

A novel iterative approach to determining compromise rankings

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Abstract—In many cases involving multi-criteria decision-making, we need compromise solutions. This is a crucial aspect due to the specific characteristics of decision problems. However, the proposed compromise approaches are often complex to verify to what extent they are reliable. Therefore, this paper proposes a new iterative approach based on decision option evaluations from selected multi-criteria decision-making methods, i.e., TOPSIS, VIKOR, and SPOTIS. The obtained results have high similarity among each other, which was measured by Spearman’s weighted correlation coefficient and WS ranking similarity coefficient. Furthermore, the proposed approach showed high efficiency and adaptability of the generated results.

I. INTRODUCTION

MANY works propose new approaches related to the topic of multi-criteria decision making, for which conflicting results are obtained compared with classical approaches. In these methods there are a number of parameters that have a great influence on the final evaluations of decision options [1]. Sałabun et al. investigated the effect of normalization and weight selection methods on the final evaluations in selected methods [2]. Ghaleb et al. evaluated the TOPSIS, AHP, and VIKOR methods based on five selected factors [3]. Baydas et al. used the stock price from the finance domain problem as a tool to compare the MCDM methods [4].

Therefore, researchers focus on improving their methods based on compromise solutions. Liao et al. presented an improved SMAA-CO method designed to determine compromise solutions [5]. The approach they presented is characterized by its ability to reflect uncertain preferences of decision-makers with conflicting criteria. Stevic et al. proposed a new compromise MARCOS approach based on ideal solutions and the utility function [6]. The method performed well in a balanced provider selection problem in the healthcare domain, where it remained stable with a large dataset.

A popular approach is voting algorithms to establish a compromise ranking based on reference rankings. One such example is the use of Borda’s approach and Copeland’s method for evaluating the performance of electric vehicle batteries [7]. Approaches based on the Borda and Copeland algorithms have also been used to create a recommendation system based on e-commerce [8] and to select online services [9].

However, these methods mainly focus on rankings in which it is difficult to observe slight differences between the obtained preferences of decision options [10]. Moreover, using methods that suffer from the problem of rank reversal paradox, it is difficult to determine the most compromise ranking.

In this paper, we propose a new method for determining compromise rankings based on reference evaluations. The evaluations of the decision alternatives obtained by the selected methods are used to determine the compromise rankings. The obtained ratings are formed into a decision matrix, where the types of attributes for the newly formed matrix depend on the ranking method. Three multi-criteria decision-making methods such as TOPSIS, VIKOR, and SPOTIS were used in this study.

The main contribution of our work is the concept of a new approach to verifying compromise solutions. Due to the number of existing MCDM approaches, it is necessary to develop compromise methods to produce consistent evaluations and rankings. Therefore, our proposed concept of a compromise approach aims to show the possibility of iteratively determining a compromise ranking based on reference rankings obtained for the original decision matrix.

The remainder of the paper is organized as follows. Section 2 presents descriptions of the TOPSIS, VIKOR, and SPOTIS methods and ranking similarity coefficients. Section 3 presents research on the proposed compromise approach. Finally, Section 4 presents a summary and outlines future research directions.

II. PRELIMINARIES

A. The TOPSIS Method

Technique of Order Preference Similarity (TOPSIS) is based on the ideal solution approach for solving multi-criteria decision problems [11]. The approach evaluates decision alternatives for the distance from a positive ideal solution and a negative ideal solution. TOPSIS is a continuously evolving method capable of solving problems involving uncertain environments [12]. Its basic version can be presented in the following steps:

Step 1. This step includes the determination of a normalized decision matrix. In the vector method used in the presented

work, the square root of all values is calculated. Equations used for profit (1) and cost criteria (2) are presented below.

$$r_{ij} = \frac{x_{ij} - \min(x_j)}{\max(x_j) - \min(x_j)} \quad (1)$$

$$r_{ij} = \frac{\max(x_j) - x_{ij}}{\max(x_j) - \min(x_j)} \quad (2)$$

Step 2. Computation of the weighted values of the normalized decision matrix v_{ij} in accordance with Equation (3).

$$v_{ij} = w_i r_{ij} \quad (3)$$

Step 3. Calculation of the positive ideal solution (PIS) and negative anti-ideal solution (NIS) vectors. The PIS represented by (4) contains the maximum values for every criterion, and the NIS expressed by (5) includes minimum values. It is unnecessary to split the criteria into benefit and cost since the normalization procedure converted the cost criteria to profit criteria.

