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Graded Logic and Professional Decision Making

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***Abstract*—This paper summarizes the basic concepts of Graded Logic (GL) and the use of GL in professional decision making. Our goal is to contrast two approaches to the development of a continuum-valued propositional logic: (1) the human-centric approach based on observing and modeling human commonsense logical reasoning in the context of decision making, and (2) the theoretical approach where logic is developed as a formal axiomatic deductive system. We show the basic advantages of human-centric approach and the applicability of this approach in the area of professional decision making.**

***Index Terms*—Graded Logic, commonsense logical reasoning, decision making, LSP method, GCD function.**

I. Introduction

LOGIC is a wide area studied in both philosophy and mathematics [1]. In this short survey paper, we are interested only in the propositional logic [2], i.e., the logic that combines degrees of truth of input statements to compute the degree of truth of a compound output statement. We briefly present a human-centric Graded Logic (GL) which is derived from observing, measuring and modeling human commonsense logical reasoning in the process of decision making. We also present a brief survey of the Logic Scoring of Preference (LSP) decision method [3] which is based on Graded Logic [22].

The classical logic [4]-[6] and its modern extensions [7], [8], as well as non-classical logics [9], [10] are created as formal axiomatic deductive systems; that is the standard theoretical approach. In such systems a mathematical theory is built on a set of axioms and axioms are assumed to be true without further consideration. Then, all other theoretical results are proved based on their consistency with the axioms. In the case of logic, axioms operate with variables that are members of a set of two or more values, but further considerations of the role and meaning of variables are not necessary and not given. So, such logics operate with anonymous real numbers, and if such numbers denote degrees of truth of specific statements, it is not necessary to know the corresponding statements, their author, and their role, meaning, and the context in which the statements are created and used. The applicability of axiomatic deductive logic systems is an independent topic outside the area of specific (logic) theory.

It is possible to develop logic in a different, human-centric way, which we propose in the case of GL. This approach is derived from logic-based applications of specific stakeholder/decision-maker who is an individual human or a human organization. The human-centric approach is based on observing, measuring, modeling and explaining natural human commonsense reasoning and decision making. Mathematical models are then developed to be consistent with observations and measurements.

The paper is organized as three sections devoted to Graded Logic, followed by a section devoted to the LSP method.

II. Human-centric Approach to Logic

A. The stakeholder/decision-maker

Logical reasoning is a human mental activity, i.e., there is no logical reasoning and no need for logic without explicit presence of a specific human (either an individual or an organization). Human logic does not exist in vacuum. Therefore, we assume that all logic problems are related to a specific human participant, identified as the stakeholder/decision-maker (SDM).

It should be self-evident that the SDM exists in a specific environment, interacts with the environment, has goals and requirements, and uses logical reasoning to make decisions necessary to satisfy requirements and attain goals.

B. Human graded percepts and graded truth

Human percepts are defined as quantifiable mental sensations/impressions of perceiving and/or reasoning. Examples of such percepts include satisfaction, importance, suitability, preference, confidence, value, and many others. The fundamental property of such percepts is that they are graded: each percept p can vary in the range from zero to its maximum value: $p \in [0, p_{max}]$. For example, each percept of satisfaction of specific requirements varies in the range from no satisfaction to the full satisfaction.

All graded percepts can be directly related to graded truth. If we define $t = p/p_{max}$, $t \in [0,1]$, then t denotes the degree of truth of statement "the percept p attained its maximum value." For example, if p is a percept of satisfaction with a family car, then t is the degree of truth of statement "the car fully satisfies all our requirements." Obviously, if the car satisfies all requirements only in 70% of cases, then $t = 0.7$ and t is a continuum-valued graded truth. Graded Logic is a propositional calculus that processes graded truth.

C. Graded Logic and decision making

Decision making is an observable human mental process based on commonsense logical reasoning. In the most frequent case, the SDM first identifies a set of m different alternatives that can be applied to attain desired goals. The decision making can then be defined as the process of comparison of alternatives and selection (and possible realization) of the best alternative. To understand the process of human decision making and its relation to GL, there is a prerequisite: it is necessary to understand the fundamental case $m = 1$.

D. The case of single alternative

The case of a single alternative is not a special case. In decision making, that is the most important essential case. It is easy to find a number of single alternative decision problems in each human life. E.g., the most important decision in most human lives is marriage, and there is a single candidate that must be carefully evaluated. Indeed, the question is how suitable a single candidate is, and not who is the best (i.e. the least unsuitable) among several candidates. The best among several candidates/options, selected using pairwise comparison methods, can still be insufficiently suitable and justifiably rejected.

Similarly, an unemployed worker can get a single job offer and the question is whether the offer is sufficiently good to be accepted. On the other hand, a company can have a single candidate applying for an open position, and it is necessary to evaluate the competence of the single candidate and then either to accept or to reject the candidate.

