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The Synergy of Interpolative Boolean Algebra and Ordinal Sums of Conjunctive and Disjunctive Functions in Stock Price Trend Prediction

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***Abstract*—Stock price prediction is crucial for accurate investment decision-making and widely regarded as one of the most important tasks in finance. Investors and financial professionals rely on a wide range of input data, such as market information, technical analysis, and fundamental analysis, to make informed decisions. When it comes to financial data, it is important to incorporate the logical dependencies of inputs into the modeling and prediction process. Therefore, logic-based approaches are considered adequate for solving such problems. This paper proposes a novel logic-based approach to stock price trend prediction based on Interpolative Boolean algebra (IBA) and ordinal sums of conjunctive and disjunctive (OSCD) functions. This is the very first paper that aims to explore the synergy of these two approaches in a real-world setting, utilizing their comparative advantages in different phases of modeling. The proposed approach is tested on a sample of 23 companies from the S&P500 over the past three years. The paper also presents the results of the application of the proposed model for the analyzed companies.**

***Index Terms*—Interpolative Boolean Algebra, Ordinal Sums of Conjunctive and Disjunctive Functions, Price Trend Forecasting, S&P 500.**

I. Introduction

FINANCIAL markets have been an attractive research field for application of artificial intelligence (AI) techniques. The most challenging task in financial markets is to forecast prices because of their dynamic, complex, evolutionary, nonlinear, nonparametric, and chaotic nature [1].

To predict stock market movements, researchers use different types of structured and unstructured inputs. Structured inputs are based on market information (such as stock prices, volumes, spread), technical analysis (including technical indicators and chart patterns), and fundamental analysis (macroeconomic indicators, financial statement ratios). The unstructured inputs include news (general financial news, company news) and posts from social networks (such as Twitter, Reddit, Facebook). Still, technical indicators and financial statement ratios are the most commonly used inputs for predicting market movements [2] – [4].

Due to its ability to recognize patterns in data, AI and machine learning (ML) techniques are extensively used for this purpose [2], [5] – [7] with deep learning models strongly emerging as the most promising predictors [8] – [11]. Being flexible and able to fit complex data, ML models don't require a theoretical understanding of the problem, nor strong assumptions about the data. However, they lack interpretability, which complicates the extraction of knowledge from the data. In the real-world setting, the predictions given by the algorithm have to be fully understandable and interpretable to financial decision makers. To overcome this issue, one could use logic for modeling to imitate human reasoning. Keeping in mind the weak theoretical background of stock price movements, researchers usually combine (fuzzy) logic with some learning/optimization algorithms to create hybrid prediction models like neuro-fuzzy systems [12], [13], deep convolutional-fuzzy systems [14], evolutionary-fuzzy systems [15], etc. Still, there are very few authors who have tried to use an expert-based (fuzzy) logic models solely for financial forecast. One of the examples is the application of Interpolative Boolean algebra (IBA) for portfolio selection [16].

On the other hand, data aggregation underpins almost every stock price trend prediction system. The careful selection of the function for aggregating financial indicators into a single indicator is of paramount importance. In other words, it is necessary to choose a function that has the desired mathematical properties, and at the same time is comprehensible even to decision makers with a not so strong mathematical background. One possible direction is to employ simple yet effective aggregation functions, such as weighted sum [17] or order weighted sum operators [18]. The other prominent approach implies using logic-based aggregation methods in a

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broader sense, e.g. Choquet integral [19]. Finally, pure logic-based approaches based on fuzzy or multi-valued are frequently used [20]. However, mixed aggregation functions adapted to data to cover conjunctive, disjunctive and averaging behavior might be beneficial.

In this study, we utilize logic-based aggregation to model future stock price movements based on a selected set of financial ratios, primarily employing fundamental analysis. We engage two logic-based aggregation methods: Interpolative Boolean algebra and Ordinal Sums of Conjunctive and Disjunctive Functions (OSCD), to construct models for price trend prediction. OSCD serves for the aggregation of financial ratios within each group. Subsequently, IBA-based aggregation is employed to amalgamate the ordinal sums of the groups. To evaluate the proposed meta-model, we utilize a dataset comprising market information and financial ratio data of the S&P 500 companies over a three-year period on a quarterly basis. The price trend prediction task is formulated as a binary classification problem.

