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An Elliptic Intuitionistic Fuzzy Model for Franchisor Selection

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Abstract—**Choosing a successful franchise company in the ever-changing business environment is a challenge for any investor. The work suggests the creation of an optimal algorithm (E-IFFr) for selecting a franchise company using the concepts of index matrices and elliptic intuitionistic fuzzy sets for modeling this variability in the business environment to optimally solve this optimal problem with elliptic intuitionistic fuzzy parameters. The E-IFFr approach involves experts with dynamic ranks performing evaluations by the selection criteria while also taking into consideration the relative importance of the criteria for each investor. The efficacy of the suggested strategy is demonstrated by a numerical example of the best franchisor selection for the courier business. In an optimistic, average, and pessimistic scenario, the investor has three options to choose from.**

I. Introduction

A PROFITABLE company strategy for entering new markets is franchising. An entrepreneur looking for a franchisor must make the best decision possible for the franchise firm. The developed theory of fuzzy logic [34] is a useful tool for working with incomplete or ambiguous information. The concept of fuzzy logic has successfully been utilized in multi-criteria decision-making problems because human judgments are usually not precise when choosing an alternative concerning multiple criteria with different levels of significance. An approach that is suitable for solving multi-criteria decision-making problems characterized by fuzzy criteria is introduced in [18], based on linguistic criteria values. An Analytic Hierarchical Process (AHP) and neural networks are used in the studies [15], [16] to develop fuzzy franchisee selection models.

Real-world situations typically involve some degree of hesitation between membership and non-membership since decision-makers frequently voice their opinions even when they are undecided about them [33]. One of the first generalizations of fuzzy sets, intuitionistic fuzzy sets (IFSs), exhibit some hesitancy. They are a more potent tool for illustrating environmental uncertainty. We have proposed a software application for the resolution of an optimal interval-valued intuitionistic fuzzy multicriteria outsourced decision-making problem in the paper [28]. Additionally, utilizing the concept of index matrices (IMs, [2]), we have developed an intuitionistic fuzzy approach (IFIMFr) and software to choose the most qualified franchise candidates (see [25], [26]). The study presents an integrated approach [19], based on stepwise weight assessment ratio analysis (SWARA) and complex proportional assessment (COPRAS) approaches, for the selection of optimal bioenergy production technology alternatives. The parameters of the contemporary economic environment are rife with uncertainty. For modeling optimal algorithms, the apparatus of intuitionistic fuzzy sets is insufficient. "Extensions" of the IFSs are detailed in the study [32] and contrasted with one another. The authors of [32] have shown that a Hesitant Fuzzy Set can be completely described by IFS [24]. In [32], the authors further demonstrate that interval-valued IFSs (IVIFSs) [6] can represent the Picture fuzzy sets [14], the Cubic set [17], the Neutrosophic fuzzy sets [22] and the Support-intuitionistic fuzzy sets [20]. Two more generalizations of intuitionistic fuzzy sets, known as circular [4], and elliptic IFSs [5], which also generalize interval-valued IFSs, have emerged in recent years.

Our work in this area is focused on creating an extension of the franchisor selection problem that can be used with circular and elliptic IFSs.

Circular and elliptic IFSs, two IFS extensions that are currently increasing in popularity, can reduce accuracy and ambiguity by enclosing the degrees of membership and non-membership in a circle or an ellipse [4], [5]. The paper [12] develops a novel current worth analysis based on interval-valued IF and C-IF sets. An integrated MCDA technique that combines the C-IF AHP and VIKOR is suggested in the work [21]. Circular IFSs are applied in Multi-Criteria Decision Making in [13]. The development of Circular Intuitionistic Fuzzy Multicriteria Analysis (C-IFFr) for Petrol Station Franchisor Selection is presented in [29].

E-IFSs are described as sets with an ellipse indicating the degrees of membership and non-membership for each element of the universe [5]. The Scopus database does not contain any established models for elliptic intuitionistic fuzzy models for franchisor selection with elliptic IF data. Using the toolset of index matrices (IMs) theories and elliptic intuitionistic fuzzy sets (E-IFSs), we enhance C-IFFr [29] in our work and create an elliptic intuitionistic fuzzy algorithm (E-IFFr) for the best

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selection of a franchise organization. The criteria values in this model are determined by experts with dynamic ranks and are expressed as E-IF numbers. The investor decides if each criterion is significant or not. The main contributions of the article are: definition of elliptic IF quads; extending comparison operations and relations on IF pairs to those on elliptic IF quads; extending the definition of three-dimensional IF index matrix (3-D IFIM) and some operations with them to those of 3-D elliptic IFIM (3-D E-IFIM); developed a model for ranking franchisors on elliptical IF data, describing to a greater extent the uncertainty in the economic environment; in this model, the evaluation of franchisors against criteria with weights set by the investor is carried out by dynamic rating experts; an application of E-IFFr in selecting a chain franchisor for the courier business in Bulgaria. The pessimistic, optimistic, and intermediate scenarios are presented to the decision maker for consideration before making a final decision. The advantage of this model is that it can be applied to both regular and elliptical IF data. Another advantage is that it can be easily extended so that it can be applied to multidimensional IF data. A numerical example of the best franchisor choice for the courier industry in Bulgaria serves as an illustration of the effectiveness of the suggested approach. The remainder of our investigation is organized as follows: Section II contains preliminary information for IM concepts and E-IF numbers. In Section III, an optimal E-IF problem with an IM creative solution for selecting a franchisor is provided, and also the actual C-IFFr problem of selecting a franchisor for the courier firm is solved by the software being developed. Future-oriented suggestions are provided in Section V.

II. IMs and Elliptic Intuitionistic Fuzzy Pairs Preliminary

In this section, the preliminaries of E-IF pairs and IMs are introduced. One of the most modern extensions of IFS is the elliptic IFSs, proposed by Atanasov in 2021. They are a powerful tool for representing data fuzziness.

