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Applications of new q-rung orthopair fuzzy rough distance measures in pattern recognition and disease dignosis problems

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***Abstract*—As the combined version of rough sets (RSs) and q-rung orthopair fuzzy sets (q-ROFSs), the idea of q-rung orthopair fuzzy rough sets (q-ROFRSs) is more flexible to deal with inaccurate, uncertain and incomplete data. In this manuscript, we propose various q-rung orthopair fuzzy rough distance measures for computing the distance between q-ROFRSs. Some examples are discussed to exemplify the efficacy of developed q-ROFR-distance measures over existing ones. We further demonstrate its utility in pattern recognition and crop disease diagnosis problems. We also establish the superiority of developed distance measures over existing distance measures on q-ROFRSs in view of the structured linguistic variables.**

***Index Terms*—q-rung orthopair fuzzy rough set; distance measure; pattern recognition; medical diagnosis.**

I. Introduction

TO HANDLE the uncertain knowledge, Pawlak (1982) introduced a mathematical approach, named as rough set theory (RST), which has been widely implemented for various purposes (Sayed et al., 2024; Hosny et al., 2024). Dubois & Prade (1990) invented an idea of fuzzy rough set (FRS) to deal with granuality, incompleteness and uncertainty of knowledge in information measures. As an extended version, Zhang et al. (2012) pioneered the intuitionistic FRSs and implemented to the decision-making area. Further, Sun & Ma (2014) combined soft set and FRSs, and developed the notion of soft fuzzy rough sets (SFRSs). A q-rung orthopair fuzzy set (q-ROFS) (Yager, 2017) is an extended version of fuzzy set (FS) in which q^{th} powers sum of membership grade (MG) and nonmembership grade (NG) is ≤ 1, where $q \geq 1$. Yager (2017) pointed out that the space of acceptable orthopairs increases as value of q increases, therefore, the q-ROFS offers more choice to scholars in stating their confidence. The doctrine of q-ROFSs is more authoritative than the FS (Zadeh, 1965), intuitionistic fuzzy set (IFS) (Atanassov, 1986), Pythagorean fuzzy set (PFS) (Yager, 2014) and Fermatean fuzzy set (FFS) (Senapati & Yager, 2020) since all types of sets are contained in the space of q-ROFSs (Yager, 2017).

Khoshaim et al. (2021) integrated the notions of rough set and q-ROFS and gave a new idea namely q-rung orthopiar fuzzy rough set (q-ROFRS). A q-ROFRS offers benefits of q-ROFS as well as rough set. For the first time, Khoshaim et al. (2021) presented the basic aggregation operators (AOs) to unite the q-ROFRNs into a single q-ROFRN. Ashraf et al. (2021) proposed some AOs based on the combination of Einstein norms and q-ROFRNs. Further, a q-ROFR Einstein AOs-based EDAS approach has been presented for robotic agrifarming assessment problem. In a study, Liu et al. (2021) gave an axiomatic definition of distance measure for q-ROFRSs. Based on the distance measure, score function and AOs, they introduced a hybrid decision support system and its application in major infrastructure projects assessment. To assess the ship energy alternatives, Qahtan et al. (2023) presented a fuzzy decision with opinion score model under q-ROFRS environment. Moreover, the weights of evaluation criteria have been determined through fuzzy-weighted zero-inconsistency model. With the use of q-ROFRSs, Mishra et al. (2024) studied a combined multiple-criteria group decision-making (MCGDM) model consisting of symmetry point of criterion (SPC) tool for objective weight of indicators, ranking comparison (RANCOM) tool for subjective weight of indicators and multi-attribute multi-objective optimization based on ratio assessment (MULTIMOORA) approach to evaluate and rank the sustainable enterprise resource planning systems.

Distance measure is a vital mathematical way to compute degree of discrimination between two objects. This concept has widely been utilized to the medical dignosis, MCGDM and pattern recognition problems (Alrasheedi et al., 2023; Gogoi et al., 2023; Rani et al., 2024). Using distance measure, Wang et al. (2019) planned distance measure and FRS-based approach for reducing the number of attributes. They developed some iterative forms to determine fuzzy rough dependency and improtance degree of attributes and introdcued iterative assessemnt framework using variable distance parameter. Based on granular distance, An et al. (2021) studied a robust FRS approach and applied it in feature selection problem. Sahu et al. (2021) studied distance measure on picture fuzzy rough sets (PFRSs) and applied for career selection of students. Tiwari & Lohani (2023) studied a conflict distance measure between interval-valued IFSs and its application in MCGDM problem. Using weighted FRSs, Wang et al. [24] presented distance measure between the sample and other samples in a feature selection problem.

In the context of q-ROFRSs, Khoshaim et al. [12] gave the distance measure for computing the dissimilarity between considered criteria during the assessment of emergency MCGDM problem. Liu et al. [14] gave an idea of Euclidean q-ROFR-distance measure and discussed its application. Khan et al. [25] presented the hamming q-ROFR-distance measure and its utility in the evaluation of positive and negative ideal solutions. Some of these measures are unable to make the difference between q-ROFRSs. To overcome drawbacks of extant distances (Liu et al. [14]; Khoshaim et al. [12], Khan et al. [25]), this work introduces some distance measures for q-ROFRSs, which take into account the lower approximation and upper approximation MG and NG functions. Further, a utility of introduced distance measures are discussed on pattern recognition, crop disease diagnosis and medical diagnosis problems.