The presented examples expose the evaluation process of a single candidate as a fundamental component of human-centric decision making. If a single candidate evaluation process is available, then the comparison of multiple candidates is automatically solved by comparing the results of evaluation of individual candidates.

E. The commonsense evaluation process and its logic components

The natural human commonsense evaluation process has the following easily visible components [22]:

1. Selection of suitability attributes.
2. Development of suitability attribute criteria.
3. Generating the attribute suitability degrees.
4. Logic aggregation of attribute suitability degrees.
5. Evaluation and comparison of alternatives.

We assume that the SDM has clearly defined goals and can specify requirements that the available objects/alternatives should satisfy. The first step performed by SDM is the selection of suitability attributes. Suitability attributes are all those characteristics of the evaluated objects that affect the overall suitability of each evaluated object. For example, if the evaluated object is a car, then the suitability attributes could include the power of engine, fuel economy, available space, the number of passengers, wheel drive, etc. It is also important to note that there are attributes of the evaluated object that according to the SDM's goals do not affect the suitability of evaluated objects/alternatives and such attributes are not considered by the SDM. Suitability attributes are denoted $a_1, \dots, a_n$, $a_i \in \mathbb{R}$, $i = 1, \dots, n$. Regularly, $n > 1$.

The second step is the definition of requirements that the suitability attributes must satisfy according to SDM's goals and needs. Such requirements are the attribute criteria, i.e. functions that specify the way SDM determines the suitability of each attribute. The suitability is a graded percept expressed as the graded truth of the statement that asserts the complete/full satisfaction of SDM's requirements. So, the attribute criteria are $g_i: \mathbb{R} \to [0,1]$, $i = 1, \dots, n$.

In the third step, the SDM separately evaluates each attribute of an evaluated object/alternative, and creates the attribute suitability degrees which are percepts of satisfaction of requirements each attribute is expected to satisfy. So, the SDM intuitively creates n degrees of truth $x_i = g_i(a_i)$, $i = 1, \dots, n$.

The availability of n individual percepts of satisfaction of the suitability attributes requirements is the result of the three initial steps of the commonsense evaluation. In the fourth step of human commonsense decision making, the individual percepts $x_1, \dots, x_n$ automatically contribute to forming a resulting graded percept of the overall satisfaction of requirements, $X = L(x_1, \dots, x_n)$. The aggregation function $L: [0,1]^n \to [0,1]$ is obviously a propositional logic formula. This function combines the models of simultaneity (graded conjunction), substitutability (graded disjunction) and negation. Such combinations of basic graded logic functions generate a wide spectrum of commonsense propositional calculus logic models used in human commonsense reasoning and decision making.

In the fifth (final) step of commonsense decision making, the overall satisfaction of requirements $X \in [0,1]$ is used to decide whether to accept or to reject a specific object/alternative. In addition, the degrees of overall suitability are used in the process of explainable commonsense comparison and selection of multiple competitive objects/alternatives.

III. The Concept of Fully Continuum-Valued Logic

A. The continuum-valued graded percepts

Human commonsense logical reasoning is based on graded percepts. The primary graded percept is the graded truth. It specifies the intensity of a specific graded percept as the degree of truth of a statement that claims the highest level of the percept. For example, if a car engine should ideally have 200 HP, then the degree of truth of the assertion that the car engine of 180 HP fully satisfies SDM's requirements could be 0.9.

Truth is not the only continuum-valued human graded percept. The second fundamental graded percept is importance. It is easy to note that in human commonsense logical reasoning some statements are more important than other statements. For example, for computationally intensive tasks, the processor speed of a laptop computer can be significantly more important than the weight of computer. So, the importance of statements aggregated by a graded logic function must also be continuum-valued. Human commonsense logical reasoning supports the "first things first" concept.

The most distinctive property of both the commonsense human logic and the Graded Logic is that both simultaneity and substitutability are *graded*: their intensity is continuously adjustable. Below, we discuss this characteristic property.

B. Unification of simultaneity and substitutability

In human commonsense logic, the simultaneity (graded conjunction) and the substitutability (graded disjunction) are not treated as two separated and different logic operations. Each human logic aggregator of two or more variables has both conjunctive and disjunctive properties. Conjunctive properties in evaluation are specified as a requirement that all inputs should simultaneously have (to some desired extent) high values. On the other hand, required disjunctive properties mean that a low satisfaction of any input can (to some desired extent) be substituted/compensated by a high value of any other input. These opposing requirements can be balanced in the case of the arithmetic mean, where the conjunctive properties are equally present as disjunctive properties. A typical example is the computation of the mean grade of students in schools (GPA), where high grades are simultaneously desired in all courses, but at the same time, a low grade in any course can be compensated by a high grade in any other course.

In Graded Logic [3], the logic aggregator that combines conjunctive and disjunctive properties is called Graded Conjunction/Disjunction (GCD) and denoted $y = x_1 \Diamond \ldots \Diamond x_k$ (the symbol $\Diamond$ is a combination of symbols $\wedge$ and $\vee$.