It's important to recognize that there are logical connections or dependencies among financial indicators within individual groups. For example, when the values of two indicators are high, they reinforce each other, leading to an upward trend (or increased satisfaction). Conversely, when values are low, it results in downward reinforcement. When some indicators are high while others are low, the satisfaction level falls somewhere in between. Therefore, for such cases, we require aggregation functions with mixed behavior [21]. One option is uninorms [21], but due to the non-continuity of representative uninorms [22], an alternative could be ordinal sums of conjunctive and disjunctive functions [23].

The next question is how to aggregate the higher-level results using ordinal sums of the indicator groups. We have chosen logical aggregation (LA) based on IBA for several reasons [24]. First, LA is a sophisticated multi-valued aggregation approach within the Boolean framework that provides clear guidelines for data aggregation, from verbal descriptions to the final mathematical aggregation model. Finally, aggregation models based on LA are fully transparent and interpretable for decision-makers.

The structure of this paper is given as follows. In the next two sections, a short overview of a theoretical background for the two aggregation methods, OSCD and logical aggregation based on IBA, is given. In Section 4 we explain the problem setup, describe the dataset, and propose the model. Finally, in the last two sections we present and discuss the experimental results and conclude the paper.

II. Interpolative Boolean Algebra

Interpolative Boolean Algebra is a consistent multi-valued realization of Boolean algebra, preserving all the laws on which Boolean algebra rests [25]. Namely, IBA is proposed as an answer for the disregard of the law of exclusion of the third and contradiction in the classical phase of logic and it serves as a fundamental component for various multi-valued techniques and methods [26]-[28].

IBA consists of two levels: the symbolic and the value levels. At the symbolic level of IBA, the structure of attributes is taken into account, while at the value level, values are assigned and the final resulting value of the expression in the Boolean framework is calculated [24]. The principle of structural functionality dictates that IBA transformations are performed at the symbolic level before introducing values. This ensures that negation is treated differently compared to traditional fuzzy approaches. Focusing on the structure preserves all Boolean laws in the multivalued case, including the laws of excluded middle and contradiction, which is the main contribution of IBA.

In the IBA framework, attributes/inputs are called primary attributes. These attributes are elements of logical functions within IBA. The value realization of primary attributes within the classical Boolean algebra implies the use of two values 0 and 1, while within the IBA primary attributes have [0,1]-valued realization. All logical functions over primary attributes represent elements of IBA. On the other hand, IBA is based on atomic elements of Boolean algebra [25]. Atomic elements are the simplest elements of Boolean algebra. They are logical functions that do not contain any other Boolean element except itself and 0 constant, e.g. Boolean algebra over two attributes has four atoms and they are $p_1 \wedge p_2, \neg p_1 \wedge p_2, p_1 \wedge \neg p_2$ and $\neg p_1 \wedge \neg p_2$.

The inclusion of atoms in a logical expression is the basis for the introduction of the structure of a logical expression and the structural level of IBA. Any logical expression is interpreted as a scalar product [16]:

$$\varphi(p_1, \dots, p_n) = P \cdot S(p_1, \dots, p_n) \tag{1}$$

where P is vector of atomic elements and $S(p_1, \dots, p_n)$ is the structural vector.

At the symbolic level, the given logical expression is transformed into a generalized Boolean polynomial (GBP) based on the transformation rules [24]:

$$(\alpha(p_1, \dots, p_n) \wedge \beta(p_1, \dots, p_n))^{\otimes} = \alpha^{\otimes}(p_1, \dots, p_n) \otimes \beta^{\otimes}(p_1, \dots, p_n) \tag{2}$$

$$(\alpha(p_1, \dots, p_n) \vee \beta(p_1, \dots, p_n))^{\otimes} = \alpha^{\otimes}(p_1, \dots, p_n) + \beta^{\otimes}(p_1, \dots, p_n) - \alpha^{\otimes}(p_1, \dots, p_n) \otimes \beta^{\otimes}(p_1, \dots, p_n) \tag{3}$$

$$(\neg\alpha(p_1, \dots, p_n))^{\otimes} = 1 - \alpha^{\otimes}(p_1, \dots, p_n) \tag{4}$$

where $\alpha(p_1, \dots, p_n)$ and $\beta(p_1, \dots, p_n)$ are complex elements of Boolean algebra.

