Outage Performance of Full-Duplex Dual-hop Relaying System With Energy Harvesting Using Fountain Codes

Nguyen Hong Viet^{*}, Dang The Hung[†], Nguyen Thi Thanh Hoai[‡], Nguyen Hoang Anh[†], Nguyen Hoanh Viet[†], and Gia-Thinh Vo[§], * Air Force Officer's College, Nha Trang, Khanh Hoa, Vietnam [†] Telecommunications University, Nha Trang, Khanh Hoa, Vietnam [‡] Le Quy Don High School, Ha Tinh, Vietnam [§] Thu Dau Mot University, Binh Duong, Vietnam

Corresponding author: danghung8384@gmail.com

Abstract—In this paper, we study a dual-hop relaying system exploiting fountain codes, where both the source node and the destination node have one antenna, while the relay node is a device with two antennas. Moreover, the relay node is a energy limited device, so it must harvest the radio frequency signals to assist the source node to transmit its data to the destination node with full-duplex mode which increases double channel capacity. We analyze the system performance through the derived exact outage probability expression over Rayleigh fading channels. Finally, Monte-Carlo simulations are performed to confirm the theoretical results of the proposed model.

Index Terms—Fountain codes, dual-hop relaying system, full-duplex, energy harvesting.

I. INTRODUCTION

Recently, the mobile users enhance connectivity quality, widen the coverage area, data transmission, and process information among devices to devices together, such as Internet of Thing (IoT) networks, smart cities, automation driving car [1]. However, multiple service platforms are supported which create many challenges as high data rates, low latency, high energy, and spectrum efficiency. To solve these problem, many new technologies have been researched and proposed to be applied in wireless communications, e.g. dual-hop/multi-hop systems [2], [3], non-orthogonal multiple access [4], short packet communications [5], massive MIMO [6], cognitive radio [7], energy harvesting (EH) from radio frequency signals [8], and full duplex (FH) transmission [9]. In particular, EH considered as a potential solution which can prolong the operation time of the devices, it is very suitable for machine with limited power sources such as sensor nodes, multihop relaying networks [10]. Moreover, FH help to improve spectrum efficiency due to the devices with FD can receive and transmit signals in band at the same time which increases double channel capacity when comparing with traditional halfduplex mode [11]. Besides, self-interference cancellation (SIC) scheme used in FD devices to improve the bit error rate (BER) performance. However, it is difficult to design and implement antennas to completely eliminate interference in FD systems which always exist the residual self-interference (RSI) level to increase the noise floor in the wireless system.

There are some literatures combining the advantages of FD and EH in relaying communications which have attracted much attention of researchers to enhance the reliability, capacity, delay time, energy efficiency and coverage area [12]- [16]. Particularly, the authors in [12] have considered a dual-hop two-way full-duplex network with EH at the relay applying amplify-and-forward (AF) scheme to forward the transmission data and spatial modulation at the source. The results showed that the BER performance and the spectrum efficiency with FD mode reach better when comparing with the normal halfduplex one. In [13], the authors studied the power allocation methods in a two-way FD system under impact of RSI via the exact expressions of optimal power allocation parameter and presented that the performance of studied protocol outperforms the regular power allocation one. The FD-EH relay system performance through the derived exact closed-form expression such as the symbol error probability has also analyzed in [14]. In the work [15], the authors considered a FD-EH relay system via the exact expressions such as outage probability (OP) and symbol error rate (SER) over cascaded Rayleigh fading channels. The results present that the performances for OP and SER which are significant lower when comparing with ones over Rayleigh fading channels. The work in [16] evaluated the outage probability in FD-EH relaying networks in which the destination has multi-antennas using selection combining (SC) or maximal ratio combining (MRC) technique to improve the quality for received signals.

Fountain codes (FC) can adapt with different channel conditions which recently has attracted the attention of many researchers [17]- [21]. Different with fixed-rate codes, a transmitter in FC do not require the knowledge of channel state information (CSI) before transmitting encoded packets, so that the receivers can maintain flexible decoding performance. The Fountain encoder can generate an unlimited number of encoded fountain packets, and then sends these encoded packets to the pre-determined receivers. If the receivers obtain enough fountain packets, they can decode the original data [18]- [21].