C. Andness and orness

Simultaneity and substitutability are graded, i.e., they also have adjustable intensity. In the case of simultaneity, the SDM may want that two (or more) requirements are simultaneously highly satisfied (e.g. all product buyers simultaneously want a high quality *and* a low price of selected product). It is easy to note that, in human reasoning, the intensity of simultaneity for conjunctive logic aggregators is continuously adjustable. In the case of high degree of simultaneity, SDMs frequently use *mandatory requirements*: if one of inputs is not satisfied, then the results of aggregation must be zero (i.e., such a function supports the annihilator 0). In the case of low intensity, the simultaneous satisfaction of inputs can be desirable but not mandatory. In such cases, the annihilator 0 must not be supported.

In the case of substitutability, the situation is similar: the disjunctive aggregators also have an adjustable intensity. High intensity disjunctive aggregators support the annihilator 1: if any of inputs is fully satisfied, then the high-intensity disjunctive aggregator is fully satisfied. In the case of lower intensity, the high degree of satisfaction of inputs is desirable, but not individually sufficient to fully satisfy a disjunctive criterion.

The intensity of simultaneity is called the conjunction degree or andness [11], [12] and denoted α. The intensity of substitutability is called the disjunction degree or orness and denoted ω. For the GCD aggregator $y = x_1 \Diamond \ldots \Diamond x_k$ they are defined as follows [11]:

$$\alpha = \frac{k}{k-1} - \frac{k+1}{k-1}\int_{[0,1]^k} (x_1 \Diamond \ldots \Diamond x_k) dx_1 \ldots \mathrm{d}x_k$$

$$\omega = 1 - \alpha \,.$$

According to [13],

$$\int_{[0,1]^k} (x_1 \wedge \ldots \wedge x_k) dx_1 \ldots \mathrm{d}x_k = \tfrac{1}{k+1} \,,$$

$$\int_{[0,1]^k} (x_1 \vee \ldots \vee x_k) dx_1 \ldots \mathrm{d}x_k = \tfrac{k}{k+1} \,.$$

Therefore, for $x_1 \wedge \ldots \wedge x_k$ we have $\alpha = 1,\ \omega = 0$, and for $x_1 \vee \ldots \vee x_k$ we have $\alpha = 0,\ \omega = 1$. Another important case is $x_1 \Diamond \ldots \Diamond x_k = (x_1 + \cdots + x_k)/k$ where $\alpha = \omega = 1/2$. Therefore, the arithmetic mean has the central logically neutral role as the centroid of GCD logic aggregators.

D. Duality of simultaneity and substitutability

Duality of simultaneity and substitutability is a natural property of commonsense human logic. Let $x_1 \Delta \ldots \Delta x_k$ denote the simultaneity (graded conjunction) of k logic variables and let $x_1 \nabla \ldots \nabla x_k$ denote the substitutability (graded disjunction) of the same logic variables. Then, the commonsense verbal interpretation of relationship between graded conjunction and graded disjunction is "*if we need simultaneously high satisfaction (truth values) of k inputs, then it is not acceptable that any one of them is not sufficiently satisfied*." In other words, $x_1 \Delta \ldots \Delta x_k = 1 - (1 - x_1) \nabla \ldots \nabla (1 - x_k)$. Similarly, "*if we need at least one sufficiently satisfied input, then it is not acceptable that all of them are simultaneously insufficiently satisfied*." Thus, $x_1 \nabla \ldots \nabla x_k = 1 - (1 - x_1) \Delta \ldots \Delta (1 - x_k)$. Therefore, it is obvious that De Morgan duality is naturally present in the commonsense human logic.

In the duality relationships we assume the same intensity of conjunctive and disjunctive aggregators Δ and ∇. If they have the highest idempotent intensity $\Delta = \wedge$ and $\nabla = \vee$ then we get the traditional De Morgan laws:

$$x_1 \wedge \ldots \wedge x_k = \overline{\bar{x}_k \vee \ldots \vee \bar{x}_k} \;;\; x_1 \vee \ldots \vee x_k = \overline{\bar{x}_k \wedge \ldots \wedge \bar{x}_k} \,.$$

In the case of low intensity where $\Delta = \nabla = \theta$ (the symbol θ denotes the arithmetic mean $x_1 \,\theta \ldots \theta\, x_k = w_1 x_1 + \cdots + w_k x_k$, $0 < w_i < 1,\ i = 1, \ldots, k\,,\ \ w_1 + \cdots + w_k = 1$) we have

$$1 - (1 - x_1)\,\theta \ldots \theta\,(1 - x_k) =$$

$$1 - [w_1(1 - x_1) + \cdots + w_k(1 - x_k)] = w_1 x_1 + \cdots + w_k x_k \,.$$

In Graded Logic duality holds for all logic aggregators: soft idempotent, hard idempotent, and nonidempotent hard hyperconjunction and hyperdisjunction [3].

