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Comparative Study: Defuzzification Functions and Their Effect on the Performance of the OFNbee Optimization Algorithm

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***Abstract*—This article explores the pivotal role of defuzzification functions in the operation of the OFNBee algorithm, which employs ordered fuzzy number arithmetic to harness the inherent dynamics within a hive. Defuzzification functions serve the purpose of representing the OFN (Ordered Fuzzy Number) as a real number, while fuzzification functions convert real numbers into OFN representations. By focusing on the defuzzification function, this study investigates its impact on the performance of the OFNBee algorithm. The research demonstrates that tailoring dedicated fuzzification functions for specific optimization problems can yield substantial improvements in algorithmic performance. It is important to note that the overall performance of the algorithm relies on both the fuzzification and defuzzification functions. Consequently, this article provides valuable insights into the effects of the defuzzification function on algorithmic outcomes.**

I. Introduction

THIS article is part of a series of research focused on the issue of fuzzy logic, and more specifically, fuzzy numbers. There are several leading models of fuzzy numbers, which will be discussed later in the text. This article focuses on the OFN model, and more specifically on the specific arithmetic resulting from the use of referral in OFN. In earlier papers [1], issues related to ordered fuzzy numbers and their application were thoroughly discussed. The aim of the current research is to create optimization algorithms based on ordered fuzzy numbers. The algorithm has been presented in many publications [2], [3], [4]. However, this article discusses the impact of the defuzzification function on the operation of this algorithm. An OFNbee algorithm has already been developed that uses the flocking behavior of bees and ordered fuzzy numbers. Due to the fact that specific defuzzyfication and fuzzyfication methods sensitive to direction are needed to move from the set of real numbers to the set of ordered fuzzy numbers and vice versa, their impact on the performance of the algorithm should be investigated. It is intuitively known that the appropriate selection of methods can improve the performance of optimization algorithms based on OFN. Until then [1], [5], the impact of the fuzzyfication function on the results of OFNBee has been examined, while the following text will present the most popular defuzzyfication methods and their impact on the results of the algorithm. Algorithms based on the behavior of insects or animals are an attempt to use natural mechanisms for optimization [6]. However, the difficulty of accurately describing the behavior of living organisms leads to oversimplification. The simplification process, although effective, limits the possibilities of such methods. Therefore, it seems reasonable to use the mechanisms of fuzzy logic, which more naturally describes phenomena occurring in the real world. Given how bee optimization algorithms are evolving and the source of their inspiration, a combination of more natural fuzzy arithmetic with bee algorithms should be considered.

II. Selected elements of fuzzy set theory

To discuss the subject of fuzzy numbers, it is necessary to start with the concept of a fuzzy set. The fuzzy set was introduced in 1965 by Lotfi Zadeh [7], [8], [9], who defined that the fuzzy set A in space X is the set of pairs described:

$$A = \{(x, \mu_A(x) : x \in X)\} \tag{1}$$

gdzie: μ_A is a membership function, assigning to each element $x \in X$ (the assumed space of considerations X) its degree of membership in the set A, where: $\mu_A : X \to [0, 1]$, therefore $\mu_A(x) \in [0, 1]$.
As in the case of classical sets, which are described by a characteristic function, in the case of fuzzy sets, the membership

function is used to describe them [10] [11]. Such a function assigns to each element of the set a real number in the interval [0,1], thus determining the degree of membership of a given element to the set. A fuzzy set must be uniquely described by its membership function, and this is the most important feature of fuzzy sets. In the theoretical model, there are no contraindications for this function to assume any shape. In the literature, however, you can find several basic functions with specific shapes [12]: h

- triangular – described by formula 2, where $a \leq b \leq c$ i and shown in 1:

$$\mu_A(x,a,b,c) = \begin{cases} 0 & , where\ x \leq a \\ \frac{x-a}{b-a} & , where\ a < x \leq b \\ \frac{c-x}{c-b} & , where\ b < x \leq c \\ 0 & , where\ x > c \end{cases} \quad (2)$$