A. Elliptic Intuitionistic Fuzzy Quads (E-IFQs)

An intuitionistic fuzzy pair (IFP) is defined as having the form $\langle a(p), b(p)\rangle$ or $\langle \mu(p), \nu(p)\rangle$: The components of an IFP are $a(p)(\mu(p)), b(p)(\nu(p)) \in [0,1]$ and $a(p)+b(p) = \mu(p)+\nu(p) \leq 1$, respectively. These components are used as an evaluation of some object or process and are interpreted as degrees of membership and non-membership, degrees of validity and non-validity, or degrees of correctness and non-correctness, etc. of a proposition p. Let us define here the elliptic IFQ (E-IFQ) as an object with the following form based on the definition of the E-IFS [5]:

$$\langle a(p), b(p); u, v\rangle = \langle \mu(p), \nu(p); u, v\rangle,$$

where $a(p)+b(p) = \mu(p)+\nu(p) \leq 1$. The "truth degree" and "falsity degree" of the statement p are considered to be $a(p)(\mu(p))$ and $b(p)(\nu(p))$ and $a(p)+b(p) \leq 1$. The ellipse's semi-major and semi-minor axes are $u, v \in [0, \sqrt{2}]$.

Let two E-IFQs be given: $x_{u_1,v_1} = \langle a, b; u_1, v_1\rangle$ and $y_{u_2,v_2} = \langle c, d; u_2, v_2\rangle$. Let us define an operation called $* \in \{\min, \max\}$. The operations over E-IFQs that come after are based on the operations for E-IFSs [5].

$$x_{u_1,v_1} \wedge_* y_{u_2,v_2} = \langle \min(a,c), \max(b,d); *(u_1,u_2), *(v_1,v_2)\rangle;$$
$$x_{u_1,v_1} \vee_* y_{u_2,v_2} = \langle \max(a,c), \min(b,d); *(u_1,u_2), *(v_1,v_2)\rangle;$$
$$x_{u_1,v_1} +_* y_{u_2,v_2} = \langle a+c-a.c, b.d; *(u_1,u_2), *(v_1,v_2)\rangle;$$
$$x_{u_1,v_1} \bullet_* y_{u_2,v_2} = \langle a.c, b+d-b.d; *(u_1,u_2), *(v_1,v_2)\rangle;$$

We suggest the following relation for comparing E-IFQs using a formula for the distance between C-IFSs [8], the relation for comparing two C-IFQs [27], and the distance from the element to the ideal positive alternative [23]:

$$x_{u_1,v_1} \geq_{R^{elliptic}} y_{u_2,v_2} \quad \text{iff } R^{elliptic}_{x_{u_1,v_1}} \leq R^{elliptic}_{y_{u_2,v_2}} \qquad (1)$$

where

$$R^{elliptic}_{x_{u_1,v_1}} = \frac{1}{6}(2-a-b)\big(|\sqrt{2}-u_1| + |\sqrt{2}-v_1| + |1-a|\big)$$

is the distance between x and the ideal positive alternative $\langle 1, 0; \sqrt{2}, \sqrt{2}\rangle$ to x.

B. Three-Dimensional Elliptic Intuitionistic Fuzzy Index Matrices (3-D E-IFIM)

In 1987, according to [1], the theory of index matrices (IMs) appeared. Over IMs, several operations, relations, and operators are defined (see [2], [31]). Assume that the set of indices $\mathscr{I}$ is fixed. Using the definition of 3-D IFIM from [2], [31], let us we define a 3-D E-IFIM $A = [K, L, H, \{\langle \mu_{k_i,l_j,h_g}, \nu_{k_i,l_j,h_g}; rf_{k_i,l_j,h_g}, rs_{k_i,l_j,h_g}\rangle\}]$ as follows:

$$\begin{array}{c|ccc}
h_g \in H & l_1 & \dots & l_n \\
\hline
k_1 & \langle \mu_{k_1,l_1,h_g}, \nu_{k_1,l_1,h_g}; rf_{k_1,l_1,h_g}, rs_{k_1,l_1,h_g}\rangle & \dots & \langle \mu_{k_1,l_n,h_g}, \nu_{k_1,l_n,h_g}; rf_{k_1,l_n,h_g}, rs_{k_1,l_n,h_g}\rangle \\
\vdots & \vdots & \dots & \vdots \\
k_m & \langle \mu_{k_m,l_1,h_g}, \nu_{k_m,l_1,h_g}; rf_{k_m,l_1,h_g}, rs_{k_m,l_1,h_g}\rangle & \dots & \langle \mu_{k_m,l_n,h_g}, \nu_{k_m,l_n,h_g}; rf_{k_m,l_n,h_g}, rs_{k_m,l_n,h_g}\rangle
\end{array},$$

where $(K, L, H \subset \mathscr{I})$ and its elements are E-IFQs.

There are many defined operations over the IMs [2]. Let E-IFIMs $A = [K, L, H, \{\langle \mu_{k_i,l_j,h_g}, \nu_{k_i,l_j,h_g}; rf_{k_i,l_j,h_g}, rs_{k_i,l_j,h_g}\rangle\}]$ and $B = [P, Q, R\{\langle \rho_{p_r,q_s,t_e}, \sigma_{p_r,q_s,t_e}; \delta f_{p_r,q_s,t_e}, \delta s_{p_r,q_s,t_e}\}]$ be given.

We for the first time introduce some operations performed over E-IFIM that are comparable to those performed over IFIMs [2].