Other sections are presented in the following way. Section 2 presents the fundamental definitions related to q-ROFRSs. Section 3 introduces three distance measures for computing the degree of distance between q-ROFRSs. Section 4 applies the developed q-ROFR-distance measures to pattern recognition and crop disease diagnosis problem. Section 5 accomplishes the whole work.

II. PRELIMINARIES

In the section, we first present basic notions related to q-ROFRSs.

Definition 2.1 [7]. Let $R=\{r_1,r_2,...,r_n\}$ be a fixed discourse set. A q-ROFS G on R is mathematically defined as

$$G=\left\{\left(r_i,\mu_G\left(r_i\right),\nu_G\left(r_i\right)\right)\middle| r_i\in R\right\}, \tag{1}$$

wherein $\mu_G:R\to[0,1]$ and $\nu_G:R\to[0,1]$ denote MG and NG of an object $r_i\in R,$ respectively, with constraints $0\le\mu_G\left(r_i\right)\le 1,\quad 0\le\nu_G\left(r_i\right)\le 1,\quad 0\le\left(\mu_G\left(r_i\right)\right)^q+\left(\nu_G\left(r_i\right)\right)^q\le 1,$ $q\ge 1,\forall\, r_i\in R.$ For $r_i\in R,$ a hesitancy grade is defined as $\pi_G\left(r_i\right)=\sqrt[q]{1-\left(\mu_G\left(r_i\right)\right)^q-\left(\nu_G\left(r_i\right)\right)^q}.$

Definition 2.2 [13]. Consider R be a fixed discourse set and $\zeta\in R\times R$ be a crisp relation. Then

(i) ζ is reflexive if $(\wp,\wp)\in\zeta,\ \forall\,\wp\in R,$

(ii) ζ is symmetric if $\wp,\partial\in R$ and $(\wp,\partial)\in\zeta,$ then $(\partial,\wp)\in\zeta,$

(iii) ζ is transitive if $\wp,\partial,\ell\in R,$ $(\wp,\partial)\in\zeta$ and $(\partial,\ell)\in\zeta,$ then $(\wp,\ell)\in\zeta.$

Definition 2.3 [12]. Let $\zeta\in R\times R$ be defined as any arbitrary relation over R. Now, define a mapping $\zeta^*:R\to P(R)$ as

$$\zeta^*\left(\wp\right)=\left\{\partial\in R:\left(\wp,\partial\right)\in\zeta\right\},\ \text{for}\ \wp\in R, \tag{2}$$

where $\zeta^*(\wp)$ is an object's successor neighborhood $\wp$ with respect to ζ. Crisp approximation space (AS) is described as a pair (R,ζ). The lower and upper approximation of $\Im$ over (R,ζ), for each $\Im\in R$ are given by

$$\underline{\zeta}\left(\Im\right)=\left\{\wp\in R:\zeta^*\left(\wp\right)\subseteq\Im\right\}, \tag{3}$$

$$\bar{\zeta}\left(\Im\right)=\left\{\wp\in R:\zeta^*\left(\wp\right)\cap\Im\neq\phi\right\}. \tag{4}$$

The pair $\left(\underline{\zeta}(\Im),\bar{\zeta}(\Im)\right)$ is stated as a rough set (RS) and $\underline{\zeta}(\Im),\bar{\zeta}(\Im):P(R)\to P(R)$ are lower and upper approximation operators, respectively.

Definition 2.4 [12]. Let R be a fixed discourse set and $\zeta\in q-ROFS\left(R\times R\right)$ be any q-ROF-relation on R. Then

(i) ζ is reflexive if $\mu_\zeta\left(\wp,\wp\right)=1$ and $\nu_\zeta\left(\wp,\wp\right)=0,\forall\,\wp\in R,$

(ii) ζ is symmetric if $\left(\wp,\partial\right)\in R\times R,$ $\mu_\zeta\left(\wp,\partial\right)=\mu_\zeta\left(\partial,\wp\right)$ and $\nu_\zeta\left(\wp,\partial\right)=\nu_\zeta\left(\partial,\wp\right),$

(iii) ζ is transitive if $\left(\wp,\ell\right)\in R\times R,$ $\mu_\zeta\left(\wp,\ell\right)\ge\vee_{\partial\in R}\left[\mu_\zeta\left(\wp,\partial\right)\vee\mu_\zeta\left(\partial,\ell\right)\right]$ and $\nu_\zeta\left(\wp,\ell\right)=\wedge_{\partial\in R}\left[\nu_\zeta\left(\wp,\partial\right)\wedge\nu_\zeta\left(\partial,\ell\right)\right].$