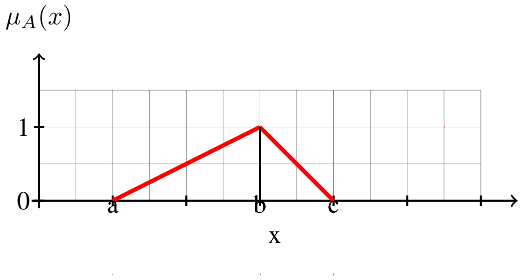

Fig. 1. Triangular

- trapezoidal – described by formula 3, where $a \leq b \leq c \leq d$ i and shown in Figure 2:

$$\mu_A(x,a,b,c,d) = \begin{cases} 0 & , where\ x \leq a \\ \frac{x-a}{b-a} & , gdzie\ a < x \leq b \\ 1 & , where\ b < x \leq c \\ \frac{d-x}{d-c} & , gdzie\ c < x \leq d \\ 0 & , where\ x > d \end{cases} \quad (3)$$

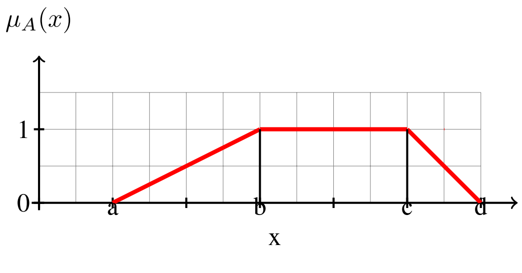

Fig. 2. Trapezoidal

- singleton – described by the formula 4, where x_0 is a parameter defining the location of the singleton, shown in 3:

$$\mu_A(x,x_0) = \begin{cases} 1 & , where\ x = x_0 \\ 0 & , where\ x \neq x \leq x_0 \end{cases} \quad (4)$$

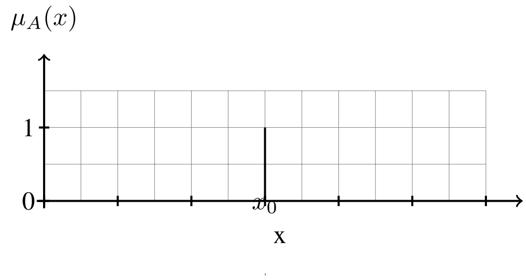

Fig. 3. Singleton

III. Fuzzy number - LR model

In 1978, Dubois and Prade proposed the LR (Left-Right) model, which was supposed to simplify quite complicated arithmetic operations performed on classical fuzzy numbers. We define a fuzzy number A of the LR type as follows: A of the LR type as follows:

$$\mu_A(x) = \begin{cases} L(\frac{m-x}{\alpha}) & , Where\ x \leq m \\ R(\frac{x-m}{\beta}) & , Where\ x \geq m \end{cases} \quad (5)$$

Where:

- m is a real number defined as an average number – $\mu_A(m) = 1$,
- $\alpha > 0$ – ufixed left-hand real number,
- $\beta > 0$ – fixed right-hand real number.

L and R are basis functions that satisfy the following conditions:

- $L(-x) = L(x), R(-x) = R(x)$,
- $L(0) = 1, R(0) = 1$,
- L and R are non-increasing functions on the interval $[0, +\infty)$.