When it comes to the primary attributes $p_1, \dots, p_n$, the following transformation rules applies [24]:

$$(p_i \wedge p_j)^{\otimes} = \begin{cases} p_i \otimes p_j, & i \neq j \\ p_i, & i = j \end{cases} \tag{5}$$

$$(p_i \vee p_j)^{\otimes} = p_i + p_j - p_i \otimes p_j \tag{6}$$

$$(\neg p_i)^{\otimes} = 1 - p_i \tag{7}$$

IBA transformations on the symbolic level enable the preservation of Boolean laws in general case. Furthermore, the first GBP transformation rule for primary attributes (5) ensures idempotency within the IBA framework.

At the value level, each element of Boolean algebra is realized by GBP. In GBP, alongside standard arithmetic addition and subtraction operations, and the generalized product (GP). GP, a binary operator on the unit interval, belongs to a subclass of t-norms that satisfies the non-negativity condition. It takes precedence as the highest priority operation within the expression. GP can be any function greater than the Lukasiewicz operator and less than the minimum [29]:

$$\max(p_1 + p_2 - 1, 0) \le p_1 \otimes p_2 \le \min(p_1, p_2) \quad (8)$$

In the IBA framework, the choice of operators for generalized products depends on the nature of the attributes and their correlation. Specifically, the Lukasiewicz operator is utilized for aggregating attributes of opposite nature, i.e., negatively correlated variables. The standard product operator is employed for uncorrelated variables. Finally, the minimum operator is applied for the aggregation of attributes of the same or similar nature, i.e., highly correlated variables.

From the practical standpoint, the two most successful application areas of IBA are logical aggregation and the IBA similarity measure, which have been used in various fields [27], [28], [30] – [33].

Logical aggregation (LA) is a Boolean consistent and fully transparent aggregation technique based on IBA [24]. It implies that the normalized values of the input variables are aggregated using GBP or weighted sum of GBPs into the resulting, globally representative value. The main advantage of LA compared to traditional aggregation methods is that it enables modelling of complex logical connections that may exist in problems where the human factor is particularly important [34].

Hence, understanding and analysing LA functions is simple, and it has found application in many areas of finance. For instance, IBA-based methods for portfolio selection showed promising results and their average monthly returns were aligned with the performance of the S&P500 index [16]. Also, a system for automatic trading on the stock market based on IBA is proposed [35]. Moreover, authors in [30], [36] used logic-based approach for financial ratio analysis of a company's performance.

III. Ordinal Sums

Mixed aggregation functions are able to adjust to data in the sense of reinforcing or averaging. One category are ordinal sums of conjunctive and disjunctive functions depicted in Fig 1.

The ordinal sum considering two attributes is an aggregation function on [0,1] [23]:

$$A(x, y) = A_1(a \wedge x, a \wedge y) + A_2(a \vee x, a \vee y) + a \quad (9)$$

where

- for $x, y \in [0, a]^2$ we get $A(x, y) = A_1(x, y)$
- for $x, y \in [a, 1]^2$ we get $A(x, y) = A_2(x, y)$
- for $x, y \in [0, a] \times [a, 1]$ we get $A(x, y) = x + y - a$
- for $x, y \in [a, 1] \times [0, a]$ we get $A(x, y) = x + y - a$