To the best of our knowledge, there are no literatures

that refer to exploit FC in FD-EH dual-hop relaying system. Therefore, in this paper we carry out research the FD dual-hop system performance using FC with EH based TS method at the relay node to enhance the power consumption, capacity level and reliable transmission. We reach the exact expression for the outage probability (OP) over Rayleigh fading channels. These mathematical expressions will be verified through Monte Carlo simulations.

The structure of our paper is arranged as follows: In section II, we represent the proposed model. In section III, we examine the system performance. Section IV gives simulation results and theoretical analysis ones. Finally, we conclude the paper in section V.

II. SYSTEM MODEL



Fig. 1. System model of the proposed protocol.

The considered model shows in Figure 1, in which a source (S) node wants to send its data to a destination (D) node with assistance a relay (R) node by decode-and-forward (DF) technique over Rayleigh fading channels. We also assume that S and D are one antenna with half-duplex (HD) action mode while R is equipped two antennas with full-duplex (FD) approach in the same frequency. Moreover, the relay node is a energy-limited device, and R must harvest the radio frequency signals energy from S through time-switching (TS) method. Therefore, the information transmission from S to D which split two time slots in which energy harvesting portion is αT , and data transmission part is $(1 - \alpha)T$, where $\alpha \in (0, 1)$ is TS factor, and T is the transmission period from S to D in the block signal.

Using the Fountain code, S divides its data into K packets. Each packet will be appropriately encoded and then sent sequentially to D in orthogonal time slots. We assume that the considered system is limited in delay time, and S will finish its transmission after transmitting N_{max} encoded packets to D. In order perform to decode the original data, D must exactly obtain at least H data packet, where $H = (1 + \varepsilon) K, H \le N_{\text{max}}$ and ε is a constant depending on the code design [19]. Let denote N_{D} as the number of encoded packets that D can receive after N_{max} time slots. If $N_{\text{D}} \ge H$, D can successfully the original information, and contrary the outage occurs if $N_{\text{D}} < H$. We denote that $h_{\rm SR}$, $h_{\rm RD}$, and $h_{\rm RR}$ are the channel factors between links S \rightarrow R, R \rightarrow D, and the residual selfinterference at R, respectively. We also assume the channels of devices as Rayleigh fading, so that the channel gains $\gamma_{\rm SR} = |h_{\rm SR}|^2$, $\gamma_{\rm RD} = |h_{\rm RD}|^2$, and $\gamma_{\rm RR} = |h_{\rm RR}|^2$, have exponential distribution with characteristic parameters $\lambda_{\rm SR}$, $\lambda_{\rm RD}$, and $\lambda_{\rm RR}$, whose cumulative distribution function (CDF) and probability density function (PDF) are expressed as

$$F_X(x) = 1 - \exp\left(-\omega x\right),\tag{1}$$

$$f_X(x) = \omega \exp\left(-\omega x\right),\tag{2}$$

where $X \in \{\gamma_{SR}, \gamma_{RD}, \gamma_{RR}\}$ and $\omega \in \{\lambda_{SR}, \lambda_{RD}, \lambda_{RR}\}$. The energy harvesting at the relay node is given as

$$E_h^R = \eta \alpha T P_{\rm S} \gamma_{\rm SR},\tag{3}$$

where $\eta \in (0, 1)$ is the conversion energy efficiency, $P_{\rm S}$ is the transmission power of the source.

The transmission power at the relay node is given by

$$P_{\rm R} = \frac{\eta \alpha T P_{\rm S} \gamma_{\rm SR}}{(1-\alpha) T} = \mu P_{\rm S} \gamma_{\rm SR},\tag{4}$$

where $\mu = \eta \alpha / (1 - \alpha)$.