IV. Ordered Fuzzy Numbers

The ordered fuzzy numbers OFN were proposed in 2002 by Witold Kosiński, Piotr Prokopowicz and Dominik Ślęzak. They focused on eliminating the shortcomings of classical fuzzy number algebra. The disadvantages in question are primarily the fact that by performing several operations on given L-R numbers, you can get numbers that are too fuzzy, which may make them less useful. This entails a large computational complexity and the inability to backward chaining. The creators of OFN also set themselves the goal of developing arithmetic, thanks to which it would be possible to perform operations on both triangular and trapezoidal numbers. They proposed a model defined as follows:

Definition 1 [13], [14], [15]
An ordered fuzzy number A is an ordered pair of functions

$$A = (f_A, g_A) \quad (6)$$

where:
$f_A, g_A : [0,1] \longrightarrow R$ are continuous functions. Accordingly, we call the functions $f_A$the increasing part (up), and the function g_A the decreasing part (down) of the ordered fuzzy number. The continuity of both parts shows that their images

are limited by intervals. They are given the names UP and DOWN respectively. To mark the limits (being real numbers) of these intervals, the following notations have been adopted: $UP = (l_A, 1_A^-)$ and $DOWN = (1_A^+, p_A)$.

V. Optimization method using ordered fuzzy numbers

A bee as a single individual shows almost no features that could be considered worth using in optimization. However, the collective work of these insects is very interesting and shows how nature deals with optimization. The communication mechanisms present in the hive allow bees to optimally manage resources and survive. Algorithms based on bee herd behavior use a space-searching mechanism to find nectar. Such algorithms treat the bee as a single solution or treat the found source as a solution. There are also more complicated behavioral adaptations. However, in all cases, the question of how the bees communicate information to each other is overlooked, or this step is reduced to some simple selection condition. The OFNBee method, using the arithmetic of directed fuzzy numbers, allowed to reflect the mechanisms of information transfer that are actually present among bees.

A new OFNBee optimization method was created by combining ordered fuzzy numbers with bee optimization. The use of OFN notation in bee optimization seems to be a natural way to describe the behavioral mechanisms observed in the hive and quoted above. These mechanisms are presented in the new method using dedicated fuzzification operators. The input data is information carried by a single bee (Figure 4), i.e.:

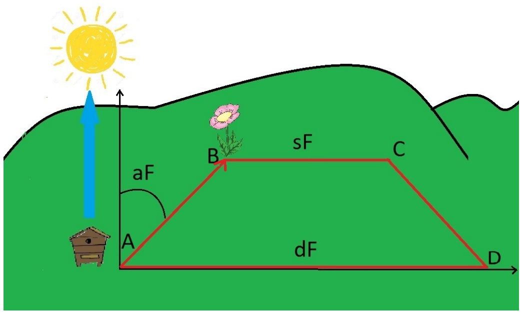

Fig. 4. Graphical interpretation of OFN in OFNBee

- the direction in which the food is located,
- navigation angle acc. sun,
- flight length,
- abundance of food source.

The determination of the directed fuzzy number A is done as follows. First, support(A) is established, which is the base of the trapezoid. Then the rising edge of f(x) is plotted at $90^o - aF$. The other base of the trapezoid is set aside from the point where the intersections of f(x) with y = 1. Finally, it remains to connect the slope g(x) to the two ends of the base of the trapezoid, as shown in Figure 4.

VI. Research methodology

In the case of checking the influence of defuzzyfication functions on the operation of the algorithm, their influence should be presented on the set of testing functions. These functions were selected due to their frequent occurrence in the literature during the verification of optimization algorithms. The OFNbee algorithm includes several configuration options. For correct operation, when running the algorithm, the fuzzyfication and defuzzyfication operator and the size of the population must be specified. In another paper [1] fuzzyfication operators were examined. Each of the two defuzzyfication functions was run with the same fuzzyfication operator and population size. Each run of the algorithm was repeated 30 times for all combinations of fuzzyfication and defuzzyfication operators, and the results are shown below.

VII. Selected OFN defuzzification operators

The new optimization method requires defuzzification operators to work. These functionals allow to represent OFN as a real number. A very important feature of OFN is number referral (direction) – this added value distinguishes ordered numbers from other solutions. That's why it's so important to use direction-sensitive defuzzification operators. The first work on defuzzification operators was undertaken by W. Kosiński [16] [17] [18] [19]. The work was continued by W. Dobrosielski in numerous articles. The functional proposed by W. Dobrosielski is described later in the document. In the operation of the new method, an important feature of the defuzzification functionals is the sensitivity to the direction of OFN, the focus was only on those methods that meet the above condition. These methods are well described in the literature and often used in control models.