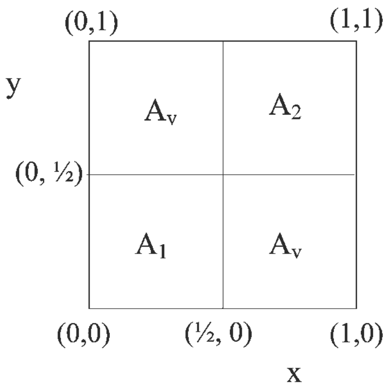

Fig. 1. Graphical illustration of ordinal sums

When A_1 is conjunctive and A_2 is disjunctive function, then in the remaining two sub squares are described with averaging function:

$$A(x, y) = A_v(x, y) = x + y - a \quad (10)$$

Observe that, for conjunctive function we should keep neutral element $A_1(a, a) = a$ [23]. For a=0.5 and product t-norm we get:

$$A_1(x, y) = 2 \cdot x \cdot y \quad (11)$$

Analogously holds for disjunctive function. Hence, for probabilistic sum t-conorm we get:

$$A_2(x, y) = -1 + 2 \cdot x + 2 \cdot y - 2 \cdot x \cdot y \quad (12)$$

The same observation holds for other t-norms and t-conorms. We introduce here only functions used in this work. Lukasiewicz t-norm (as a representative of nilpotent functions) is:

$$A_1(x, y) = \max(0, x + y - 0.5) \quad (13)$$

while its dual t-conorm is:

$$A_2(x, y) = \min(0, x + y - 0.5) \quad (14)$$

In this way, we are able to upwardly reinforce high values (or assign them value 1), downwardly reinforce low values (or even assign them value 0) and assign average value for the other cases. In the case of averaging behavior, we can manage inclination towards conjunctive, or disjunctive behavior. In the case of careful or pessimistic evaluation (conjunctive inclination), we adopt, e.g., geometric mean for averaging part as:

$$A_v(x, y) = 2 \cdot x \cdot y \quad (15)$$

Observe that, even though (11) and (15) have the same structure, they are applied on different sub squares and therefore behave differently.

IV. Materials and Methods

A. The problem setup

Financial prediction poses a complex and challenging problem, inherently fraught with uncertainty and risk. Consequently, it may not always accurately foresee future outcomes due to unforeseen events, market volatility, or changes in underlying assumptions. Achieving accurate predictions is a primary challenge, alongside the imperative of rendering them comprehensible to decision-makers and fully interpretable.

Various analytical techniques are employed in financial prediction, encompassing fundamental, technical, and sentiment analysis, alongside machine learning or statistical tools. However, despite the breadth of methodologies, financial predictions remain vulnerable to uncertainty and risk. Thus, ensuring their accuracy, comprehensibility, and interpretability becomes paramount.

In this paper, we aim to address a stock trend prediction problem, i.e., forecasting the future direction of stock prices by identifying patterns in historical stock price data. For the purpose of this paper, we will consider this problem as a binary classification. Namely, the main goal is to determine which companies' stock should be bought and which stock should be sold. Stocks that should be bought are the ones which will increase in price in the future, while the stocks with decreasing value are considered to be sold.

The success of the proposed models will be measured using standard metrics for binary classification: receiver operating characteristic (ROC) curve / area under the curve (AUC), together with precision, recall and F1 metric.

B. Dataset

This study employs a dataset consisting of financial ratios for 23 companies that constitute the S&P500 index. Companies are selected to cover different industries, such as the Energy and Materials sectors, to Media & Entertainment companies. Furthermore, companies are chosen as a balanced sample in terms of upward and downward price trends during the observed period of time. Finally, each company is described with fundamental financial indicators collected on a quarterly basis for the period of three years (from December 2021 to December 2023). The final dataset consists of 201 instances, reflecting the availability of data and the fact that some of the chosen companies were not in the S&P 500 index throughout the entire observed period. A detailed overview of the companies and the industries they belong to is provided in Table I.

Based on the literature review and recommendations of experts in the field, eight financial indicators were chosen as inputs for the experiment. The selected attributes differ in nature and cover four aspects of a company's performance and financial health: activity, liquidity, cash flow, and investment ratios. From each group, the two most significant indicators were chosen. Net operating assets to total assets (a_1) and asset turnover (a_2) are identified as representatives of activity ratios, cash and cash equivalents to total assets (l_1) and working capital to total assets (l_2) from liquidity indicators, ratio of operating cash flow to total assets (c_1) and free cash flow yield (c_2) from cash flow indicators and earnings per share to price (i_1) and enterprise value growth rate (i_2) from investment ratios.