$$v_j^+ = \{v_1^+, v_2^+, \dots, v_n^+\} = \left\{ \max_j(v_{ij}) \right\} \quad (4)$$

$$v_j^- = \{v_1^-, v_2^-, \dots, v_n^-\} = \left\{ \min_j(v_{ij}) \right\} \quad (5)$$

Step 4. Calculation of distance from PIS by (6) and NIS, by (7) for every alternative under consideration [2].

$$D_i^+ = \sqrt{\sum_{j=1}^n (v_{ij} - v_j^+)^2} \quad (6)$$

$$D_i^- = \sqrt{\sum_{j=1}^n (v_{ij} - v_j^-)^2} \quad (7)$$

Step 5. Calculation of the result for every considered variant by (8). The score ranges from 0 to 1. An alternative that has a preference value closer to 1 is better.

$$C_i = \frac{D_i^-}{D_i^- + D_i^+} \quad (8)$$

B. The VIKOR Method

ViseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR) is a method based on the compromise approach, which evaluates alternatives with conflicting types of criteria [13]. The compromise solution in this method is considered to be the solution that is closest to the ideal. On the other hand, compromise is achieved through mutual concessions. This method can be presented in the following steps:

Step 1. Determination of the best f_j^* and the worst f_j^- value for the function of individual criteria. For benefit criteria, formula (9) is performed

$$f_j^* = \max_i f_{ij}, \quad f_j^- = \min_i f_{ij} \quad (9)$$

while for the cost criteria, formula presented below is applied (10).

$$f_j^* = \min_i f_{ij}, \quad f_j^- = \max_i f_{ij} \quad (10)$$

Step 2. Calculation of S_i and R_i using formulas (11) and (12).

$$S_i = \sum_{j=1}^n w_j \frac{(f_j^* - f_{ij})}{(f_j^* - f_j^-)} \quad (11)$$

$$R_i = \max_j w_j \frac{(f_j^* - f_{ij})}{(f_j^* - f_j^-)} \quad (12)$$

Step 3. Calculation of Q_i using Equation (13).

$$Q_i = v \frac{(S_i - S^*)}{(S^- - S^*)} + (1 - v) \frac{(R_i - R^*)}{(R^- - R^*)} \quad (13)$$

where

$$S^* = \min_i S_i, \quad S^- = \max_i S_i$$

$$R^* = \min_i R_i, \quad R^- = \max_i R_i$$

v means the weight adopted for the strategy of "most criteria". In the calculations in this study v equal to 0.5 was set.

Step 4. Graded variants in S , R and Q are ordering ascending. The outcome is provided in 3 ranked lists.

Step 5. The consensus is suggested regarding the good advantage and acceptable stability concerning vectors received in the preceding stage. The most favourable variant has the lowest value and is the leader of the ranking Q .

C. The SPOTIS Method

Stable Preference Ordering Towards Ideal Solution (SPOTIS) is a newly developed approach robust to the reverse ranking paradox. The approach is based on evaluation concerning the distance from the Ideal Solution Point of the given decision alternatives [14]. Additionally, it allows the introduction of an expert point against which the alternatives are evaluated. It can be presented as follows:

Step 1. Determine the normalized distances computed for Ideal Solution Point as Equation (14) demonstrates.

$$d_{ij}(A_i, S_j^*) = \frac{|S_{ij} - S_j^*|}{|S_j^{max} - S_j^{min}|} \quad (14)$$

Step 2. Calculate weighted normalized distances represented by $d(A_i, S^*) \in [0, 1]$ as Equation (15) shows.

$$d(A_i, S^*) = \sum_{j=1}^N w_j d_{ij}(A_i, S_j^*) \quad (15)$$

The resulting ranking calculated according to $d(A_i, S^*)$ values. Alternatives with lower values of $d(A_i, S^*)$ receive better positions in the ranking. The technique presented in this paper can be demonstrated by an alternative algorithm, included in [14]. However, the authors of this work provided and applied this option because it seems straightforward. Moreover, both versions supply identical outcomes.