Addition-$(\circ_1, \circ_2, *)$:

$$A \oplus_{(\circ_1,\circ_2,*)} B$$
$$= [K \cup P, L \cup Q, H \cup R, \{\langle \phi_{t_u,v_w,x_y}, \psi_{t_u,v_w,x_y}; \eta_{t_u,v_w,x_y}\rangle\}],$$

where $\langle \circ_1, \circ_2\rangle \in \{\langle \max, \min\rangle, \langle \min, \max\rangle, \langle average, average\rangle\}$ and $* \in \{\max, \min\}$.

$$\langle \phi_{t_u,v_w,x_y}, \psi_{t_u,v_w,x_y}; \eta_{t_u,v_w,x_y}\rangle$$
$$= \langle \circ_1(\mu_{k_i,l_j,x_y}, \rho_{p_r,q_s,x_y}), \circ_2(\nu_{k_i,l_j,x_y}, \sigma_{p_r,q_s,x_y});$$
$$*(rf_{t_u,v_w,x_y}, \delta f_{t_u,v_w,x_y}, *(rs_{t_u,v_w,x_y}, \delta s_{t_u,v_w,x_y})\rangle.$$

Multiplication:

$$A \odot_{(\circ_1,\circ_2,*)} B$$
$$= [K \cup (P-L), Q \cup (L-P), H \cup R, \{\langle \phi_{t_u,v_w,x_y}, \psi_{t_u,v_w,x_y};$$
$$\eta f_{t_u,v_w,x_y}, \eta s_{t_u,v_w,x_y}\rangle\}],$$

where

$$\langle \phi_{t_u,v_w,x_y}, \psi_{t_u,v_w,x_y}\rangle$$

is defined in [2],

$$\eta f_{t_u,v_w,x_y} = *(rf_{t_u,v_w,x_y}, \delta f_{t_u,v_w,x_y})$$
$$\text{and } \eta s_{t_u,v_w,x_y} = *(rs_{t_u,v_w,x_y}, \delta s_{t_u,v_w,x_y}).$$

The following operations cannot be performed on these conventional verso matrices. They are designed with the ability to automate specific IM operations to implement different models and algorithms.

Aggregation operation by one dimension:

Let us extend the operations $\#_q, (q \leq i \leq 3)$ from [30] such that they can be applied over E-IFQs $x = \langle a, b; rf_1, rs_1\rangle$ and $y = \langle c, d; rf_2, rs_2\rangle$:

$$x\#_1, *y = \langle min(a,c), max(b,d); *(rf_1, rf_2), *(rs_1, rs_2)\rangle;$$
$$x\#_2, *y = \langle average(a,c), average(b,d); *(rf_1, rf_2), *(rs_1, rs_2)\rangle;$$
$$x\#_3, *y = \langle max(a,c), min(b,d); *(rf_1, rf_2), *(rs_1, rs_2)\rangle.$$

Let the fixed index be $k_0 \notin K$. The expanded definition of the aggregation operation $\alpha_{K,\#_q,*}(A, k_0)$ by the dimension K over 3-D E-IFIM A utilizing that of [27], [30] is as follows:

$h_g \in H$	l_1	$\ldots$
k_0	$\underset{i=1}{\overset{m}{\#_q,*}} \langle \mu_{k_i,l_1,h_g}, \nu_{k_i,l_1,h_g}; rf_{k_i,l_1,h_g}, rs_{k_i,l_1,h_g}\rangle$	$\ldots$

$\ldots$	l_n
$\ldots$	$\underset{i=1}{\overset{m}{\#_q,*}} \langle \mu_{k_i,l_n,h_g}, \nu_{k_i,l_n,h_g}; rf_{k_i,l_1,h_g}, rs_{k_i,l_1,h_g}\rangle$.

We may perform a super pessimistic aggregation operation in conditions of high inflation using $\#_1^*$, an average aggregation operation in anticipation of slight fluctuations in the market situation using $\#_2^*$, and a super optimistic aggregation operation in conditions of stability of the market parameters using $\#_3^*$.

Projection [2]: Let $W \subseteq K$, $V \subseteq L$ and $U \subseteq H$. Then,

$$pr_{W,V,U}A = [W, V, U, \{\langle R_{p_r,q_s,e_d}, S_{p_r,q_s,e_d}\rangle\}],$$

where for each $k_i \in W, l_j \in V$ and $t_g \in U$,

$$\langle R_{p_r,q_s,e_d}, S_{p_r,q_s,e_d}\rangle = \langle \mu_{k_i,l_j,h_g}, \nu_{k_i,l_j,h_g}\rangle.$$

Reduction [2]: An IM A's operations-reduction $(k, \perp, \perp)$ is defined as follows:

$$A_{(k,\perp,\perp)} = [K - \{k\}, L, H, \{c_{t_u,v_w,e_d}\}], \text{where}$$
$$c_{t_u,v_w,e_d} = a_{k_i,l_j,h_g} (t_u = k_i \in K - \{k\}, v_w = l_j \in L, e_d = h_g \in H).$$

Substitution [2]:

$$\left[\frac{p}{k_i}; \perp, \perp\right] A = \left[(K - \{k_i\}) \cup \{p\}, L, H, \{a_{k_i,l_j,h_g}\}\right]$$

A Level Operator for Decreasing the Number of Elements of E-IFIM: Let $\langle \alpha, \beta; r_1, r_2\rangle$ is an E-IFQ and $A = [K, L, H, \{a_{k_i,l_j,h_g}\}] = [K, L, H, \{\langle \mu_{k_i,l_j,h_g}, \nu_{k_i,l_j,h_g}; rf_{k_i,l_j,h_g}, rs_{k_i,l_j,h_g}\rangle\}$ is a 3-D E-IFIM, then according to [10] let us define the operator $N^{>_{R^{elliptic}}}_{\langle \alpha,\beta,r_1,r_2\rangle}(A) = [K, L, H, \{\langle \rho_{k_i,l_j,h_g}, \sigma_{k_i,l_j,h_g}; rf_{k_i,l_j,h_g}, rs_{k_i,l_j,h_g}\rangle\}]$, where

$$\langle \rho_{k_i,l_j,h_g}, \sigma_{k_i,l_j,h_g}; rf^n_{k_i,l_j,h_g}, rs^n_{k_i,l_j,h_g}\rangle$$

$$= \begin{cases} a_{k_i,l_j,h_g} & \text{if } a_{k_i,l_j,h_g} >_{R^{elliptic}} \langle \alpha, \beta; r_1, r_2\rangle \\ \langle 0, 1; 0, 0\rangle & \text{otherwise} \end{cases} \quad (2)$$