Definition 2.5 [12]. Let R be a fixed discourse set and $\zeta\in q-ROFS\left(R\times R\right)$ be any non-empty q-rung orthopair fuzzy relation on R. The pair (R,ζ) is therefore stated as a q-rung orthopair fuzzy approximation space (q-ROFAS). The lower and upper approximation of $\Im$ over AS (R,ζ) are two q-ROFSs for any $\Im\subseteq q-ROFS(R),$ given by

$$\underline{\zeta}\left(\Im\right)=\left\{\left(\wp,\mu_{\underline{\zeta}}\left(\wp\right),\nu_{\underline{\zeta}}\left(\wp\right)\right):\wp\in R\right\}, \tag{5}$$

$$\bar{\zeta}\left(\Im\right)=\left\{\left(\wp,\mu_{\bar{\zeta}}\left(\wp\right),\nu_{\bar{\zeta}}\left(\wp\right)\right):\wp\in R\right\}, \tag{6}$$

where $\mu_{\underline{\zeta}(\Im)}\left(\wp\right)=\underset{\partial\in R}{\wedge}\left[\mu_\zeta\left(\wp,\partial\right)\wedge\mu_\Im\left(\partial\right)\right],$ $\nu_{\underline{\zeta}(\Im)}\left(\wp\right)=\underset{\partial\in R}{\vee}\left[\nu_\zeta\left(\wp,\partial\right)\vee\nu_\Im\left(\partial\right)\right],$ $\mu_{\bar{\zeta}(\Im)}\left(\wp\right)=\underset{\partial\in R}{\vee}\left[\mu_\zeta\left(\wp,\partial\right)\vee\mu_\Im\left(\partial\right)\right],$ $\nu_{\bar{\zeta}(\Im)}\left(\wp\right)=\underset{\partial\in R}{\wedge}\left[\nu_\zeta\left(\wp,\partial\right)\wedge\nu_\Im\left(\partial\right)\right],$ satisfying $0\le\left(\mu_{\underline{\zeta}(\Im)}\left(\wp\right)\right)^q+\left(\nu_{\underline{\zeta}(\Im)}\left(\wp\right)\right)^q\le 1$ and $0\le\left(\mu_{\bar{\zeta}(\Im)}\left(\wp\right)\right)^q+\left(\nu_{\bar{\zeta}(\Im)}\left(\wp\right)\right)^q\le 1,\ q\ge 1.$ As $\bar{\zeta}(\Im)$ and $\bar{\zeta}(\Im)$ are q-ROFSs, $\underline{\zeta}\left(\Im\right),\bar{\zeta}\left(\Im\right):q-ROFS\left(R\right)\to q-ROFS\left(R\right)$ are lower and upper approximation operators. Thus, a pair $\zeta\left(\Im\right)=\left(\underline{\zeta}\left(\Im\right),\bar{\zeta}\left(\Im\right)\right)$ $=\left\{\left(\wp,\left(\mu_{\underline{\zeta}(\Im)}\left(\wp\right),\nu_{\underline{\zeta}(\Im)}\left(\wp\right)\right),\left(\mu_{\bar{\zeta}(\Im)}\left(\wp\right),\nu_{\bar{\zeta}(\Im)}\left(\wp\right)\right)\right):\wp\in R\right\}$ is referred as q-ROFRS. For ease,

$$\zeta(\Im)=\left\{\left\langle \wp,\left(\mu_{\underline{\zeta}(\Im)}(\wp),\nu_{\underline{\zeta}(\Im)}(\wp)\right),\left(\mu_{\bar{\zeta}(\Im)}(\wp),\nu_{\bar{\zeta}(\Im)}(\wp)\right)\right\rangle:\wp\in R\right\}$$

is defined as $\zeta(\Im)=\left((\underline{\mu},\underline{\nu}),(\bar{\mu},\bar{\nu})\right)$ and named as q-rung orthopair fuzzy rough number (q-ROFRN) and its collection is acknowledged as q-ROFRS(R).

Definition 2.6 [14]. Let $\zeta(\Im_1)=\left(\underline{\zeta}(\Im_1),\bar{\zeta}(\Im_1)\right)=\left((\underline{\mu}_1,\underline{\nu}_1),(\bar{\mu}_1,\bar{\nu}_1)\right)$ and $\zeta(\Im_2)=\left(\underline{\zeta}(\Im_2),\bar{\zeta}(\Im_2)\right)=\left((\underline{\mu}_2,\underline{\nu}_2),(\bar{\mu}_2,\bar{\nu}_2)\right)$ be two q-ROFRNs and $\alpha>0$ be a real number. Then, Liu et al. [14] defined some operations on q-ROFRNs, given as

(i) $\left(\zeta(\Im_j)\right)^c=\left(\left(\bar{\zeta}(\Im_j)\right)^c\times\left(\underline{\zeta}(\Im_j)\right)^c\right)=\left((\bar{\nu}_j,\bar{\mu}_j),(\underline{\nu}_j,\underline{\mu}_j)\right)$,

(ii) $\zeta(\Im_1)+\zeta(\Im_2)=\left(\underline{\zeta}(\Im_1)\oplus\underline{\zeta}(\Im_2),\bar{\zeta}(\Im_1)\oplus\bar{\zeta}(\Im_2)\right)$,

(iii) $\zeta(\Im_1)\times\zeta(\Im_2)=\left(\underline{\zeta}(\Im_1)\otimes\underline{\zeta}(\Im_2),\bar{\zeta}(\Im_1)\otimes\bar{\zeta}(\Im_2)\right)$,

(iv) $\alpha\zeta(\Im_j)=\left(\alpha\underline{\zeta}(\Im_j),\alpha\bar{\zeta}(\Im_j)\right),\ j=1,2,$

(v) $\left(\zeta(\Im_j)\right)^{\alpha}=\left(\left(\underline{\zeta}(\Im_j)\right)^{\alpha},\left(\bar{\zeta}(\Im_j)\right)^{\alpha}\right),\ j=1,2,$