E. The drastic conjunction

What is the strongest possible conjunction? The highest level of simultaneity of high values of k inputs is obviously the extreme requirement that all inputs must be fully satisfied $x_1 = \cdots = x_k = 1$. In all other cases the result of conjunctive aggregation is 0. Such a conjunctive function is called drastic conjunction and its analytic form is $y = \lfloor \prod_{i=1}^{k} x_i \rfloor$. Since $\int_{[0,1]^k} \lfloor \prod_{i=1}^{k} x_i \rfloor dx_1 \ldots dx_k = 0$, it follows that the highest possible andness is $\alpha = \frac{k}{k-1}$. The lowest possible orness for drastic conjunction is $\omega = 1 - \alpha = \frac{-1}{k-1}$.

F. The drastic disjunction

The drastic disjunction is a function that is the De Morgan dual of drastic conjunction: $y = 1 - \lfloor \prod_{i=1}^{k} (1 - x_i) \rfloor$. So, the strongest possible disjunction is the case where any nonzero input can fully satisfy the disjunctive criterion which is not satisfied if and only if all inputs are zero. Then, we have $\int_{[0,1]^k} \{1 - \lfloor \prod_{i=1}^{k} (1 - x_i) \rfloor\} dx_1 \ldots dx_k = 1$ and therefore, $\alpha = -\frac{1}{k-1}$, and $\omega = \frac{k}{k-1}$.

G. The interpolative GCD logic aggregator

The extreme drastic conjunction and drastic disjunction aggregators show that the GCD aggregator must cover the full range of andness and orness $\left[-\frac{1}{k-1}, \frac{k}{k-1}\right]$. To provide a continuous transition in this wide range of andness, we use interpolative logic aggregators [14]. We select a sequence of conjunctive "anchor aggregators:"

- Logic neutrality (arithmetic mean, $\alpha = 0.5$)
- Threshold hard conjunction ($\alpha = 0.75$)
- Pure conjunction ($\alpha = 1$)
- Product t-norm ($\alpha = (k2^k - k - 1)/(k - 1)2^k$)
- Drastic conjunction ($\alpha = k/(k - 1)$)

Between the anchor aggregators with andness α_p and α_q we use interpolation:

$$GCD(\boldsymbol{x};\alpha) = \frac{\alpha_q - \alpha}{\alpha_q - \alpha_p} GCD(\boldsymbol{x};\alpha_p) + \frac{\alpha - \alpha_p}{\alpha_q - \alpha_p} GCD(\boldsymbol{x};\alpha_q)$$

$$\alpha_p \le \alpha \le \alpha_q, \quad \boldsymbol{x} = (x_1, \ldots, x_k).$$

Between the logic neutrality and threshold conjunction the interpolated GCD aggregators are soft (the annihilator 0 is not supported). Above the threshold andness the interpolated GCD aggregators are hard (the annihilator 0 is supported). Below the pure conjunction the GCD aggregators are idempotent and above the pure conjunction they are nonidempotent. We use this interpolative form of GCD for $\alpha \ge 0.5$. In the disjunctive range of andness ($\alpha < 0.5$) we recursively use De Morgan duals of the conjunctive GCD:

$$GCD(\boldsymbol{x};\alpha) = 1 - GCD(\boldsymbol{1} - \boldsymbol{x}; 1 - \alpha), \quad \alpha < \frac{1}{2}.$$

H. Fully continuum-valued propositional logic

Graded Logic is a fully continuum-valued propositional logic of human commonsense reasoning. Same as in natural human commonsense reasoning, everything is a mater of degree: truth, importance, conjunction (simultaneity), and disjunction (substitutability) are continuum-valued (graded).

The concept of making a propositional logic consistent with natural commonsense human reasoning is easily justifiable by the fact that decision making is a human mental activity. Logic models that are not consistent with observable properties of human reasoning cannot generate results that are explainable and acceptable with confidence. Indeed, the credibility of decision methods that are not consistent with human commonsense decision making is generally questionable.

IV. Main Properties of Graded Logic

A. The postulates of Graded Logic

Graded Logic is not a formalized axiomatic theory, but it is built on a set of strict postulates that reflect the observable properties of human commonsense logic. There are ten such postulates [22]:

1. The truth of statements must be continuum-valued (graded).
2. The importance of statements must be continuum-valued (graded).
3. The simultaneity of statements must be continuum-valued (graded) in the full range $\frac{1}{2} \le \alpha \le \frac{k}{k-1}$.
4. The substitutability of statements must be continuum-valued (graded) in the full range $\frac{1}{2} \le \omega \le \frac{k}{k-1}$.
5. The simultaneity and substitutability must be complementary and unified.
6. Logic neutrality (arithmetic mean) must be available as a balance of simultaneity and substitutability.
7. The idempotency of logic aggregators must be selectable.
8. The annihilator support for idempotent simultaneity must be selectable.
9. The annihilator support for idempotent substitutability must be selectable.
10. The simultaneity and substitutability models must be dual.