III. An Elliptic Intuitionistic Fuzzy Method for Selecting the Most Beneficial Franchisor (E-IFFr)

This section will extend the intuitionistic fuzzy (IF) algorithm for the best franchisee selection [26] to suggest an algorithm for a specific type of E-IF franchisor selection problem (E-IFFr). The Elliptical IFS is a better tool for characterizing this fuzziness than the IFS in situations of galloping inflation and quick changes in the economic environment because in these situations, the degrees of truth and falsity of a given element shift in the shape of an ellipse.

The optimal E-IF franchisor selection problem is posed: An entrepreneurial company has created an evaluation system with criteria $\{c_1, \ldots, c_j, \ldots, c_n\}$ (for $j = 1, ..., n$) for franchise companies from a certain business. The business wants to choose a successful business franchisor. It is necessary to do a professional evaluation by experts $\{d_1, \ldots, d_s, \ldots, d_D\}$ of franchise businesses $\{k_1, \ldots, k_i, \ldots, k_m\}$ in the pertinent business sector. The ranking coefficients of the experts $\{r_1, \ldots, r_s, \ldots, r_D\}$ are calculated based on their qualitative involvement in the evaluation of franchise procedures and are given to the experts in the form of IFPs $\langle \delta_s, \varepsilon_s\rangle (1 \leq s \leq D)$. The interpretation of the elements δ_s and ε_s is the level of competence and incompetence of the s-th expert, respectively. The expert evaluations of the franchise chains are made and presented as IF data ev_{k_i,c_j,d_s} (for $1 \leq i \leq m, 1 \leq j \leq n, 1 \leq s \leq D$). The final estimates of the franchisors are calculated in the form of E-IFQs fi_{k_i,v_e,h_f} (for $1 \leq i \leq m$), taking into consideration the E-IF priorities pk_{c_j,v_e} of the criteria c_j (for $j = 1, ..., n$) from the view of the entrepreneur v_e in a given moment h_f. The optimal aim is to determine which franchise chain is the most suitable for the entrepreneurial company.

A. Index-matrix Interpretation of the Optimal Elliptic Intuitionistic Fuzzy Franchisor Selection Problem

The following operations are part of the index-matrix approach to the optimal elliptic intuitionistic fuzzy franchisor selection problem (E-IFFr), defined above:

Step 1. An IF index matrix $EV[K, C, E, \{ev_{k_i,c_j,d_s}\}]$, $K = \{k_1, k_2, \ldots, k_m\}$, $C = \{c_1, c_2, \ldots, c_n\}$ and $E = \{d_1, d_2, \ldots, d_D\}$ is constructed. Due to the uncertainty of the economic environment, the elements $\{ev_{k_i,c_j,d_s}\} = \langle \mu_{k_i,c_j,d_s}, \nu_{k_i,c_j,d_s}\rangle$ (for $1 \leq i \leq m, 1 \leq j \leq n, 1 \leq s \leq D$) of the matrix EV are the IF valuations of the d_s-th expert for the k_i-th franchisor by the c_j-th criterion. Next, we go on to *Step 2*.

Step 2. An IFP $r_s = \langle \delta_s, \varepsilon_s\rangle, (s \in E)$, whose components might be interpreted as showing how competent or incompetent experts are, should be used to specify each expert's score coefficient.

The IM has been built by:

$$EV^*[K, C, E, \{ev^*_{k_i,c_j,d_s}\}]$$
$$= r_1 pr_{K,C,d_1} EV \oplus_{(\circ_1,\circ_2)} r_2 pr_{K,C,d_2} EV \ldots \oplus_{(\circ_1,\circ_2)} r_D pr_{K,C,d_D} EV.$$
$$EV := EV^*(ev_{k_i,l_j,d_s} = ev^*_{k_i,l_j,d_s}, \ \forall k_i \in K, \forall l_j \in L, \forall d_s \in E).$$

The degrees of membership and non-membership of the E-IFQs are determined by the elements of the matrix EV using the three aggregating operations $\alpha_{K,\#_1,*}, \alpha_{K,\#_3,*}$ and $\alpha_{K,\#_2,*}$, which provide the evaluations of the k_i-th franchisor against the c_j-th criterion in a present moment $h_f \notin E$:

$$PI_{min}[K, h_f, C, \{pi_{min_{k_i,h_f,c_g}}\}] = \alpha_{E,\#_1}(EV^*, h_f)$$

$$= \left\{ \begin{array}{c|c} c_j & h_f \\ \hline k_1 & \underset{s=1}{\overset{D}{\#_1}} \langle \mu_{k_1,c_j,d_s}, \nu_{k_1,c_j,d_s} \rangle \\ \vdots & \vdots \\ k_m & \underset{s=1}{\overset{D}{\#_1}} \langle \mu_{k_m,c_j,d_s}, \nu_{k_m,c_j,d_s} \rangle \end{array} \mid c_j \in C \right\};$$

$$PI_{max}[K, h_f, C, \{pi_{max\,k_i,h_f,c_g}\}] = \alpha_{E,\#_3}(EV^*, h_f)$$

$$= \left\{ \begin{array}{c|c} c_j & h_f \\ \hline k_1 & \underset{s=1}{\overset{D}{\#_3}} \langle \mu_{k_1,c_j,d_s}, \nu_{k_1,c_j,d_s} \rangle \\ \vdots & \vdots \\ k_m & \underset{s=3}{\overset{D}{\#_1}} \langle \mu_{k_m,c_j,d_s}, \nu_{k_m,c_j,d_s} \rangle \end{array} \mid c_j \in C \right\}$$

$$PI* = PI_{min} \oplus_{(\circ_1,\circ_2,*)} PI_{max}$$

Then the centers of the E-IFQs used to evaluate the franchise companies are represented as elements in a matrix as follows: $PI[K, h_f, C, \{pi_{k_i,h_f,c_g}\}] = \alpha_{E,\#_2}(PI^*, h_f), (h_f \notin E)$. Next, we continue to *Step 3*.