(vi) $\dfrac{\zeta(\Im_1)}{\zeta(\Im_2)}=\zeta(\Im_1)\times\left(\zeta(\Im_2)\right)^c$

$$=\left(\underline{\zeta}(\Im_1)\otimes\left(\bar{\zeta}(\Im_2)\right)^c,\bar{\zeta}(\Im_1)\otimes\left(\underline{\zeta}(\Im_2)\right)^c\right)$$

Definition 2.7 [14]. Let G, H and T be three q-ROFRSs. A q-ROFR distance measure $d: q-ROFRSs(R)\times q-ROFRSs(R)\to[0,1]$ is a real-valued mapping which satisfies the given axioms:

(i) $d(G,H)\geq 0,$

(ii) $d(G,H)=0$ iff $G=H,$

(iii) $d(G,H)=d(H,G),$

(iv) If $G\subseteq H\subseteq T,$ then $d(G,H)\leq d(G,T)$ and $d(H,T)\leq d(G,T).$

III. Proposed q-ROFR-Distance Measures

This section develops some distance measures to calculate the degree of dissimilarity between q-ROFRSs. Moreover, some examples are discussed to illustrate the usefulness of developed distance measures over extant distance measures (Khoshaim et al. [12], Liu et al. [14], Khan et al. [25]).

Let G and H be the q-ROFRSs. Then three q-ROFR-distance measures are given as

$$d_1(G,H)=\sqrt{\frac{1}{4n}\sum_{i=1}^{n}\left(\begin{array}{l}\left|\sqrt{\underline{\mu}_G^q(r_i)}-\sqrt{\underline{\mu}_H^q(r_i)}\right|+\left|\sqrt{\underline{\nu}_G^q(r_i)}-\sqrt{\underline{\nu}_H^q(r_i)}\right|\\+\left|\sqrt{\bar{\mu}_G^q(r_i)}-\sqrt{\bar{\mu}_H^q(r_i)}\right|+\left|\sqrt{\bar{\nu}_G^q(r_i)}-\sqrt{\bar{\nu}_H^q(r_i)}\right|\end{array}\right)}. \quad (7)$$

$$d_2(G,H)=\sqrt{\frac{1}{4n}\sum_{i=1}^{n}\left(\begin{array}{l}\left|\sqrt{\underline{\mu}_G^q(r_i)}-\sqrt{\underline{\mu}_H^q(r_i)}\right|+\left|\sqrt{\underline{\nu}_G^q(r_i)}-\sqrt{\underline{\nu}_H^q(r_i)}\right|\\+\left|\sqrt{\underline{\pi}_G^q(r_i)}-\sqrt{\underline{\pi}_H^q(r_i)}\right|+\left|\sqrt{\bar{\mu}_G^q(r_i)}-\sqrt{\bar{\mu}_H^q(r_i)}\right|\\+\left|\sqrt{\bar{\nu}_G^q(r_i)}-\sqrt{\bar{\nu}_H^q(r_i)}\right|+\left|\sqrt{\bar{\pi}_G^q(r_i)}-\sqrt{\bar{\pi}_H^q(r_i)}\right|\end{array}\right)}. \quad (8)$$

$$d_3(G,H)=\sqrt{\frac{3}{4n}\sum_{i=1}^{n}\left(\begin{array}{l}\dfrac{\left(\underline{\mu}_G^q(r_i)-\underline{\mu}_H^q(r_i)\right)^2}{\underline{\mu}_G^q(r_i)+\underline{\mu}_H^q(r_i)+2}+\dfrac{\left(\underline{\nu}_G^q(r_i)-\underline{\nu}_H^q(r_i)\right)^2}{\underline{\nu}_G^q(r_i)+\underline{\nu}_H^q(r_i)+2}\\+\dfrac{\left(\bar{\mu}_G^q(r_i)-\bar{\mu}_H^q(r_i)\right)^2}{\bar{\mu}_G^q(r_i)+\bar{\mu}_H^q(r_i)+2}+\dfrac{\left(\bar{\nu}_G^q(r_i)-\bar{\nu}_H^q(r_i)\right)^2}{\bar{\nu}_G^q(r_i)+\bar{\nu}_H^q(r_i)+2}\end{array}\right)}. \quad (9)$$

Property 3.1. For two q-ROFRSs G and H, $0\leq d_j(G,H)\leq 1,$ where j = 1, 2, 3.

Property 3.2. For two q-ROFRSs G and H, $d_j(G,H)=0$ iff $G=H,$ where j = 1, 2, 3.

Property 3.2. $d_j(G,H)=d_j(H,G),$ where G and H are two q-ROFRSs and j = 1, 2, 3.

Property 3.4. Let G, H and T be three q-ROFRSs. If $G\subseteq H\subseteq T,$ then $d_j(G,H)\leq d_j(G,T)$ and $d_j(H,T)\leq d_j(G,T),$ where j = 1, 2, 3.