B. The Graded Logic Conjecture

Each propositional calculus uses a set of basic logic functions to create compound logic formulas. GL is a propositional logic and therefore the fundamental question is to select the necessary and sufficient basic logic functions of graded propositional calculus. According to *Graded Logic Conjecture (GLC)* [3], necessary and sufficient basic Graded Logic functions include ten characteristic functions: nine characteristic special cases of the GCD aggregator and negation. Following are the GLC functions, classified by andness and their support of GL postulates [22]:

1. Graded hyperconjunction ($\alpha > 1$) [C/A0/NI]
2. Pure conjunction – minimum ($\alpha = 1$) [C/A0/ID]
3. Hard graded conjunction ($0.75 \le \alpha < 1$) [C/A0/ID]
4. Soft graded conjunction ($0.5 < \alpha < 0.75$) [C/NA/ID]

5. Logic neutrality ($\alpha = 0.5$) [N/NA/ID]
6. Soft graded disjunction ($0.25 < \alpha < 0.5$) [D/NA/ID]
7. Hard graded disjunction ($0 < \alpha \leq 0.25$) [D/A1/ID]
8. Pure disjunction - maximum ($\alpha = 0$) [D/A1/ID]
9. Graded hyperdisjunction ($\alpha < 0$) [D/A1/NI]
10. Negation (which is not an aggregator)

The classification codes [type/annihilators/idempotence] are the following: C = conjunctive, D = disjunctive, N = neutral, A0 = supports annihilator 0, A1 = supports annihilator 1, NA = no support for annihilators, ID = idempotent, NI = nonidempotent.

The GLC is supported by the following properties/facts:

- The GLC functions explicitly support the functionality requested in the postulates of Graded Logic
- All ten GLC functions are provably used in human commonsense logical reasoning.
- Observations of human commonsense logical reasoning have not detected logical reasoning patterns that are not modellable by the presented GLC list of basic GL functions. In particular, all canonical logic aggregation structures detected in the area of decision making, and reported in [3], are modellable using combinations of GLC functions.
- The GLC functions include pure conjunction, disjunction and negation which are necessary and sufficient in classical Boolean logic, making GL a generalization of the classical Boolean logic.

Since GL is not an axiomatic formal system, the satisfaction of GL postulates and consistency with observable commonsense human logic can be used as a sufficient support for conclusion that ten GLC functions are necessary and sufficient to create all formulas of the graded propositional calculus used in natural human reasoning. Hyperconjunction, hard and soft conjunctive and disjunctive GCD, and hyperdisjunction have continuously adjustable andness/orness in their respective ranges.

GCD functions that support annihilators (A0, A1) are denoted as *hard*, and GCD functions that do not support annihilators (NA) are denoted as *soft*. These logic aggregators have the following verbalized interpretation:

- *Must have all inputs*: hard conjunctive.
- *Nice to have most inputs*: soft conjunctive.
- *Nice to have some inputs*: soft disjunctive.
- *Enough to have any input*: hard disjunctive.

According to interpolative method of GCD design, the anchor aggregators have a constant andness, and the inter-anchor aggregators cover a range of andness. The locations of anchor aggregators that are thresholds between soft and hard GCD aggregators are freely adjustable, but in [14] the uniform distribution of hard and soft properties based on thresholds $\alpha = 0.75$ and $\alpha = 0.25$ is experimentally verified to be the closest to the commonsense human logical reasoning. The properties of the GCD function in the full range of andness (from the drastic conjunction CC to the drastic disjunction DD) are shown in Fig. 1 (dark gray area = soft conjunction/disjunction, light gray area = hard conjunction/disjunction, white area = hyperconjunction and hyperdisjunction).

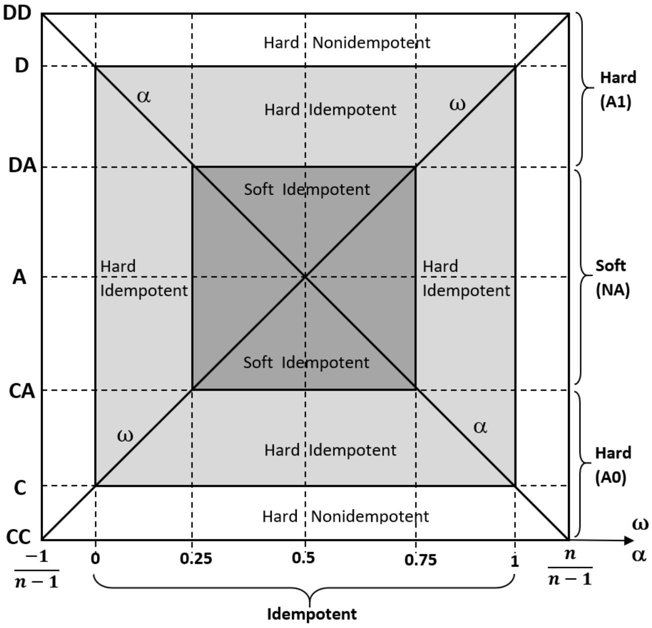

Fig 1. GCD logic aggregator in the full range of andness/orness

C. Advanced Graded Logic constructs

Combinations of GLC functions yield an infinite number of possible propositional calculus formulas. Some of them have a frequent use in logic decision models. Three most important constructs are (1) the partial absorption, (2) the selector-based formulas, and (3) the nonstationary formulas.

