, vol. 25, no. 7, pp. 5791-5805, 2021.
- S. Al-Humairi, A. Hizami, A. Zaidan, B. Zaidan, H. Alsattar, S. Qahtan, [15] O. Albahri, M. Talal, A. Alamoodi, and R. Mohammed, "Towards sustainable transportation: A pavement strategy selection based on the extension of dual-hesitant fuzzy multi-criteria decision-making methods." IEEE Transactions on Fuzzy Systems, 2022.
- [16] S.-M. Chen and J.-M. Tan, "Handling multicriteria fuzzy decisionmaking problems based on vague set theory," Fuzzy sets and systems, vol. 67, no. 2, pp. 163-172, 1994.
- [17] T.-Y. Chen, "A comparative analysis of score functions for multiple criteria decision making in intuitionistic fuzzy settings," Information Sciences, vol. 181, no. 17, pp. 3652-3676, 2011
- [18] S. K. De, R. Biswas, and A. R. Roy, "An application of intuitionistic fuzzy sets in medical diagnosis," Fuzzy sets and Systems, vol. 117, no. 2, pp. 209-213, 2001.
- [19] A. Kharal, "Homeopathic drug selection using intuitionistic fuzzy sets," Homeopathy, vol. 98, no. 1, pp. 35-39, 2009.
- [20] Ž. Stević, D. Pamučar, A. Puška, and P. Chatterjee, "Sustainable supplier selection in healthcare industries using a new mcdm method: Measurement of alternatives and ranking according to compromise solution (marcos)," Computers & Industrial Engineering, vol. 140, p. 106231, 2020.
- [21] P. Ziemba, "Selection of electric vehicles for the needs of sustainable transport under conditions of uncertainty-a comparative study on fuzzy mcda methods," Energies, vol. 14, no. 22, p. 7786, 2021.