A. Golden Ratio defuzzification operator

This section presents the golden ratio defuzzification functional developed by W. Dobrosielski [20]. The method from the fuzzy number A allows to determine the real value from it in accordance with the formula 7. A graphical interpretation of GR is shown in Figure 5[20]:

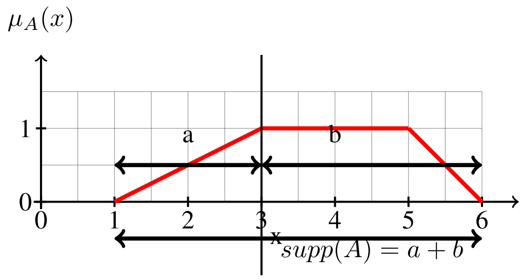

Fig. 5. Graphic interpretation of GR

$$GR = \frac{min(supp(A)) + |supp(A)|}{\Phi} \quad (7)$$
$$where\ \Phi = 1,618033998875\dots$$

where:
GR is the defuzzification operator,
$supp(A)$ means the support of the fuzzy number A in the X universe.

Equation 8 allows to find a crisp (defuzzification) value for an ordered OFN using the GR method[20]:

$$GR(A) = \begin{cases} min(supp(A)) + \frac{|supp(A)|}{\Phi}, \\ \ for\ (A)\ positively\_directed \\ \\ max(supp(A)) - \frac{|supp(A)|}{\Phi}, \\ \ for\ (A)\ negatively\_directed \end{cases} \quad (8)$$

B. Mandala Factor Defuzzification Operator

Another operator used in the new method is the Mandala Factor operator [21] [22] proposed by J. Czerniak. Mandala is a painting composed of colorful grains of sand, arranged by Buddhist monks. The inspiration of grains of sand forming a mandala is at the heart of the Mandala Factor. Given the trapezoidal OFN A shown in Figure 6, fill the contour marked by the sides of the OFN number and the OX axis with virtual grains of sand as in Figure 7. Then a rectangle is built, which is filled with virtual grains of sand. Backfilling consists in pouring sand vertically in columns until it is exhausted. The crisp value of the number is obtained in the place where the last of the columns poured ended (Figure 8). Mathematically, the notation is as follows 9[21]:

$$MF(A) = \begin{cases} c + r & , for\ (A)\ positively \\ c - r & , for\ (A)\ negatively \end{cases} \quad (9)$$

w:

$$r = \frac{1}{d-c}\int_c^d x\,dx - \frac{c}{d-c}\int_c^d dx + \frac{f}{f-e}\int_f^e dx - \frac{1}{f-e}\int_e^f x\,dx + \int_d^e dx$$

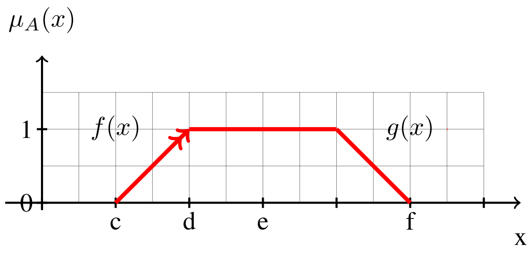

Fig. 6. OFN number A

VIII. SELECTED MATHEMATICAL TESTING FUNCTIONS

For the purpose of the experiment, at this stage, the functions that are most often used as testing were selected. Literature results for various optimization algorithms are also available [23]. Selected mathematical functions:

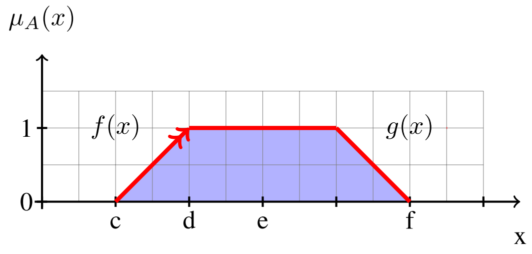

Fig. 7. Visualization of the Mandela Factor operation - step one

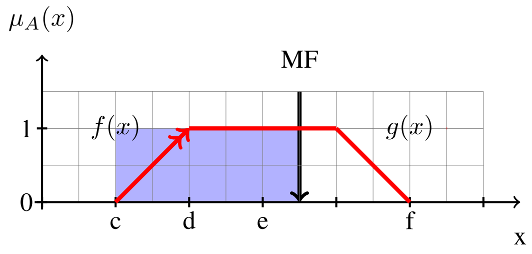

Fig. 8. Visualization of the Mandela Factor operation - step two

- The Sphere function is described by Equation 10.

$$f(x) = \sum_{i=1}^{n} x_i^2 \quad (10)$$

  - Recommended variable values: $-5.12 \leq x_i \leq 5.12$ $i = 1, 2, ..., n$
  - Global minimum: $x = (0, ..., 0), f(x) = 0$
- The Rosenbrock function is described by Equation 11.

$$f(X) = \sum_{i=1}^{d-1}[100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2] \quad (11)$$

  - Recommended variable values: $-2,048 \leq x_i \leq 2,048$ $i = 1, 2, ..., n$
  - Global minimum: $x = (1, ..., 1), f(x) = 0$
- The Rastrigin function is described by Equation 12.

$$f(X) = An + \sum_{i=1}^{n}[x_i^2 - Acos(2\pi x_i)] \quad (12)$$

  - Recommended variable values: $-5,12 \leq x_i \leq 5,12$ $i = 1, 2, ..., n$
  - Global minimum: $x = (0, ..., 0), f(x) = 0$
- The Griewank function is described by Equation 13.

$$f_n(x_1, ..., x_n) = 1 + \frac{1}{4000}\sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n} cos(\frac{x_i}{\sqrt{i}}) \quad (13)$$

  - Recommended variable values: $-600 \leq x_i \leq 600$ $i = 1, 2, ..., n$
  - Global minimum: $x = (0, ..., 0), f(x) = 0$

- The Schwefel function is described by Equation 14.

$$f(x) = \sum_{i=1}^{n} \left[-x_i \sin(\sqrt{|x_i|}) \right] \tag{14}$$

The test area is usually limited to a hypercube $-500 \leq x_i \leq 500,\ i = 1, ..., n$.

  - Recommended variable values: $-500 \leq x_i \leq 500$ $i = 1, 2, ..., n$
  - Global minimum $f(x) = -n \cdot 418.9829;\quad x_i = 420.9687,\ i = 1, ..., n$.

- The Ackley function is described by Equation 15.

$$f(x) = -aexp(-b\sqrt{\frac{1}{d}\sum_{i=1}^{d} x_i^2}) \tag{15}$$

  - Recommended variable values: $a = 20,\ b = 0.2,\ c = 2\pi$.
  - Global minimum: $x = (0, ..., 0), f(x) = 0$

IX. Results

The defuzzification operator is necessary for the new method to work, because it is used to defuzzify the OFN number - to get the crisp value in the form of a real number. Tables 1 compares the results generated by the algorithm for the GR and MF defuzzification functionals. In order to select the appropriate and optimal combination of defuzzification and fuzzification operators, the results obtained by the algorithm for the selected functions should be compared. Each of the test functions was run 30 times and the results are presented in Table I.

As can be seen, the results presented in Table I show that the defuzzification method has an impact on the results of the OFNBee algorithm. Although the method works better and its optimization result is good for both defuzzification functions presented in the article, it can be observed that we get a more accurate result for the MF. The results presented in the table are the average results for each of the mathematical functions and the fuzzification function. The algorithm was run 30 times for each math function and for each defuzzification function. Therefore, it can be considered that the algorithm has a high repeatability. Thanks to the stable operation of the method, it is possible to assess the effect of the defuzzification method on the result.