Next, we present an example as Example 3.1 consisting of six different pairs of q-ROFRSs. Through this example, we highlight the drawbacks of extant q-ROFR-distance measures (Khoshaim et al. [12], Liu et al. [14], Khan et al. [25]). To this aim, we firstly recall the existing measures by Khoshaim et al. [12], Liu et al. [14], Khan et al. [25], given as follows:

Khoshaim et al.'s q-ROFR-DM [12]:

$$d_4(G,H)=\frac{1}{2}\left(\begin{array}{l}\left|\left(\underline{\mu}_G\right)^2-\left(\underline{\mu}_H\right)^2\right|^p+\left|\left(\underline{\nu}_G\right)^2-\left(\underline{\nu}_H\right)^2\right|^p\\+\left|\left(\bar{\mu}_G\right)^2-\left(\bar{\mu}_H\right)^2\right|^p+\left|\left(\bar{\nu}_G\right)^2-\left(\bar{\nu}_H\right)^2\right|^p\end{array}\right)^{1/p}. \quad (10)$$

Liu et al.'s q-ROFR-DM [14]:

$$d_5(G,H)=\frac{1}{4}\left(\begin{array}{l}\left(\left(\underline{\mu}_G^q\right)-\left(\underline{\mu}_H^q\right)\right)^2+\left(\left(\underline{\nu}_G^q\right)^2-\left(\underline{\nu}_H^q\right)\right)^2\\+\left(\left(\bar{\mu}_G^q\right)^2-\left(\bar{\mu}_H^q\right)\right)^2+\left(\left(\bar{\nu}_G^q\right)-\left(\bar{\nu}_H^q\right)\right)^2\\+\left(\left(\underline{\pi}_G^q\right)-\left(\underline{\pi}_H^q\right)\right)^2+\left(\left(\bar{\pi}_G^q\right)-\left(\bar{\pi}_H^q\right)\right)^2\end{array}\right)^{1/2}. \quad (11)$$

Khan et al.'s q-ROFR-DM [25]:

$$d_6(G,H)=\frac{1}{4}\left(\begin{array}{l}\left|\left(\underline{\mu}_G\right)-\left(\underline{\mu}_H\right)\right|+\left|\left(\underline{\nu}_G\right)-\left(\underline{\nu}_H\right)\right|\\+\left|\left(\bar{\mu}_G\right)-\left(\bar{\mu}_H\right)\right|+\left|\left(\bar{\nu}_G\right)-\left(\bar{\nu}_H\right)\right|\\+\left|\left(\underline{\pi}_G\right)-\left(\underline{\pi}_H\right)\right|+\left|\left(\bar{\pi}_G\right)-\left(\bar{\pi}_H\right)\right|\end{array}\right). \quad (12)$$

Example 3.1. Consider the six different pairs of q-ROFRSs, which are given as *Set-1: {G =(0.26,0.36), (0.36,0.46), H =(0.36,0.26), (0.46,0.36)}, Set-2: {G = ((1,0), (1,0)), H = (0,1), (0,1)}, Set-3: {G = ((1,0), (1,0)), H =*

((0,0), (0,0))}, Set-4: {G = ((0.5,0.5), (0.5,0.5)), H = ((0,0), (0,0))}, Set-5: {G = (0.36,0.16), (0.46,0.26), H = (0.46,0.26), (0.56,0.36)} and *Set-6: {G = ((0.36,0.16), (0.46,0.26)), H = ((0.46,0.16), (0.56,0.26))}*. Next, we compute the degree of distance between these pairs of sets through the proposed and existing q-ROFR-distance measures (Khoshaim et al. [12], Liu et al. [14], Khan et al. [25]).

Table I presents required computational results of the q-ROFR-distance measures. On account of the obtained results, we draw the following conclusions:

- For two sets (Set-2 and Set-3), it can be observed that the q-ROFR-distance measure by Liu et al. [14] obtains the same value "1.803". It means that Liu et al.'s distance measure does not fulfil the postulate (*i*) of Definition 2.7.
- The distance measure by Khan et al. [25] is unable to differentiate two different pairs of sets (Set-1 and Set-6) as it obtains the same value "0.1". For three sets (Set-2, Set-3 and Set-4), Khan et al.'s [25] distance measure is unable to describe the difference between two different q-ROFRSs.
- The proposed q-ROFR-distance measure satisfies the axiomatic requirements of distance measure, given in Definition 2.7. For very similar but different q-ROFRSs, the proposed distance measure provides clear and rational data, which shows its effectiveness and rationality over extant measures (Khoshaim et al. [12], Liu et al. [14], Khan et al. [25]).

IV. Various Applications on q-ROFR Environment

To verify the rationality of introduced q-ROFR-distance measure given in Eq. (7)-Eq. (9), we present their utility in the field of pattern recognition and crop disease diagnosis.

A. Application to Pattern Recognition

Let us assume four known patterns R_1, R_2, R_3 and R_4, which have classifications S_1, S_2, S_3 and S_4, respectively.
The known patterns are characterized by given q-ROFRSs in $\boldsymbol{R} = \{r_1, r_2\}$:

$$R_1 = \{(r_1,(0.9,0.5),(0.6,0.7)),(r_2,(0.8,0.6),(0.5,0.7))\}, \quad (13)$$

$$R_2 = \{(r_1,(0.4,0.7),(0.5,0.6)),(r_2,(0.7,0.5),(0.4,0.6))\}, \quad (14)$$

$$R_3 = \{(r_1,(0.3,0.6),(0.6,0.5)),(r_2,(0.5,0.7),(0.7,0.3))\}, \quad (15)$$

$$R_4 = \{(r_1,(0.5,0.8),(0.2,0.6)),(r_2,(0.7,0.5),(0.6,0.6))\} \quad (16)$$

Given an unknown pattern is defined as

$$T = \{(r_1,(0.6,0.5),(0.5,0.5)),(r_2,(0.3,0.7),(0.5,0.6))\}. \quad (17)$$

The objective is to identify that which class does the unknown pattern's T belong to. In accordance with the doctrine of minimum distance measure between q-ROFRSs, the procedure of assigning T to S_{k^*} is defined as