Step 3. At this point, the evaluation system for the franchise business candidate will be optimized. We recommend removing slower or more expensive criteria to measure that has been found to closely connect with other criteria under intuitionistic fuzzy settings from the franchisee evaluation system utilizing inter-criteria analysis (ICrA, [7], [9]). Let $\langle \alpha, \beta \rangle$ be an IFP. The criteria C_k and C_l are in
(α, β)-positive consonance, if $\mu_{C_k,C_l} > \alpha$ and $\nu_{C_k,C_l} < \beta$;
(α, β)-negative consonance, if $\mu_{C_k,C_l} < \beta$ and $\nu_{C_k,C_l} > \alpha$;
(α, β)-dissonance, otherwise.

The transposed IM $PI^T = [K, C, h_f, \{pi^T{}_{k_i,c_g,h_f}\}]$ is searched for consonant criteria using the ICrA algorithm. More expensive, slower, or more complicated criteria are eliminated from the evaluation franchise system using the IM reduction operation over matrix PI^T. The following step is *Step 4*.

Step 4. Now we can calculate E-IFIM $A[K, C, h_f\{a_{k_i,c_g,h_f}\}]$, which represents current assessments of the franchisors using the methodology from [5] according to the system of criteria:

h_f	c_1	$\dots$	c_n
k_1	$\langle \mu^a_{k_1,c_1}, \nu^a_{k_1,c_1}; rf^a_{k_1,c_1}, rs^a_{k_1,c_1} \rangle$	$\dots$	$\langle \mu^a_{k_1,c_n}, \nu^a_{k_1,c_n}; rf^a_{k_1,c_n}, rs^a_{k_1,c_n} \rangle$
$\vdots$	$\vdots$	$\dots$	$\vdots$
k_m	$\langle \mu^a_{k_m,c_1}, \nu^a_{k_m,c_1}; rf^a_{k_m,c_1}, rs^a_{k_m,c_1} \rangle$	$\dots$	$\langle \mu^a_{k_m,c_n}, \nu^a_{k_m,c_n}; rf^a_{k_m,c_n}, rs^a_{k_m,c_n} \rangle$

,

where $K = \{k_1, \dots, k_i, \dots, k_m\}, i = 1, \dots, m$; $C = \{c_1, \dots, c_j, \dots, c_n\}, j = 1, \dots, n$; its elements a_{k_i,c_g,h_f} (for $i = 1, \dots, m; g = 1, \dots, n$) are created as E-IFQs by transforming the IFPs $pi^T{}_{k_i,c_j,h_f}$ using the following steps

for $g = 1$ to n, $i = 1$ to m

$$\Big\{ \mu^a_{k_i,c_g,h_f} = \mu^{pi^T}_{k_i,c_g,h_f}; \nu^a_{k_i,c_g,h_f} = \nu^{pi^T}_{k_i,c_g,h_f},$$

$$rf^a_{k_i,c_g,h_f} =$$

$$\sqrt{\min_{1\le s\le D} \mu^{ev}_{k_i,c_g,d_s}{}^2 + \left\{ \frac{\max\limits_{1\le s\le D} \mu^{ev}_{k_i,c_g,d_s}{}^2 - \min\limits_{1\le s\le D} \mu^{ev}_{k_i,c_g,d_s}{}^2}{\max\limits_{1\le s\le D} \nu^{ev}_{k_i,c_g,d_s}{}^2 - \min\limits_{1\le s\le D} \nu^{ev}_{k_i,c_g,d_s}{}^2} \right\}^2 \cdot \min_{1\le s\le D} \nu^{ev}_{k_i,c_g,d_s}{}^2}$$

$$\text{and } rs^a_{k_i,c_g} =$$

$$\sqrt{\min_{1\le s\le D} \mu^{ev}_{k_i,c_g,d_s}{}^2 \cdot \left\{ \frac{\max\limits_{1\le s\le D} \nu^{ev}_{k_i,c_g,d_s}{}^2 - \min\limits_{1\le s\le D} \nu^{ev}_{k_i,c_g,d_s}{}^2}{\max\limits_{1\le s\le D} \mu^{ev}_{k_i,c_g,d_s}{}^2 - \min\limits_{1\le s\le D} \mu^{ev}_{k_i,c_g,d_s}{}^2} \right\}^2 + \min_{1\le s\le D} \nu^{ev}_{k_i,c_g,d_s}{}^2}$$

$$\Big\}$$

Next, we go on to *Step 5*.

Step 5. At this stage, a 3-D E-IFIM *PK* is created, and the coefficients used in the following operation determine the weighting of each evaluation criterion from the view of the entrepreneur v_e for the franchise business:

$$PK[C, v_e, h_f, \{pk_{c_j,v_e,h_f}\}] = \begin{array}{c|c} h_f & v_e \\ \hline c_1 & pk_{c_1,v_e,h_f} \\ \vdots & \vdots \\ c_j & pk_{c_j,v_e,h_f} \\ \vdots & \vdots \\ c_n & pk_{c_n,v_e,h_f} \end{array},$$

where $C = \{c_1, c_2, \dots, c_n\}$. The evaluation E-IFIM $FI[K, v_e, h_f, \{fi_{k_i,v_e,h_f}\}] = A \odot_{(\circ_1,\circ_2,*)} PK$ (for $1 \le i \le m$) for the entrepreneur v_e includes all of the E-IF estimates for k_i-th franchisor. Go to *Step 6*.