The *partial absorption function* [12], [15], [16], [3], [22] aggregates two asymmetric inputs: mandatory/optional and sufficient/optional. It uses the weighted arithmetic mean $\diamond$, the hard partial conjunction Δ, and the hard partial disjunction ∇ to create the *conjunctive partial absorption* function $CPA(x,y) = x\Delta(x \diamond y)$ and the *disjunctive partial absorption* $DPA(x,y) = x\nabla(x \diamond y)$. The basic property of CPA is $CPA(0,y) = 0$, showing that x is a mandatory input that must be satisfied. The basic property of DPA is $DPA(1,y) = 1$, showing that x is a sufficient input and it is enough to fully satisfy this input. In both cases, y is an optional input. If $0 < x < 1$, then $CPA(x,0) = x - P$, $CPA(x,1) = x + R$, $P > R$. Similarly, $DPA(x,0) = x - P$, $DPA(x,1) = x + R$, $R > P$. The parameter P is called penalty, and the parameter R is called reward. Users must select the desired mean values of P and R, and the detailed organization of CPA and DPA aggregators can be obtained using appropriate software tools [17]-[19].

In some cases, propositional logic formulas must be combined with the ***if-then-else*** control structures [22]. That can be achieved using the selector function which compares the input degree of truth x with a threshold value T as follows:

$$b = SEL(x,T) = \begin{cases} 1, & x \geq T \\ 0 & x < T \end{cases} .$$

The selector function can be combined with the GL conjunction (C), disjunction (D), and negation (not) as shown in Fig. 2 to achieve the following general if-then-else construct:

$$z = \begin{cases} L_1(\boldsymbol{X}_1), & x \geq T \\ L_2(\boldsymbol{X}_2), & x < T \end{cases} .$$

Here $y_1 = L_1(\boldsymbol{X}_1) \in [0,1]$ denotes a graded propositional calculus formula based on an array of input degrees of truth $\boldsymbol{X}_1$. Likewise, $y_2 = L_2(\boldsymbol{X}_2) \in [0,1]$ denotes a graded propositional calculus formula based on an array of input degrees of truth $\boldsymbol{X}_2$. Generally, $\boldsymbol{X}_1 \neq \boldsymbol{X}_2$, but frequently we can have $\boldsymbol{X}_1 = \boldsymbol{X}_2 = \boldsymbol{X}$. Similar reasoning can be applied to L_1 and L_2; e.g., these can be the same propositional formulas that differ only in weights or only in selected inputs. The selector variables x and T can be independent inputs or selected components of arrays $\boldsymbol{X}_1$ and $\boldsymbol{X}_2$. Obviously, the if-then-else construct provides a very high flexibility for the development of sophisticated graded propositional calculus formulas.

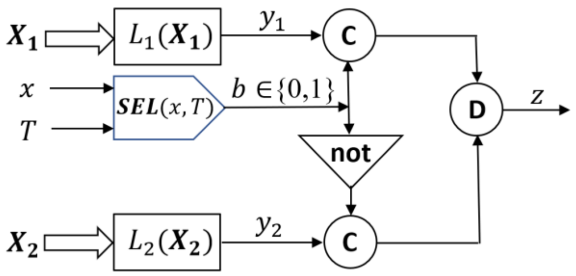

Fig 2. A general if-then-else construct implemented in GL [22]

Graded propositional calculus formulas with fixed structure and constant parameters (weights and andness/orness) are called *stationary* GL models. They are most frequently used in decision-making models that provide the overall suitability of various competitive objects/alternatives. Much less frequently we can use *nonstationary* GL models [3], where the parameters (weights, andness, orness) can be functions of input attributes. The if-then-else constructs are a special case of the nonstationary graded logic models. Of course, while the design of stationary decision models is simple and requires very modest effort, the design of nonstationary models needs significantly higher effort and a thorough justification.