$$k^* = \arg\min_k \{d_\alpha(R_k,T)\}, \alpha = 1,2,3. \quad (18)$$

Table II shows computational outcomes of q-ROFR-distance measures. Based on the obtained results, it has been observed that the pattern T is being classified to S_3 as it has least degree of distance on known pattern R_k and unknown pattern T.

Table III.

Degree of distance measure $d_\alpha(R_k,T)$, $k \in \{1,2,3,4\}$

Pattern	R_1	R_2	R_3	R_4
T	0.437	0.699	**0.393**	0.449

B. Application to Crop Disease Diagnosis

Here, we apply the proposed q-ROFR-distance measures for diagnosing the crop disease in an Indian region. This study consists of sets of crops, diseases and factors, which are represented by P = {Wheat, Rice, Carrot, Onion red}, H = {Viroid, Fungal, Nematodes, Bacterial, Phytoplasmal} and V = {Temperature, Soil moisture, Insect, pH value, Humidity}, respectively. Table III displays related features of considered diseases and Table IV presents the symptoms features of given crops in terms of q-ROFRNs.

In order to do a proper diagnosis, we compute for each crop $p_i \in P$, where $i \in \{1,2,3,4\}$, the degree of q-ROFR-distance measure $d_\alpha(f(p_i),h_k)$ on crop symptoms and set of symptoms that are feature for each diagnosis $h_k \in H$ with $k \in \{1,2,3,4,5\}$. Similar to Eq. (18), the proper diagnosis h_{k^*} for i^{th} crop is determined as follows:

Table I.

Comparative results by different q-ROFR-distance measures

Sets	Distance measures	d_1	d_2	d_3	d_4	d_5	d_6
Set-1	G = ((0.26,0.36), (0.36,0.46)) H = ((0.36,0.26), (0.46,0.36))	0.3	0.333	0.1	0.092	0.079	0.1
Set-2	G = ((1,0), (1,0)) H = ((0,1), (0,1))	1.0	1.307	1.0	1.260	**1.803**	**1.0**
Set-3	G = ((1,0), (1,0)) H = ((0,0), (0,0))	0.707	0.924	0.707	1.0	**1.803**	**1.0**
Set-4	G = ((0.5,0.5), (0.5,0.5)) H = ((0,0), (0,0))	0.595	0.648	0.399	0.315	0.419	**1.0**
Set-5	G = ((0.36,0.16), (0.46,0.26)) H = ((0.46,0.26), (0.56,0.36))	0.298	0.335	0.099	0.099	0.153	0.2
Set-6	G = ((0.36,0.16), (0.46,0.26)) H = ((0.46,0.16), (0.56,0.26))	0.225	0.262	0.077	0.093	0.124	0.1

$$k^* = \arg\min_k \left\{ d_\alpha \left(f(p_i), h_k \right) \right\}, \alpha = 1,2,3. \tag{19}$$

We allocate to the i^{th} crop the diagnosis whose symptoms have lowest degree of distance measure from crop symptoms. Table V shows the required computational results of crop disease diagnosis.

It can be observed from Table V that "Wheat" is most affected by Bacterial, "Rice" is most affected by Fungal disease, "Carrot" is affected by Nematodes and "Onion red" is most affected by Fungal disease.

C. Applications, gaps and future directions of q-ROFRSs

In the thematic assessment, numerous emerging ideas have been surfaced, contributed to the developing landscape of q-ROFRSs literature. These concepts incorporate various disciplines namely correlation coefficient, similarity measure assessment on q-ROFRSs, calculations relating q-ROFRNs, the generalization of q-ROFRS with 2-Tuple linguistic approach and the application of q-ROFRSs in healthcare, digital technology mainly in the evaluation of challenges and barriers. Briefing understudied regions in q-ROFRSs literature, Table VI is presented by the authors, helps as a concise reference for future direction.

V. Conclusion

In the paper, we have introduced three new distance measures for q-ROFRSs with their enviable properties in the context of q-ROFRSs. We have discussed the consistency and efficacy of the developed distance measures through a comparative example consisting of six different pairs of q-ROFRSs. In addition, we have highlighted the counter-intuitive cases of Khoshaim et al. [12], Liu et al. [14] and Khan et al. [25] q-ROFR-distance measures. It has been obtained that in some circumstances, developed q-ROFR-distance measures perform better than some of the existent distance measures for some sets of q-ROFRSs. Further, the developed q-ROFR-distance measures has been implemented to the pattern recognition and crop disease diagnosis problems. In future, the developed q-ROFR-distance measures can be used to solve texture extraction and medical diagnosis problems. In addition, the proposed measures can be extended under different fuzzy environments such as interval-valued q-ROFRSs, linear Diophantine fuzzy rough sets, hypersoft rough sets and others.