D. Similarity coefficients

Ranking similarity coefficients such as the weighted Spearman coefficient r_w and the ranking similarity coefficient WS were used to examine the similarity of the rankings [15], [16]. In the case of the weighted Spearman coefficient, it was designed to reflect the most relevant alternatives that were rated the best. In contrast, the ranking similarity coefficient WS is an asymmetric ranking similarity coefficient where the alternatives at the top are given the most consideration. These coefficients can be represented by the formulas (16) and (17).

$$r_w = 1 - \frac{6 \cdot \sum_{i=1}^n (x_i - y_i)^2 ((N - x_i + 1) + (N - y_i + 1))}{n \cdot (n^3 + n^2 - n - 1)} \tag{16}$$

$$WS = 1 - \sum_{i=1}^n \left(2^{-x_i} \frac{|x_i - y_i|}{\max\{|x_i - 1|, |x_i - N|\}} \right) \tag{17}$$

III. STUDY CASE

Taking advantage of compromise ranking methods is an important topic that often arises in multi-criteria decision-making. We can determine the most flexible ranking under multiple conflicting criteria with them. In the following, we propose a new iterative approach based on the preferences of reference MCDM methods. The whole procedure can be described as follows:

Step 1. Evaluation of the formed decision matrix n by MCDM methods

Step 2. Create a decision matrix based on the ratings of the MCDM methods. Criteria types are determined based on the method's ranking type.

Step 3. Return to step 1 until the ranking $i - 1$ and the ranking i of the selected methods are the same, where ranking $i - 1$ denotes the ranking obtained from the evaluations entering the current decision matrix, and ranking i denotes the currently created ranking.

A decision matrix whose values were randomly generated from a uniform distribution $[0, 1]$ was used for the study. It is created from 4 criteria and 10 alternatives. For each of the considered criteria, the same weight was assigned, and it was assumed that the first two criteria are profit type while the last two criteria are cost type. Finally, the created decision matrix is presented using the table I.

TABLE I: Generated decision matrix consisting of alternatives $A_1 - A_{10}$ and criteria $C_1 - C_4$.

A_i	C_1	C_2	C_3	C_4
A_1	0.989490	0.592018	0.818605	0.031060
A_2	0.083740	0.507472	0.378180	0.976184
A_3	0.445502	0.029106	0.196421	0.897797
A_4	0.408798	0.982134	0.428897	0.126954
A_5	0.471495	0.042197	0.154756	0.379112
A_6	0.205953	0.113477	0.537154	0.396152
A_7	0.481553	0.793211	0.541687	0.265333
A_8	0.515628	0.270621	0.392020	0.281762
A_9	0.789527	0.050453	0.277638	0.179735
A_{10}	0.507601	0.545130	0.644899	0.954265

The obtained preferences of the different alternatives for each iteration are shown using the illustration 1, where 1a represents the preference for the TOPSIS approach, 1b represents the preference for the VIKOR approach, and 1c represents the preference for the SPOTIS approach.

For the TOPSIS approach, stabilization of ratings was faster than for the rest of the methods. For the TOPSIS approach, the range relative to the evaluated decision options increased. The first ratings obtained were in the range $[0.33748, 0.65286]$, while the last ratings were $[0, 1]$. Additionally, the spread of the obtained ratings also changed, where in the first iteration, the standard deviation was 0.11151, while in the last iteration, it was 0.354886.

The stabilization of SPOTIS approach ratings was slightly faster. As with the TOPSIS approach, the range of obtained ratings for subsequent iterations in the VIKOR approach increased. For the first iteration, the obtained preference values were in the range $[0.03838, 1]$, while for the last iteration, the values were in the range $[0, 1]$. The change in scatter of these values, on the other hand, is not as prominent as for the TOPSIS approach. Indeed, for the VIKOR approach, a standard deviation value of 0.32609 was obtained for the first iteration and 0.35500 for the last iteration.

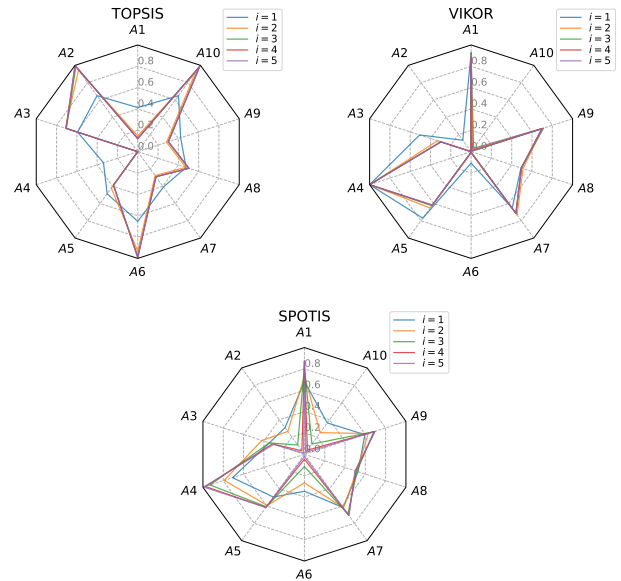


Fig. 1: Obtained preferences for TOPSIS, VIKOR, and SPOTIS methods in subsequent iterations of the proposed approach.