Next, we have the received signal at R and D, respectively as

$$y_{\rm R} = h_{\rm SR} \sqrt{P_{\rm S}} x_{\rm S} + h_{\rm RR} \sqrt{P_{\rm R}} x_{\rm R} + n_{\rm R}, \qquad (5)$$

$$y_{\rm D} = h_{\rm RD} \sqrt{P_{\rm R} x_{\rm R}} + n_{\rm D},\tag{6}$$

where $x_{\rm S}$ and $x_{\rm R}$ are transmission signals of the source node and the relay node, $n_{\rm R}$ and $n_{\rm D}$ are additive white Gaussian noise (AWGN) with the zero mean and variance N_0 (also assume that all devices of the system which have the same variance N_0). From (5), we can determine the power of signal self-interference at R as

$$E\{P_{R}\gamma_{RR}\} = \mu P_{S}E\{\gamma_{RR}\gamma_{SR}\},\qquad(7)$$

where $E\{.\}$ is the statistical expectation operator.

We assume that the relay node is equipped with selfinterference cancellation (SIC). However, in practice the signal self-interference at R will not completely eliminate due to the imperfection of hardware that will still have a part, called residual self-interference (RSI), it is a Gaussian random variable which has variance as

$$\delta_{\rm RSI} = \kappa \mu P_{\rm S},\tag{8}$$

where κ is the SIC efficiency at R.

The signal-to-interference-plus-noise-ratio (SINR) at R and D is shown as

$$\Psi_{\rm SR} = \frac{\gamma_{\rm SR} P_{\rm S}}{\delta_{\rm RSI} + N_0}, \Psi_{\rm RD} = \frac{\gamma_{\rm RD} P_{\rm R}}{N_0},\tag{9}$$

For the DF relaying protocol, the end-to-end of the system depends on the lowest SINR level. The equivalent SINR of the system is as follows:

$$\Psi_{e2e} = \min\left(\Psi_{SR}, \Psi_{RD}\right). \tag{10}$$

Next, the outage probability (OP) value at D is presented as

$$OP = Pr(N_D < H | N_{max}).$$
(11)

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III. OUTAGE PERFORMANCE

Each encoded packet will be decoded successfully if the end-to-end data rate is larger than a predetermined target rate which is given by $C_{\rm th}$. Otherwise, the encoded packet will not be obtained correctly. Therefore, the probability that the destination cannot accurately decode one encoded packet as

$$\rho_{\rm D} = \Pr\left[(1 - \alpha) \log_2 \left(1 + \Psi_{\rm e2e} \right) < C_{\rm th} \right]$$
$$= \Pr\left(\Psi_{\rm e2e} < \gamma_{\rm th} \right), \tag{12}$$

where $C_{\rm th}$ is the desired transmission rate of the system, and $\gamma_{\rm th} = 2^{C_{\rm th}/(1-\alpha)} - 1$.

We have the probability of a encoded packet which is decoded correctly as $Pr(\Psi_{e2e} > \gamma_{th}) = 1 - \rho_D$. Combining (9), (10), and (12), ρ_D is rewritten as follows:

$$\rho_{\rm D} = \Pr\left[\min\left(\Psi_{\rm SR}, \Psi_{\rm RD}\right) < \gamma_{\rm th}\right]. \tag{13}$$

We can implement (13) as below

$$\rho_{\rm D} = 1 - \Pr\left(\Psi_{\rm SR} > \gamma_{\rm th}, \Psi_{\rm RD} > \gamma_{\rm th}\right). \tag{14}$$

Let $X = |h_{\rm SR}|^2$ and $Y = |h_{\rm RD}|^2$, we can rewrite (14) follow as

$$\rho_{\rm D} = 1 - \Pr\left(\frac{P_{\rm S}}{\delta_{\rm RSI} + N_0} X > \gamma_{\rm th}, \frac{\mu P_{\rm S}}{N_0} XY > \gamma_{\rm th}\right)$$
$$= 1 - \underbrace{\Pr\left(X > \gamma_{\rm th} \frac{(\delta_{\rm RSI} + N_0)}{P_{\rm S}}, XY > \gamma_{\rm th} \frac{N_0}{\mu P_{\rm S}}\right)}_{I(a,b)}.$$
(15)

To calculate ρ_D , the first we need to find out integral of I(a,b). Applying the conditional probability theorem [22], we have

$$I(a,b) = \Pr(X > a, XY > b)$$

=
$$\int_{a}^{\infty} \left[1 - F_Y\left(\frac{b}{x}\right)\right] f_X(x) \, dx, \qquad (16)$$

where $a = \frac{\gamma_{\text{th}}(\delta_{\text{RSI}}+N_0)}{P_{\text{S}}}$, $b = \frac{\gamma_{\text{th}}}{\Delta \mu}$, and $\Delta = P_{\text{S}}/N_0$ is transmit signal-to-noise ratio (SNR).