Step 6. At this stage, based on the aggregation operation $\alpha_{K,\#_q,*}(FI, k_0)$, the business owner v_e chooses the franchisor that is the most advantageous. Depending on the value of q, utilizing pessimistic, average, or optimistic scenarios:

$$\alpha_{K,\#_q,*}(FI, k_0)$$
$$= \begin{array}{c|c} h_f & v_e \\ \hline k_0 & \underset{i=1}{\overset{m}{\#_{q,*}}} \langle \mu_{k_i,v_e,h_f}, \nu_{k_i,v_e,h_f}, rf_{k_i,v_e,h_f}, rs_{k_i,v_e,h_f} \rangle \end{array}, \quad (3)$$

where $k_0 \notin K, 1 \le q \le 3$. Go to *Step 7*.

Step 7. The updated rating coefficients for the experts who participated in the evaluation process are obtained in this stage. The expert's new score will be altered by the method used in the work after he participates in the present procedure. Let's assume the expert d_s $(s = 1, ..., D)$ has participated in γ_s evaluation procedures for the selection of a franchisee, based on which his score $r_s = \langle \delta_s, \varepsilon_s, \phi^1_s, \phi^2_s \rangle$ is determined, then after his participation in the current procedure, his new score will be changed by ideas from [3]:

$$\langle \delta_s^{'}, \varepsilon_s^{'}; {\phi^1}^{'}_s, {\phi^2}^{'}_s \rangle$$

$$= \begin{cases} \langle \frac{\delta\gamma+1}{\gamma+1}, \frac{\varepsilon\gamma}{\gamma+1}; *({\phi^1}^{'}_s, \phi_s^1), *({\phi^2}^{'}_s, \phi_s^2) \rangle, & \text{if the expert's assessment was accurate} \\ \langle \frac{\delta\gamma}{\gamma+1}, \frac{\varepsilon\gamma}{\gamma+1}; *({\phi^1}^{'}_s, \phi_s^1), *({\phi^2}^{'}_s, \phi_s^2) \rangle, & \text{if the expert has not provided any estimates} \\ \langle \frac{\delta\gamma}{\gamma+1}, \frac{\varepsilon\gamma+1}{\gamma+1}; *({\phi^1}^{'}_s, \phi_s^1), *({\phi^2}^{'}_s, \phi_s^2) \rangle, & \text{if the expert has made an inaccurate assessment} \end{cases}$$

The algorithm is complete.

If the operations $\langle \circ_1, \circ_2 \rangle = \langle \min, \max \rangle$ are used in the E-IFFr, the pessimistic scenario has been utilized.

If $\langle \circ_1, \circ_2 \rangle = \langle \max, \min \rangle$ are applied, the optimistic scenario has been obtained, and if the operations $\langle \circ_1, \circ_2 \rangle = \langle average, average \rangle$ are used, the averaged scenario has been obtained. The operation "$* = \max$" is used in cases of more ambiguity, otherwise "$* = \min$." As a result, different ideal solutions could be produced based on the investor's viewpoint. As a result, an investor may have more faith in the discovered solution.

Based on the complexity of ICrA [11]), the suggested E-IFFr method has a complexity $O(Dm^2n^2)$. Crisp and E-IF data can be used using the suggested E-IFFr approach. It can be used without limitations and is easy to adapt to the different kinds of data that are present in a fuzzy environment.

There is no model for the best franchise chain selection based on elliptic IF fuzzy data in the scientific literature that can represent ambiguous or incomplete data under situations of significant market parameter volatility. There are three scenarios in this model, each based on the decision-makers attitudes. Experts evaluate franchisors to make the best choice, and their ratings as well as the importance of the criteria are taken into consideration during the review process. Because of this, the proposed E-IF optimum franchisor selection task is socially oriented and takes into account the preferences of the decision-maker as well as the experts.

IV. Using E-IFFr Approach to Overcome the Difficulty of Selecting a Courier Franchise Company

Finding the ideal franchisor in the courier services industry is possible with the help of the E-IFFr technique of Sect. III. Let us formulate the following problem for this purpose:

A business investor v_e wants to make an optimal choice of a courier brand offering a franchise such as Econt, Leo Express, Fasto Courier, and T-Post. For this purpose, he creates a system of criteria for evaluating potential franchisors k_i (for $1 \le i \le 4$) using the expertise of experts d_1, d_2, and d_3. The four groups of criteria that make the system for franchisee selection using the requirements of the four courier franchise companies are as follows:

- C_1 - the choice of a well-known, respected brand with a significant market share with reputation, market share, and corporate capabilities;
- C_2 - expected profit in the form of commission, which is a dynamic % of the realized turnover of the office depending on the quality and volume of the courier services provided.
- C_3 - operational and initial costs for starting the business model: to calculate initial and ongoing operational costs, such as initial and monthly franchise fees; royalties and marketing expenses; costs for satisfying the brand's requirements for the look of the offices and their equipment, for the look of the cars and their number; costs of providing office security measures; costs of providing equipment for servicing large-volume shipments
- C_4 - evaluation of the franchisor's level of training and support, including ongoing marketing and technical support.