TABLE IVII.
SYMPTOMS-DISEASES Q-RUNG ORTHOPAIR FUZZY ROUGH RELATION

Symptoms	Viroid	Fungal	Nematodes	Bactarial	Phytoplasmal
Temperature	((0.6, 0.7), (0.2, 0.5))	((0.8, 0.4), (0.4, 0.6))	((0.9, 0.2), (0.4, 0.3))	((0.6, 0.5), (0.5, 0.2))	((0.8, 0.5), (0.3, 0.6))
Soil Moisture	((0.9, 0.4), (0.6, 0.3))	((0.7, 0.6), (0.4, 0.2))	((0.5, 0.7), (0.5, 0.1))	((0.8, 0.4), (0.5, 0.2))	((0.7, 0.6), (0.5, 0.1))
Insect	((0.7, 0.5), (0.4, 0.2))	((0.8, 0.3), (0.5, 0.2))	((0.6, 0.5), (0.5, 0.3))	((0.4, 0.9), (0.4, 0.3))	((0.9, 0.4), (0.5, 0.3))
pH value	((0.9, 0.3), (0.5, 0.3))	((0.6, 0.5), (0.3, 0.5))	((0.8, 0.4), (0.5, 0.2))	((0.7, 0.6), (0.5, 0.2))	((0.6, 0.8), (0.5, 0.4))
Humidity	((0.3, 0.9), (0.2, 0.7))	((0.8, 0.4), (0.5, 0.3))	((0.7, 0.6), (0.6, 0.2))	((0.6, 0.5), (0.2, 0.6))	((0.4, 0.7), (0.6, 0.2))

TABLE IIIV.
CROPS-SYMPTOMS Q-RUNG ORTHOPAIR FUZZY ROUGH RELATION

Crops	Temperature	Soil moisture	Insect	pH value	Humidity
Wheat	((0.3, 0.6), (0.2, 0.8))	((0.4, 0.8), (0.5, 0.7))	((0.5, 0.9), (0.3, 0.4))	((0.4, 0.9), (0.4, 0.7))	((0.6, 0.5), (0.5, 0.4)
Rice	((0.4, 0.5), (0.5, 0.9))	((0.2, 0.6), (0.5, 0.4))	((0.7, 0.4), (0.2, 0.8))	((0.5, 0.7), (0.3, 0.5))	((0.4, 0.4), (0.5, 0.6))
Carrot	((0.4, 0.6), (0.3, 0.5))	((0.3, 0.6), (0.2, 0.5))	((0.5, 0.6), (0.7, 0.4))	((0.5, 0.4), (0.6, 0.3))	((0.4, 0.6), (0.3, 0.4))
Onion red	((0.5, 0.7), (0.5, 0.4))	((0.6, 0.4), (0.5, 0.5))	((0.7, 0.2), (0.6, 0.5))	((0.4, 0.6), (0.5, 0.3))	((0.8, 0.4), (0.5, 0.2))

TABLE V.
DEGREE OF DISTANCE MEASURE ON EACH CROP SYMPTOMS AND CONSIDERED SET OF POSSIBLE DIAGNOSES

Crops	Viroid	Fungal	Nematodes	Bacterial	Phytoplasmal
Wheat	0.567	0.504	0.513	**0.47**	0.497
Rice	0.53	**0.441**	0.514	0.493	0.454
Carrot	0.454	0.47	**0.44**	0.455	0.449
Onion red	0.471	**0.371**	0.429	0.429	0.436

TABLE VI.
OUTLINE OF UNDERSTUDIED REGIONS IN q-ROFRSS LITERATURE OFFERING FOUNDATION FOR FUTURE RESEARCH DIRECTIONS

q-ROFRSs dimensions	Understudied Regions
Aggregation operators (AOs)	a) New AOs defined on q-ROFRS or its generalizations, b) Integration of some extant AOs for finding more powerful and flexible AOs.
q-ROFRSs linguistic rating	Linguistic rating mapping to deal linguistic information of q-ROFRSs generalizations
MCDM methods	a) Proposing hybrid MCDM approaches to evade the drawbacks of single MCDM models, b) Integrating mathematical model or optimization with q-ROFRS MCDM models.
Application regions	a) Healthcare, b) Agriculture/agro-farming, c) Finance/Economy, d) Manufacturing, e) Technology innovation, f) Emergency decision-making, and others.
Application objectives	a) Advancing and designing urban areas, b) Manufacturing robot design and evaluation, c) Digital technology evaluation, and others.

REFERENCES

[1] Z. A. Pawlak, “Rough sets”, *International Journal of Computer & Information Sciences*, vol. 11, no. 5, pp. 341–356, 1982.

[2] G. I. Sayed, E. I. A. El-Latif, A. E. Hassanien, V. Snasel, “Optimized long short-term memory with rough set for sustainable forecasting renewable energy generation”, *Energy Reports*, vol. 11, pp. 6208-6222, 2024.

[3] R. A. Hosny, R. Abu-Gdairi, M. K. El-Bably, “Enhancing Dengue fever diagnosis with generalized rough sets: Utilizing initial-neighbourhoods and ideals”, *Alexandria Engineering Journal*, vol. 94, pp. 68-79, 2024.

[4] D. Dubois, H. Prade, “Rough fuzzy sets and fuzzy rough sets”, *International Journal of General Systems*, vol. 17, no. 2–3, pp. 191–209, 1990.