From (1) and (2), the integral I(a, b) can be rewritten as

$$I(a,b) = \lambda_{\rm SR} \int_{a}^{\infty} \exp\left(-\lambda_{\rm SR} x\right) \exp\left(-\lambda_{\rm RD} \frac{b}{x}\right) dx. \quad (17)$$

With the help of [22, eq. (1.211.1)], we have

$$\exp\left(-\lambda_{\rm RD}\frac{b}{x}\right) = \sum_{m=0}^{\infty} \frac{\left(-\lambda_{\rm RD}b\right)^m}{m!} x^{-m}.$$
 (18)

Substituting [18] into [17], which yields

$$I(a,b) = \lambda_{\rm SR} \sum_{m=0}^{\infty} \frac{(-\lambda_{\rm RD}b)^m}{m!} \times \int_a^{\infty} x^{-m} \exp\left(-\lambda_{\rm SR}x\right) dx.$$
(19)

Applying [22, eq. (3.351.2)], we have

$$\int_{a}^{\infty} x^{-m} \exp\left(-\lambda_{\rm SR} x\right) dx = \left(\lambda_{\rm SR}\right)^{m-1} \Gamma\left(1-m, \lambda_{\rm SR} a\right),$$
(20)

where $\Gamma(\cdot, \cdot)$ is the upper incomplete Gamma function [22, eq. (8.350.2)]. Replacing [20] into [19], we obtain the integral of I(a, b) as

$$I(a,b) = \lambda_{\rm SR} \sum_{m=0}^{\infty} \frac{\left(-\lambda_{\rm RD}b\right)^m}{m!} \times \left(\lambda_{\rm SR}\right)^{m-1} \Gamma\left(1-m,\lambda_{\rm SR}a\right).$$
(21)

Similarly, replacing [21] into [15], we get the exact probability of ρ_D as follows:

$$\rho_{\rm D} = 1 - \lambda_{\rm SR} \sum_{m=0}^{\infty} \frac{\left(-\lambda_{\rm RD}b\right)^m}{m!} \times \left(\lambda_{\rm SR}\right)^{m-1} \Gamma\left(1 - m, \lambda_{\rm SR}a\right).$$
(22)

As mentioned in (11), the exact expression of OP can be provided as

$$OP = Pr (N_{\rm D} < H | N_{\rm max})$$

= $\sum_{N_{\rm D}=0}^{H-1} C_{N_{\rm max}}^{N_{\rm D}} \rho_{\rm D}^{N_{\rm D}} (1 - \rho_{\rm D})^{N_{\rm max} - N_{\rm D}}.$ (23)

We observed from (23) that the possible values of $N_{\rm D}$ are 0 to H-1, and $C_{N_{\rm max}}^{N_{\rm D}}$ are cases for each value of $N_{\rm D}$.

IV. SIMULATION RESULTS

In this section, we perform Monte-Carlo simulations based Matlab software which validate the math expressions in Section III and evaluate the system's features. In all simulations, we set the parameters as the number of required Fountain packets that need to be reached to restore the original data (H), target rate $(C_{\rm th})$, path loss exponent factor (β) , the energy conversion efficiency of energy harvesting phase (η) , total block time (T) by 3, 1, 3, 1, 1, and the coordinates of the node in a two-dimensional plane Oxy as source, relay, and destination node are (0,0), (0.5,0), and (1,0), respectively. For all of the figures, we use 5×10^5 trials over Rayleigh fading channels, and the simulation results and theoretical ones are notated by Sim and Theory.