V. Professional Decision Making

A. Characteristics of professional decision making

Professional decision-making problems can be found in many areas. A detailed survey of such problems and corresponding examples can be found in [3] and [22]. Following are the main characteristics of such problems:

1. *The need for domain experts.* Graded logic decision problems cannot be successfully solved without expertise in the area of organization and functioning of evaluated objects. For example, all medical decision problems (e.g. evaluation of patient priority for organ transplantation) need the collaboration with medical doctors. Evaluation teams that evaluate, compare, and select aircrafts for specific stakeholder, should include pilots. In these examples, medical doctors and pilots play the role of domain experts: they provide knowledge that is necessary to select suitability attributes, suggest importance weights, decide about the hard and soft aggregation, etc.
2. *The need for decision engineers.* Decision engineers are professional evaluators specialized in decision methods, experienced in solving decision problems, and familiar with the use of specialized software tools that are necessary for development and use of decision models. Decision engineers are central participants in evaluation teams, responsible for logic methodology, scheduling of activities and for communication with other participants in the professional decision-making team.
3. *The role of stakeholder.* In professional decision making, the stakeholder is an organization that makes decisions about selecting an object/alternative that will contribute to attaining stakeholder's goals. The stakeholder decides about development goals, provides all financing, accepts or rejects the selected best alternative, and bears the consequences of the realized decision.
4. *Organization of professional decision-making teams.* Professional decision-making teams consist of three participants: (1) stakeholder, (2) decision engineer (evaluator), and (3) domain expert. Each participant can be a single person or a group of people. In some cases, a single person can play more than one role (e.g. a decision engineer can also be a domain expert).
5. *Large number of suitability attributes.* Decision problems based on logical reasoning/evaluation can be classified according to the number of input suitability attributes. The cases below 10 inputs can be considered toy problems, popular as examples in theoretical papers. The problems below 50 inputs are frequent in individual decision making (comparison and selection of cars, homes, educational institutions, etc.). Low to medium complexity professional problems include 50 to 150 inputs. Higher complexity problems can have 150 to 600 input attributes.
6. *Precision of logic decision models.* The professional decision models should be as precise as possible. The main goal in this area is to include all suitability attributes, i.e., all components that provably affect the overall suitability of evaluated object/alternative. Equally important is to avoid considering attributes that characterize the evaluated object, but do not affect its suitability (e.g., the color of a chassis of electronic equipment is an attribute that does not affect suitability).
7. *Sensitivity and tradeoff analysis.* Before starting the use of professional decision models, it is advisable to perform a sensitivity analysis (the analysis of the impact of individual inputs on the final decision result)

and a tradeoff analysis (an analysis of compensatory properties of inputs of decision models – the capability of selected input to compensate the deficiency of another input).

8. *Reliability analysis.* The parameters of decision models (e.g., andness and importance weights) are determined by decision-making teams with limited accuracy. Reliability analysis is necessary to assess possible errors of final evaluation and selection results, and the reliability of the ranking of competitors.
9. *Optimization of evaluated objects.* For objects that have cost, stakeholders are frequently interested in solving the following optimization problems: (1) find the minimum cost necessary for achieving a specific degree of overall suitability of an evaluated object; (2) find the highest overall suitability that can be achieved with specific approved financial resources; (3) find the configuration of an evaluated object so that it yields the highest suitability obtained per invested monetary unit.
10. *Tolerance of missing data.* In some cases, the values of some suitability attributes are not available. In such cases there are two possibilities: (1) disqualify the object/alternative that has incomplete inputs, or (2) perform the decision process replacing nonexistent inputs with neutral values [20]. In most applications, the preferred method is the missingness-tolerant aggregation.
11. *The need for explainability of results.* All decisions need and can be explained in a simple verbal way. That is particularly important in professional decision making where decisions must be understood and accepted by many people in the stakeholder organization. A quantitative explainability method for evaluation decision results can be found in [21].

B. The Logic Scoring of Preference Method

Our basic concept in professional decision problems is that such problems should be solved using methods fully consistent with commonsense human logical reasoning and decision making. The method that we propose is the Logic Scoring of Preference (LSP), which is presented in detail in [3]. LSP is a human-centric decision method based on Graded Logic, organized according to observable patterns of human commonsense decision making. Consequently, it consists of the following five major steps.