Ranking coefficients of the experts $\{r_1, r_2, r_3\}$ are given in the form of IFPs $\langle \delta_s, \varepsilon_s \rangle (1 \le s \le 3)$. The four courier franchise chains' expert assessments were made by the criteria and presented as IF data ev_{k_i,c_j,d_s} (for $1 \le i \le 4, 1 \le j \le 4, 1 \le s \le 3$). The final E-IF evaluations fi_{k_i,v_e,h_f} (for $1 \le i \le 4$) of the courier brands are based on the priorities pk_{c_j,v_e} of criteria c_j (for $j = 1,...,4$) from the point of view of entrepreneur v_e at time h_f. The optimal purpose is to determine which franchise structure for courier services is the best for the startup business.

Solution of the problem:

Step 1. At this stage, the 3-D expert assessment IFIM $EV[K, C, E, \{es_{k_i,c_j,d_s}\}]$ is created with the expert's estimations in the c_j-th criterion for the k_i-th franchisor (for $1 \le i \le 4, 1 \le j \le 4, 1 \le s \le 3$), and its form is:

d_1	c_1	c_2	c_3	c_4
k_1	$\langle 0.30, 0.30 \rangle$	$\langle 0.20, 0.50 \rangle$	$\langle 0.60, 0.20 \rangle$	$\langle 0.20, 0.50 \rangle$
k_2	$\langle 0.10, 0.60 \rangle$	$\langle 0.40, 0.40 \rangle$	$\langle 0.40, 0.50 \rangle$	$\langle 0.40, 0.40 \rangle$
k_3	$\langle 0.40, 0.20 \rangle$	$\langle 0.10, 0.70 \rangle$	$\langle 0.20, 0.40 \rangle$	$\langle 0.60, 0.20 \rangle$
k_4	$\langle 0.10, 0.75 \rangle$	$\langle 0.20, 0.70 \rangle$	$\langle 0.205, 0.70 \rangle$	$\langle 0.40, 0.50 \rangle$

d_2	c_1	c_2	c_3	c_4
k_1	$\langle 0.40, 0.40 \rangle$	$\langle 0.10, 0.70 \rangle$	$\langle 0.70, 0.10 \rangle$	$\langle 0.30, 0.50 \rangle$
k_2	$\langle 0.20, 0.80 \rangle$	$\langle 0.30, 0.50 \rangle$	$\langle 0.60, 0.20 \rangle$	$\langle 0.60, 0.10 \rangle$
k_3	$\langle 0.30, 0.40 \rangle$	$\langle 0.30, 0.60 \rangle$	$\langle 0.10, 0.70 \rangle$	$\langle 0.40, 0.40 \rangle$
k_4	$\langle 0.15, 0.60 \rangle$	$0.25, 0.30 \rangle$	$\langle 0.20, 0.60 \rangle$	$\langle 0.30, 0.30 \rangle$

d_3	c_1	c_2	c_3	c_4
k_1	$\langle 0.10, 0.70 \rangle$	$\langle 0.20, 0.70 \rangle$	$\langle 0.40, 0.40 \rangle$	$\langle 0.40, 0.40 \rangle$
k_2	$\langle 0.10, 0.80 \rangle$	$\langle 0.30, 0.60 \rangle$	$\langle 0.20, 0.60 \rangle$	$\langle 0.50, 0.20 \rangle$
k_3	$\langle 0.30, 0.50 \rangle$	$\langle 0.20, 0.70 \rangle$	$\langle 0.30, 0.60 \rangle$	$\langle 0.40, 0.50 \rangle$
k_4	$\langle 0.10, 0.80 \rangle$	$\langle 0.30, 0.50 \rangle$	$\langle 0.10, 0.70 \rangle$	$\langle 0.30, 0.60 \rangle$

Step 2. These are the ranking experts' rank coefficients: $\{r_1, r_2, r_3\} = \{\langle 0.80, 0.10 \rangle, \langle 0.70, 0.10 \rangle, \langle 0.90, 0.10 \rangle\}$.

The evaluation IM $EV^*[K, C, E, \{ev^*\}]$ is made using the subsequent procedures:

$$EV^* = r_1 pr_{K,C,d_1} EV \oplus_{(\circ_1,\circ_2)} r_2 pr_{K,C,d_2} EV \oplus_{(\circ_1,\circ_2)} r_3 pr_{K,C,d_3} EV;$$

$$EV := EV^*$$

Then the IMs are created:

$$PI* = PI_{min} \oplus_{(\circ_1,\circ_2,*)} PI_{max}$$

and

$$PI[K, h_f, C, \{pi_{k_i,h_f,c_g}\}] = \alpha_{E,\#_2}(PI^*, h_f), (h_f \notin E)$$

whose elements are the coordinates of the centers of the E-IFQs evaluating the courier brands.

Step 3. At this step, we ran the ICrA over the matrix PI^T with $\alpha = 0.80$ and $\beta = 0.10$. Following ICrA, it is concluded that

TABLE I
THE IFPS PROVIDE THE INTERCRITERIA CORRELATIONS

	C_1	C_2	C_3	C_4
C_1	—	$\langle 0.76;0.20\rangle$	$\langle 0.79;0.17\rangle$	$\langle 0.71;0.20\rangle$
C_2	$\langle 0.76;0.20\rangle$	—	$\langle 0.73;0.23\rangle$	$\langle 0.76;0.12\rangle$
C_3	$\langle 0.79;0.17\rangle$	$\langle 0.73;0.23\rangle$	—	$\langle 0.65;0.26\rangle$
C_4	$\langle 0.71;0.20\rangle$	$\langle 0.76;0.12\rangle$	$\langle 0.65;0.26\rangle$	—

no consonant-dependent criteria exist. The results are shown as an IM in μ - ν view result matrix (cf. table I):