Figure 2 shows the value of OP in terms of Δ ($\Delta = P_{\rm S}/N_0$) (dB). In this figure, the SIC efficiency at the relay node ($\kappa = -30$ (dB)), maximum number of time slots for transmission from S to D ($N_{\rm max} = 5$). Observing at this figure, we can see that OP value decreases when increasing the transmit SNR (Δ). Because transmit power level of the source increases which increases the ability receiving of enough required encoded packets to reconstruct the original information at the destination. The second remarkable point in Figure 2 is the value of OP increases when decreasing the time duration of energy harvesting α , for example, the value of $\alpha = 0.1$, OP is the highest when comparing with OP at the values of $\alpha \in (0.2, 0.3)$, and OP is the smallest when $\alpha = 0.3$.



Fig. 2. OP plots versus Δ (dB) when $\kappa=-30$ (dB), $N_{\rm max}=5.$



Fig. 4. OP plots versus Δ (dB) when $\alpha=0.2,~N_{\rm max}=5.$



Fig. 3. OP plots versus Δ (dB) when $\kappa = -30$ (dB), $\alpha = 0.3$.

Fig. 5. OP plots versus α when $\kappa = -30$ (dB), $\Delta = 30$ (dB).



In figure 3 shows OP based on Δ in dB when $\kappa = -30$ (dB) and $\alpha = 0.3$. We can see that the value of OP decreases when increasing the levels of Δ . Moreover, OP significantly decreases as increasing the value of $N_{\rm max}$. However, when $N_{\rm max}$ increases which makes to increase the system delay time.

In figure 4 investigates the impact of the SIC level on the performance of the considered protocol, i.e., $\kappa = -10$ dB, $\kappa = -20$ dB, $\kappa = -30$ dB, and $\alpha = 0.2$, $N_{\rm max} = 5$. It can seen that OP value significantly decreases when increasing SIC level and the transmit SNR (Δ). Therefore, RSI has seriously affect on the system performance.

In figure 5, we present OP as a function of α when $\kappa = -30$ dB, and $\Delta = 30$ dB. We can realize that OP decreases when increasing the value of $N_{\rm max}$, and OP is the lowest at the value $\alpha = 0.5$. Moreover, when increasing the maximum number of time slots $(N_{\rm max})$ that means the delay time of the system increases which is an important measurement standard in wireless communication systems.

V. CONCLUSIONS

In this paper, we evaluated the system performance of the considered FD dual-hop relaying protocol using Fountain codes, in which the relay node collects the RF signals from the source, in terms of outage probability expression over Rayleigh fading channels. The analytical results show that the performance of the SIC, time switching ratio factor α , choosing a suitable transmission power level, and the number of maximum time slots for data transmission are important parameters that determine the main performance of the system and should be carefully considered when designing and setting the system.

References

- S. Malathy, et al. "Routing constraints in the device-to-device communication for beyond IoT 5G networks: a review," *Wirel. Netw.*, vol. 27, no. 5, pp. 3207-3231, 2021.
- [2] M. O. Hasna and M.-S. Alouini, "End-to-end performance of transmission systems with relays over Rayleigh-fading channels," *IEEE Trans. Wirel. Commun.*, vol. 2, no. 6, pp. 1126-1131, 2003.
- [3] M. Hasna and M.-S. Alouini, "A performance study of dual-hop transmissions with fixed gain relays," *IEEE Trans. Wirel. Commun.*, vol. 3, no. 6, pp. 1963-1968, Nov. 2004.
- [4] L. Dai, B. Wang, Z. Ding, Z. Wang, S. Chen, and L. Hanzo, "A survey of non-orthogonal multiple access for 5G," *IEEE Commun. Surv. Tutor.*, vol. 20, no. 3, pp. 2294-2323, 2018.