[5] X. Zhang, B. Zhou, P. Li, “A general frame for intuitionistic fuzzy rough sets”, *Information Sciences*, vol. 216, pp. 34–49, 2012.

[6] B. Sun, W. Ma, “Soft fuzzy rough sets and its application in decision making”, *Artificial Intelligence Review*, vol. 41, no. 1, pp. 67–80, 2014.

[7] R. R. Yager, “Generalized orthopair fuzzy sets”, *IEEE Transactions on Fuzzy Systems*, vol. 25, pp. 1222-1230, 2017.

[8] L. A. Zadeh, “Fuzzy sets”, *Information and Control*, vol. 8, no. 3, pp. 338–353, 1965.

[9] K. T. Atanassov, “Intuitionistic fuzzy sets”, *Fuzzy Sets and Systems*, vol. 20, no. 1, pp. 87–96, 1986.

[10] R. R. Yager, “Pythagorean membership grades in multicriteria decision making”, *IEEE Transactions on Fuzzy Systems*, vol. 22, no. 4, pp. 958–965, 2014.

[11] T. Senapati, R. R. Yager, “Fermatean fuzzy sets”, *J Ambient Intell Human Comput.*, vol. 11, pp. 663–674, 2020. https://doi.org/10.1007/s12652-019-01377-0.

[12] A. B. Khoshaim, S. Abdullah, S. Ashraf, M. Naeem, “Emergency decision-making based on q-rung orthopair fuzzy rough aggregation information. Computers”, *Materials & Continua*, vol. 69, no. 3, pp. 4077-4094, 2021.

[13] S. Ashraf, N. Rehman, A. Hussain, A. Al-Salman, A. H. Gumaei, “q-Rung orthopair fuzzy rough einstein aggregation information-based EDAS method: applications in robotic agrifarming”, *Computational Intelligence and Neuroscience*, vol. 2021, Article ID 5520264), pp. 01-27, 2021.

[14] F. Liu, T. Li, J. Wu, T. Liu, „Modification of the BWM and MABAC method for MAGDM based on q-rung orthopair fuzzy rough numbers”, *International Journal of Machine Learning and Cybernetics*, vol. 12, no. 9, pp. 2693–2715, 2021.

[15] S. Qahtan, H. A. Alsattar, A. A., Zaidan, M. Deveci, D. Pamucar, D. Delen, “Performance assessment of sustainable transportation in the shipping industry using a q-rung orthopair fuzzy rough sets-based decision-making methodology”, *Expert Systems with Applications*, vol. 223, 119958, 2023, https://doi.org/10.1016/j.eswa.2023.119958.

[16] A. R. Mishra, P. Rani, D. Pamucar, V. Simic, “Evaluation and prioritization of sustainable enterprise resource planning in SMEs using q-rung orthopair fuzzy rough set-based decision support model”, *IEEE Transactions on Fuzzy Systems*, vol. 32, no. 5, pp. 3260-3273, 2024.

[17] A. F. Alrasheedi, A. R. Mishra, P. Rani, E. K. Zavadskas, F. Cavallaro, “Multicriteria group decision making approach based on an improved distance measure, the SWARA method and the WASPAS method”, *Granular Computing*, vol. 8, pp. 1867–1885, 2023.

[18] S. Gogoi, B. Gohain, R. Chutia, “Distance measures on intuitionistic fuzzy sets based on cross-information dissimilarity and their diverse applications”, *Artificial Intelligence Review*, vol. 56, pp. 3471–3514, 2023.

[19] P. Rani, A. R. Mishra, F. Cavallaro, A. F. Alrasheedi, “Location selection for offshore wind power station using interval-valued intuitionistic fuzzy distance measure-RANCOM-WISP method”, *Scientific Reports*, vol. 14, no. 4706, 2024, https://doi.org/10.1038/s41598-024-54929-6.

[20] C. Wang, Y. Huang, M. Shao, X. Fan, “Fuzzy rough set-based attribute reduction using distance measures”, *Knowledge-Based Systems*, vol. 164, pp. 205-212, 2019.

[21] S. An, Q. Hu, C. Wang, “Probability granular distance-based fuzzy rough set model”, *Applied Soft Computing*, vol. 102, no. 107064, 2021, https://doi.org/10.1016/j.asoc.2020.107064.

[22] R. Sahu, S. R. Dash, S. Das, “Career selection of students using hybridized distance measure based on picture fuzzy set and rough set theory”, *Decision Making: Applications in Management and Engineering*, vol. 4, no. 1, pp. 104–126, 2021.

[23] A. Tiwari, Q. M. D. Lohani, “Interval-valued intuitionistic fuzzy rough set system over a novel conflict distance measure with application to decision-making”, *MethodsX*, vol. 10, no. 102012, 2023, https://doi.org/10.1016/j.mex.2023.102012.

[24] C. Wang, C. Wang, Y. Qian, Q. Leng, “Feature Selection Based on Weighted Fuzzy Rough Sets”, *IEEE Transactions on Fuzzy Systems*, vol. 32, no. 7, pp. 4027-4037, 2024.

[25] S. Khan, M. Khan, M. S. A. Khan, S. Abdullah, F. Khan, “A novel approach toward q-rung orthopair fuzzy rough Dombi aggregation operators and their application to decision-making problems”, *IEEE Access*, vol. 11, pp. 35770-35783, 2023.