The table shows that the algorithm achieves the expected value of 0 for the functions Sphere, Rastrigin, Griewank and Ackley. For the functions Schwefel and Rosenbrock, the results are close to the expected 0, but do not reach it. However, you can see that the results for the MF function are closer to the expected value of 0. It follows that this defuzzification function will be suitable for this set of functions. Results presented

TABLE I
Average results for the defuzzification function

Function name	Statistic data	GR	MF	GR	MF
		Trapezoid		Rectangular triangle	
Sphere	SD	0	0	0	0
	AV	0	0	0	0
	SD time	0,564357941	0,026869202	0,043654864	0,035467
	AV time	0,443666667	0,344333333	0,336666667	0,322
Rosenbrock	SD	0,004398444	0,004238917	0,004300068	0,00308
	AV	0,003596884	0,002850368	0,00267976	0,003214
	SD time	1,119912815	0,081095942	0,063162807	0,020525
	AV time	0,592333333	0,454	0,399666667	0,361667
Rastrigin	SD	0	0	0	0
	AV	0	0	0	0
	SD time	0,020899321	0,068043259	0,086659593	0,022816
	AV time	0,356666667	0,406666667	0,420666667	0,336333
Griewank	SD	0	0	0	0
	AV	0	0	0	0
	SD time	0,021890532	0,033314938	0,05339185	0,020457
	AV time	0,369666667	0,392666667	0,361	0,342333
Schwefel	SD	0,015795763	0,041012122	0,015873975	0,059517
	AV	0,003475821	0,011620559	0,004544122	0,01207
	SD time	0,041811014	0,038639209	0,055403432	0,034709
	AV time	0,469666667	0,483666667	0,441666667	0,435667
Ackley	SD	0	0	0	0
	AV	4,44E-16	4,44E-16	4,44E-16	4,44E-16
	SD time	0,020117471	0,030026808	0,088617089	0,020126
	AV time	0,394333333	0,395333333	0,504333333	0,361333

X. Summary

The new hybrid OFNBee method is characterized by the use of three groups of bees, but thanks to the use of OFN notation and the use of the basic feature of OFN, i.e. referral, the algorithm finds a solution much faster when using a smaller population size - fewer bees in groups. Directed fuzzy numbers allow very well to reflect the sense of decision-making occurring in the hive during the dance of real bees. A very important part of OFN are the fuzzyfication and defuzzyfication operators, which become essential elements when using directed fuzzy numbers to solve real-world problems. Defuzzyfication methods are available in the literature, but the specificity of OFN and bee optimization algorithms does not always allow the use of existing functionals. In the experimental part, two existing fuzzyfication methods were used, i.e. Golden Ratio and Mandala Factor. They were created in the AIRLAB Artificial Intelligence and Robotics Research Laboratory at Kazimierz Wielki Univesrity.

A new optimization method using ordered fuzzy numbers is exceptionally good at optimizing mathematical functions. And thanks to the use of ordered fuzzy numbers, it is possible to accurately reproduce the natural mechanisms occurring in the hive. As can be seen in Table I, the accuracy and speed of the method are greatly influenced by the defuzzification functions. It should be noted that these functions are available in the literature, so they are not adapted to the method. However, it can be seen that the MF method returns better results than the GR method. It can therefore be concluded that the use of a dedicated defuzzification method could further increase its efficiency. In-depth research in this area may result in the creation of dedicated defuzzification functions for the OFNBee optimization method.

There are few defuzzification methods dedicated to OFN in the literature. The main reason for this is that such methods must be direction sensitive. Directing is the main characteristic of OFN, therefore the use of defuzzification methods other than those dedicated to OFN could disturb the specificity of OFNBee operation that uses the arithmetic of these numbers. The next stage of research will be the creation of further defuzzification methods, but dedicated to selected optimization problems and comparing them with the methods described in this article.

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