Among the considered methods, the SPOTIS approach had the slowest stabilization of ratings. Ranges of obtained assessments were increasing much slower than in the case of TOPSIS and VIKOR. In addition, the intervals obtained in the last iteration did not reach the limits of the SPOTIS method domain. For the first iteration, the score interval $[0.30921, 0.70877]$ was achieved, while the score interval $[0.01359, 0.99223]$ was achieved for the last iteration. Regarding the

scatter of values, it is similar to the TOPSIS method. In the first iteration, the standard deviation for the scores was 0.14258, while in the last iteration, it was 0.34732.

Using the Figure 3 the obtained rankings for each iteration are shown, where 3a shows the obtained ranking for the TOPSIS method, 3b shows the obtained ranking for the VIKOR method, and 3c shows the obtained ranking for the SPOTIS method.

The TOPSIS method, besides alternative A_4 , also obtained invariant positions in all iterations for two alternatives, i.e., alternative A_3 and alternative A_9 , which obtained fourth and seventh ranking positions, respectively. The only noticeable changes in the ranking positions for this method occurred between iteration one and iteration two.

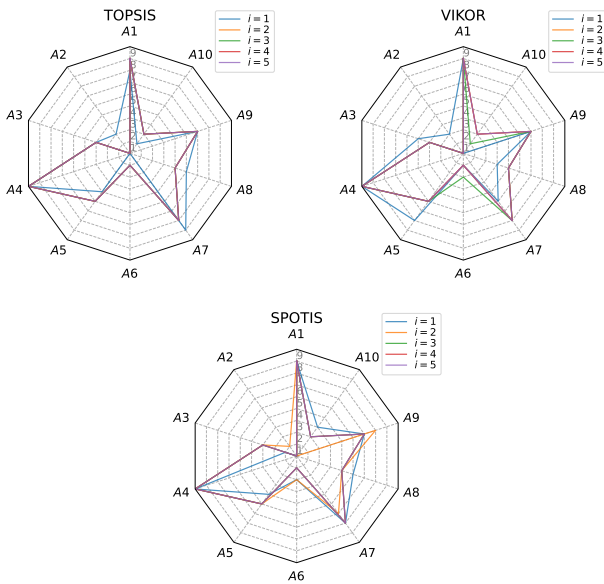


Fig. 3: Obtained rankings for TOPSIS, VIKOR, and SPOTIS methods in subsequent iterations of the proposed approach.

In the case of the VIKOR method, besides alternative A_4 , invariability of position for each iteration is also obtained by alternative A_1 . The alternative A_{10} received the most changes in its ranking position during all iterations, obtaining the first ranking position in the first iteration, obtaining the third-ranking position in the second, fourth, and fifth iterations, and obtaining the second-ranking position in the third iteration.

For the SPOTIS method, in addition to alternative A_4 , alternative A_1 also received an unchanged ranking position of 9. The most significant apparent change in ranking position for this method occurred for alternative A_{10} . In the first iteration, it achieved the fourth-ranking position. The second iteration achieved the first ranking position, and in the rest of the iterations, it achieved the third-ranking position.

A comparison of the rankings from the first and last iteration for the considered TOPSIS, VIKOR, and SPOTIS methods has been shown with the help of Figure 2. Referring to the TOPSIS method, only three alternatives, i.e., A_3 , A_4 , and A_9 are in

the same positions in the first and last rankings. The biggest difference is seen for alternative A_2 , where in the case of the first iteration, it reached the third-ranking position, while in the last iteration, it reached the first ranking position. In the case of the method, the same positions in both considered rankings were achieved by four alternatives A_1 , A_4 , A_6 , and A_9 . Only two alternatives differed by one ranking position. In contrast, four alternatives differed by two ranking positions. The SPOTIS method has the same ranking positions from the last iteration relative to the first iteration. Alternatives A_1 , A_2 , A_4 , A_7 , and A_9 remained in their ranking positions. Only one alternative differed by two positions among the rankings considered.

The Spearman's weighted correlation coefficient values for the individual rankings from iterations $i-1$ and i are presented using the table II. The TOPSIS method achieved the fastest, equally similar rankings, where as early as the third iteration, the value of r_w was 1.0. However, the VIKOR method only achieved this value in the fifth iteration, while the SPOTIS method achieved it in the fourth iteration. The most significant ranking differences are seen in the SPOTIS method, where in the first iteration, the value of r_w between the rankings was 0.85013, and in the second iteration, it was 0.93278.