Step 4. Now we can calculate E-IFIM $A[K,C,h_f\{a_{k_i,c_g,h_f}\}]$, which represents recent assessments of the courier brands using the following criteria:

h_f	c_1	c_2	$\cdots$
k_1	$\langle 0.21,0.54;0.06,0.04\rangle$	$\langle 0.14,0.68;0.04,0.02\rangle$	$\cdots$
k_2	$\langle 0.11,0.77;0.04,0.02\rangle$	$\langle 0.26,0.57;0.04,0.02\rangle$	$\cdots$
k_3	$\langle 0.26,0.45;0.03,0.01\rangle$	$\langle 0.16,0.62;0.04,0.02\rangle$	$\cdots$
k_4	$\langle 0.10,0.74;0.03,0.01\rangle$	$\langle 0.20,0.57;0.03,0.01\rangle$	$\cdots$

$\cdots$	c_3	c_4
$\cdots$	$\langle 0.44,0.34;0.05,0.03\rangle$	$\langle 0.24,0.55;0.05,0.03\rangle$
$\cdots$	$\langle 0.31,0.51;0.04,0.02\rangle$	$\langle 0.39,0.34;0.04,0.02\rangle$
$\cdots$	$\langle 0.16,0.62;0.03,0.01\rangle$	$\langle 0.36,0.45;0.03,0.01\rangle$
$\cdots$	$\langle 0.14,0.74;0.03,0.01\rangle$	$\langle 0.26,0.54;0.04,0.02\rangle$

Step 5. At this stage, the priority of each evaluation criterion from the viewpoint of the franchisor v_e is determined by the coefficients employed in the subsequent process on a 3-D E-IFIM PK:

$$PK[C,v_e,h_f,\{pk_{c_j,v_e,h_f}\}]=\begin{array}{c|c} h_f & v_e \\ \hline c_1 & \langle 0.90,0.10;0.02,0.01\rangle \\ c_2 & \langle 0.80,0.10;0.02,0.01\rangle \\ c_3 & \langle 0.60,0.20;0.02,0.01\rangle \\ c_4 & \langle 0.80,0.10;0.02,0.01\rangle \end{array} \quad (4)$$

The evaluation E-IFIM

$$FI[K,v_e,h_f,\{fi_{k_i,v_e,h_f}\}]=A\odot_{(\circ_1,\circ_2,min}PK$$

(for $1\le i\le m$) includes the full estimates of the k_i-th franchise chain for the business owner v_e based on the optimistic case:

$$\text{and } FI=\begin{array}{c|c} h_f & v_e \\ \hline k_1 & \langle 0.674,0.046;0.02,0.01\rangle \\ k_2 & \langle 0.688,0.040;0.02,0.01\rangle \\ k_3 & \langle 0.694,0.047;0.02,0.01\rangle \\ k_4 & \langle 0.539,0.170;0.01,0.01\rangle \end{array} \quad (5)$$

Step 6. According to the optimistic aggregation operation $\alpha_{K,\#_3,min}(FI,k_0)$, k_3 is the best courier franchise brand in Bulgaria from the point of view of the entrepreneur, with a maximum acceptance degree of 0.694 and a minimum rejection degree of 0.047. The decision-makers will select the candidate k_4 with the minimum degree of membership 0.539 and the greatest degree of non-membership 0.17 in a pessimistic scenario if the future is unclear and the decision-making environment is unpredictable.

Step 7. At the last step, we assume that the correctness of the experts' evaluations was evaluated by senior experts and they are correct from the point of view of intuitionistic fuzzy logic [3] and their new rating coefficients are equal to $\{\langle 0.82,0.09;0.02,0.01\rangle,\langle 0.73,0.09;0.02,0.01\rangle,$ $\langle 0.91,0.09;0.02,0.01\rangle\}$.

The decision-maker will favor the pessimistic scenario when there is high inflation and significant uncertainty, the averaged scenario when there are only minor variations in the market parameters, and the optimistic scenario when the market parameters are stable. We were unable to locate techniques for issues of a comparable type under circumstances of high uncertainty characterized by E-IF logic, preventing us from performing a comparative analysis between the suggested E-IFKP approach for franchisor optimization.

The question of whether minor variations in the values of the input parameters utilized impact the outcomes of the model emerges following the results of the E-IFKP franchisor approach. A crucial step in solving the E-IFKP franchisor problem is to evaluate the robustness of the findings in the constructed model and their sensitivity to changes in the input variables.

The weights of the criteria are of great importance to the results of the E-IFKP franchisor problem. A sensitivity analysis consisting of 8 different scenarios have been conducted to analyze the effect of the change in weight of each criterion on the ranking results by $\pm10\%,\pm25\%,\pm50\%$ and $\pm75\%$ respectively. In the analysis, a total of 8 different changes have been applied to the weights of the criteria included in the study, and the final results are different in the cases indicated.

The sensitivity of franchisor ranking to criteria weights is explored, and four distinct weighting methodologies have been taken into consideration to reveal different answers.

The results of the sensitivity analysis show that there is sensitivity in the output results when criteria are assigned different weights assets. The best option is candidate k_3 if the first criterion is given top priority; the best option is candidate k_1 if the second criterion is given top priority; candidate k_3 if the third criterion is given top priority; and the best option is candidate k_2 if the fourth criterion is given top priority.

V. CONCLUSION

The instruments of E-IF logic and the theory of index matrices were used in the study to construct an optimal algorithm (E-IFFr) for the most effective selection of franchise firms in conditions of significant parameter uncertainty. Additionally, it considered the opinions of the experts as well as the order in which entrepreneurs should rank the evaluation criteria. A case study of the franchisor selection for a courier brand is used to illustrate the proposed strategy based on the criteria of 4 leading courier franchise companies in Bulgaria. The created E-IFFr method can be used with both crisp and elliptic values. There are no restrictions on its use, and it is simple to modify to various types of data that are present in a fuzzy environment. In the future, research will continue with the development of a franchisor selection software program to automate the proposed approach and with its extension so that it can be applied to three-dimensional data and with its extension so that it can be applied to three-dimensional elliptic intuitionistic fuzzy data.

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