1. *Identification of stakeholders and their goals.* The goal of decision making is to find the best way to satisfy goals and requirements of specific SDM. So, the initial step in the LSP method is to clearly identify the stakeholder, the purpose of evaluated objects/alternatives and the goals of evaluation and selection process. From precise identification of SDM goals and requirements it is possible to create analytic LSP decision models in correct and fully justifiable way.
2. *Development of the suitability attribute tree.* In natural commonsense decision making the number of suitability attributes is small and SDM can identify them easily and in any order. As opposed to that, in professional decision making, it is necessary to develop a large number of attributes and that must be done in an organized and systematic way. The LSP method develops suitability attributes using a hierarchical stepwise decomposition process that starts with a single root node (overall suitability). This node is then decomposed in main components (e.g. a complex computer system can be decomposed into four main components: hardware, software, performance, and vendor support). In the next step, each component is further decomposed, creating a tree structure. At the end of decomposition process we reach components that cannot be further decomposed (e.g. the computer memory capacity is directly measurable and cannot be further decomposed). These leaves of the suitability attribute tree are suitability attributes. In a special case of a binary attribute tree with n suitability attributes, the total number of decomposable nodes is $\frac{n}{2}+\frac{n}{4}+\cdots+2+1=n-1$. So, the effort for creating a binary suitability attribute tree is proportional to $n-1$. For non-binary trees the effort is less than the effort for the binary tree.
3. *Definition of elementary suitability attribute criteria.* For each of n suitability attributes it is necessary to create an evaluation function called the attribute criterion. E.g., if M is the memory capacity, and if $M \le M_{min}$ is not acceptable and $M \ge M_{max}$ completely satisfies SDM's requirements, then the memory attribute suitability criterion could be the following: $x = g(M) = min\big(1, max(0, (M-M_{min})/(M_{max}-M_{min}))\big)$. The total effort of creating attribute criteria is proportional to n.
4. *Development of the graded logic aggregation structure.* The logic aggregation of n attribute suitability degrees follows the attribute suitability tree, going node by node from the leaves towards the root of the tree. In this process it is necessary to create $n-1$ (or less) graded logic functions. Using these aggregation functions, the LSP method provides the graded logic criterion for computing the overall suitability $X = L(g_1(a_1),\dots,g_n(a_n))$ as a graded propositional calculus formula. The effort to complete this step is proportional to n-1.
5. *Computation of the overall suitability and value.* Suppose that we have $m > 1$ objects/alternatives that have costs $C_1,\dots,C_m$. If the approved budget is limited to C_{max}, then we assume that $C_i \le C_{max}$, $i = 1,\dots,m$. We also assume that the overall suitability must be above the minimum threshold X_{min}, and consequently $X_i \ge X_{min}$, $i = 1,\dots,m$. All SDMs are interested in high overall suitability achieved simultaneously with the low cost. Consequently, the overall value V_i of each alternative is a hard graded conjunction of the relative suitability and the relative cost:

$$V_i = \frac{X_i}{\max(X_1,\dots,X_m)} \Delta \frac{\min(C_1,\dots,C_m)}{C_i},\ \ i = 1,\dots,m\,.$$

Obviously, $V_i \in [0,1]$, and such an aggregator can be a weighted geometric mean:

$$V_i = \left(\frac{X_i}{\max(X_1,\dots,X_m)}\right)^w \left(\frac{\min(C_1,\dots,C_m)}{C_i}\right)^{1-w}, \; i = 1,\dots,m.$$

If the SDM is in situation where the suitability is more important then affordability, then $w > 1/2$. If the affordability is more important, then $w < 1/2$.

The best alternative (and the proposed decision) is the alternative/object that has the maximum value: $V^* = \max(V_1,\dots,V_m)$. In the special case $m = 1$, the single alternative is considered acceptable if $X_1 \geq X_{min}$ and $C_1 \leq C_{max}$.

The LSP method in steps 2 and 4 needs effort proportional to $n - 1$ and in step 3 the effort proportional to n. Therefore, LSP is a linear algorithm: the overall LSP effort is $O(n)$. This is a very important property because it shows that the LSP method strictly supports the human commonsense logical reasoning, but expands the applicability of this form of reasoning far beyond the natural limitations of human intuitive mental processes. That justifies the use of the LSP method in the sensitive area of professional decision making.

VI. Conclusion

We presented, contrasted, and confronted two approaches to the development of continuum-valued propositional logic: (1) the formal axiomatic deductive approach used in development of logic theories, and (2) the human-centric approach based on human commonsense logical reasoning with graded percepts. Our goal is to show that methods for professional decision making must be consistent with human commonsense decision making, and that human commonsense decision making is based on Graded Logic, which is the logic of natural human logical reasoning with graded percepts. Observations and applications show that the human-centric approach to logic and decision methods is a natural way to develop methods for professional decision making. In this paper we presented a condensed survey of the Graded Logic and its use in the development and use of the LSP method for professional decision making.

Graded Logic is a fully continuum-valued propositional logic: the continuum-valued variables and parameters include the graded truth, the graded importance, and the graded conjunction/disjunction. These three graded percepts are provably present in human commonsense logical reasoning. Unique properties of GL are (1) continuous transition in the whole range from the drastic conjunction to the drastic disjunction, (2) unification of complementary models of simultaneity and substitutability in a single general logic aggregator GCD, (3) selectability of annihilators, (4) selectability of idempotent or non-idempotent logic aggregators, (5) andness-directedness: visibility and adjustability of andness/orness as input parameters of the GCD function, and (6) support for stationary and nonstationary graded logic aggregators.

The LSP decision method uses all unique properties of GL to provide advanced professional decision methodology that is fully consistent with human commonsense logical reasoning. Both GL and the LSP method have a history of successful applications, but they also offer a variety of topics for future work. These topics include LSP applications in new (particularly medical) areas, experiments with human subjects to verify and expand GL models (particularly in the areas of hyperconjunction and hyperdisjunction), the comparison of LSP with other similar methods, the development and study of a variety of nonstationary criteria, as well as the development of new decision-support software tools and their applications.

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