TABLE II: Spearman weighted coefficient r_w values for $Rank_{i-1}$ and $Rank_i$.

Iteration	Methods		
	TOPSIS	VIKOR	SPOTIS
$i = 2$	0.92286	0.87107	0.85013
$i = 3$	1.0	0.98126	0.93278
$i = 4$	1.0	0.98126	1.0
$i = 5$	1.0	1.0	1.0

The values of the ranking similarity coefficient for the individual rankings from iterations $i-1$ and i are shown using the table III. The differences between the methods show a higher number of iterations in reaching the upper bound value of this coefficient. For example, the TOPSIS method reached the value of 1.0 in 3 iterations, the VIKOR method in 5 iterations, and the SPOTIS method in 4 iterations. Interestingly, the SPOTIS method obtained a lower value of the WS coefficient in iteration 3 than the value of the WS coefficient in iteration 2.

TABLE III: Rank similarity coefficient values WS for $Rank_{i-1}$ and $Rank_i$.

Iteration	Methods		
	TOPSIS	VIKOR	SPOTIS
$i = 2$	0.86730	0.82914	0.83945
$i = 3$	1.0	0.95089	0.83792
$i = 4$	1.0	0.95089	1.0
$i = 5$	1.0	1.0	1.0

IV. CONCLUSION

One of the essential issues of considering multiple MCDM methods evaluating the same set of decision alternatives is

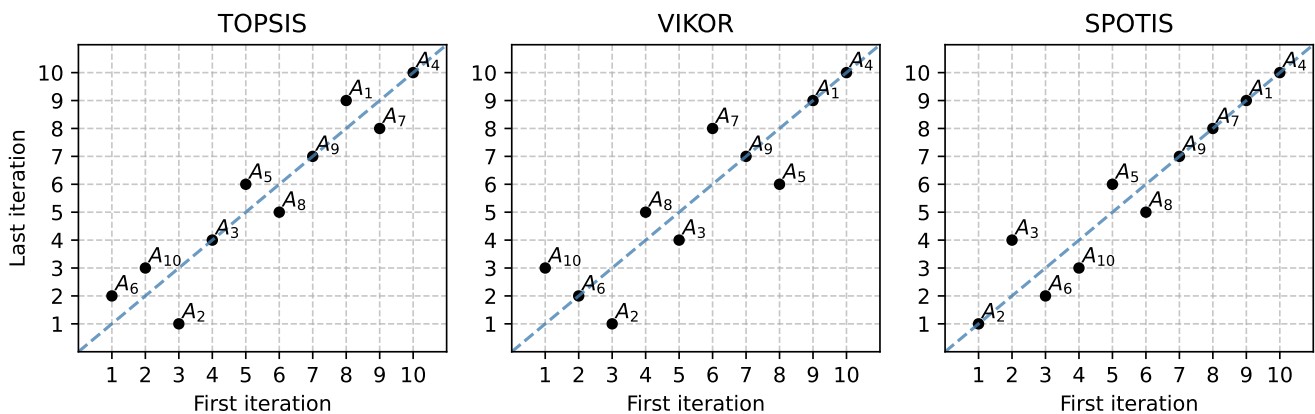


Fig. 2: Visualization of ranking similarity from first iteration and last iteration for TOPSIS, VIKOR and SPOTIS approaches.

the compromise. This paper presents a new iterative approach related to the output values of TOPSIS, VIKOR, and SPOTIS methods to create successive decision matrices. The proposed approach is very flexible and guarantees the reliability of the results, as shown by the tests performed. All considered methods obtained a compromise ranking in a minimal number of iterations. Additionally, each of the considered methods in the last iteration obtained the same ranking as the other methods. High reliability of the obtained results is also guaranteed by the used similarity coefficients of the rankings r_w and WS , which reached the upper values of their ranges.

In future research, it would be helpful to consider the effect of the number of criteria on the number of iterations needed to reach a compromise for the methods. Additionally, it would be helpful to investigate the effect of different weight allocation methods on the final values of the proposed approach. Finally, to adapt the approach in future research, it would need to be verified with the reference ranking. Future research would also need to include a broader validation of the proposed approach with more MCDM methods. In addition, an aspect related to the impact of the number of alternatives and criteria on the final results would need to be addressed.

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