- [5] G. Durisi, T. Koch, and P. Popovski, "Toward massive, ultrareliable, and low-latency wireless communication with short packets," *Proceedings of the IEEE*, vol. 104, no. 9, pp. 1711-1726, 2016.
- [6] D. Borges, P. Montezuma, R. Dinis, and M. Beko, "Massive MIMO techniques for 5G and beyond-opportunities and challenges," *Electronics*, vol. 10, no. 14, pp. 1667, 2021.
- [7] J. Mitola and G. Q. Maguire, "Cognitive radio: Making software radios more personal," *IEEE Pers. Commun.*, vol. 6, no. 4, pp. 13-18, Aug. 1999.
- [8] T. X. Doan, T. M. Hoang, T. Q. Duong, and H. Q. Ngo, "Energy harvesting-based D2D networks in the presence of interference and ambient RF sources," *IEEE Access*, Mar. 2017.
- [9] Z. Zhongshan, C. Xiaomeng, L. Keping, A. V. Vasilakos, and L. Hanzo, "Full duplex techniques for 5G networks: self-interference cancellation, protocol design, and relay selection," *IEEE Communications Magazine*, vol. 53, no. 5, pp. 128-137, 2015.
 [10] S. Ulukus et al., "Energy harvesting wireless communications: A review
- [10] S. Ulukus et al., "Energy harvesting wireless communications: A review of recent advances," *IEEE Journal on Selected Areas in Communications*, vol. PP, no. 99, pp. 1-1, 2015.
- [11] V.-D. Nguyen, T. Q. Duong, H. D. Tuan, O.-S. Shin, and H. V. Poor, "Spectral and energy efficiencies in full-duplex wireless information and power transfer," *IEEE Trans. on Commun.*, vol. 65, no. 5, pp. 2220-2233, 2017.
- [12] A. Koc, I. Altunbas, and E. Basar, "Two-way full-duplex spatial modulation systems with wireless powered AF relaying," *IEEE Wirel. Commun. Lett.*, vol. 7, no. 3, pp. 444-447, 2018.
- [13] Y. Jingrui, L. Xuefang, and Y. Qinghai, "Power allocation of two way full-duplex AF relay under residual self-interference," in *Communications and Information Technologies (ISCIT)*, 2014 14th International Symposium on, 2014, pp. 213-217.
- [14] B. C. Nguyen, T. M. Hoang, and P. T. Tran, "Performance analysis of full-duplex decode-and-forward relay system with energy harvesting over nakagami-*m* fading channels," *International Journal of Electronics and Communications*, vol. 98, pp. 114-122, 2019.
 [15] B. C. Nguyen, X. N. Tran, T. M. Hoang, "Performance analysis of
- [15] B. C. Nguyen, X. N. Tran, T. M. Hoang, "Performance analysis of full-duplex vehicle-to-vehicle relay system over double-rayleigh fading channels," *Mob. Netw. Appl.*, vol. 25, pp. 363-372, 2020.
- [16] P. T. Tin, T. N. Nguyen, D. H. Tran, M. Voznak, V. D. Phan, and S. Chatzinotas, "Performance enhancement for full-duplex relaying with time-switching-based SWIPT in wireless sensors networks," *Sensors*, vol. 21, no. 11, pp. 3847, 2021.
- [17] D. J. C. Mackay, "Fountain codes," *IEEE Proc. Commun.*, vol. 152, pp. 1062-1068, Dec. 2005.
- [18] J. Castura and Y. Mao, "Rateless coding for wireless relay channels," *IEEE Trans. Wirel. Commun.*, vol. 6, no. 5, pp. 1638-1642, May 2007.
- [19] T. T. Duy and H.Y. Kong, "Secondary spectrum access in cognitive radio networks using rateless codes over rayleigh fading channels," *Wirel. Pers. Commun.*, vol. 77, no. 2, pp. 963-978, Jul. 2014.
- [20] X. Wang, W. Chen, and Z. Cao, "A rateless coding based multi-relay cooperative transmission scheme for cognitive radio networks," in *IEEE Globecom, Honolulu*, HI, USA, Nov. 2009, pp. 164-169.
 [21] X. Di, K. Xiong, P. Fan, and H. C. Yang, "Simultaneous wireless
- [21] X. Di, K. Xiong, P. Fan, and H. C. Yang, "Simultaneous wireless information and power transfer in cooperative relay networks with rateless codes," *IEEE Trans. Veh. Technol.*, vol. 66, no. 4, pp. 2981-2996, Apr. 2017.
- [22] D. Zwillinger, Table of integrals, series, and products. Elsevier, 2014.