TABLE I.
OVERVIEW OF COMPANIES AND INDUSTRIES

Company	**Industry**
Accenture (ACN)	Software & Services
Align Technology (ALGN)	Health Care Equipment & Services
Amcor (AMCR)	Materials – Containers & Packaging
Broadcom Inc. (AVGO)	Semiconductors & Semiconductor Equipment
Bristol Myers Squibb (BMY)	Pharmaceuticals, Biotechnology & Life Sciences
Corteva (CTVA)	Materials - Chemicals
Dow Inc. (DOW)	Materials – Chemicals
Fox Corp. Class B (FOX)	Media & Entertainment
Fox Corp. Class A (FOXA)	Media & Entertainment
Fortinet (FTNT)	Software & Services
Hasbro (HAS)	Consumer Durables & Apparel
Gartner (IT)	Materials – Containers & Packaging
Mastercard (MA)	Financial Services
3M (MMM)	Capital Goods
Paramount Global (PARA)	Media & Entertainment
Paychex (PAYX)	Commercial & Professional Services
Pool Corp. (POOL)	Consumer Discretionary Distribution & Retail
Phillips 66 (PSX)	Energy – Oil, Gas & Consumable Fuels
Qorvo (QRVO)	Semiconductors & Semiconductor Equipment
Uber (UBER)	Transportation
Vici Properties (VICI)	Equity Real Estate Investment Trusts (REITs)
Warner Bros. Discovery (WBD)	Media & Entertainment
Welltower Inc (WELL)	Equity Real Estate Investment Trusts (REITs)

As data are on different scales, the usual step for any data mining task is normalization. It is also relevant in mining patterns from data by logic aggregation usually working on the unit interval. A simple normalization might cause outliers skew the normalization [37]. Hence, we applied normalization based on the inter quartile distribution in the following way. All values lower than $L = \mathrm{L} = \mathrm{Q1} - 1.5 \cdot (\mathrm{Q3} - \mathrm{Q1})$ are transformed to 0, while all values greater than $\mathrm{H} = \mathrm{Q3} + 1.5 \cdot (\mathrm{Q3} - \mathrm{Q1})$ are transformed into 1, where *Q1* and *Q3* are the first and third quartile, respectively.

Inner values are normalized as (see Fig. 2).

$$x^* = \frac{x-L}{H-L} \tag{16}$$

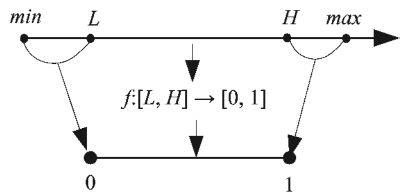

Fig. 2. Normalization into the unit interval adopted from **[35]**

The output variable in our case are returns of a stock price, i.e. the change in the price of a stock over a certain period of time. According to the returns, instances are divided into two classes depending on whether they are positive or negative. Accordingly, 1 is assigned to positive and 0 for negative returns.

C. Meta-model

The majority of models for financial prediction are based on a black-box approach, lacking interpretability, which is often crucial for decision-makers. Additionally, data on financial markets often incorporate a certain extent of uncertainty and vagueness that may hinder decision-making. Therefore, logic-based approaches seem to be a natural direction for the development of our model.

The proposed model respects the hierarchy of selected inputs; that is, first, we will obtain aggregated indicators of activity (A), liquidity (L), cash flow (C) and investment ratios (I), and then the final assessment for a company ($score$). The model is entirely based on logic-based techniques.

On one hand, we have chosen OC as an aggregation operation at the group level, owing to its capability to generate a minimized aggregation score for small attribute values and maximize aggregation scores for attributes with large values. This would result in a clear separation of preferred values from those representing potentially risky situations for further aggregation at the group level. Therefore, scores at the level of the groups are calculated as follows:

$$A = OSDC(a_1, a_2) \quad (17)$$

$$L = OSDC(l_1, l_2) \quad (18)$$

$$C = OSDC(c_1, c_2) \quad (19)$$

$$I = OSDC(i_1, i_2) \quad (20)$$

On the other hand, we have implemented IBA-based logical aggregation as a final aggregation function due to the mathematical characteristics of IBA, along with its explainability. The final score is obtained as an LA of the scores at the level of the group:

$$score = LA(A, L, C, I) \quad (21)$$

From a mathematical standpoint, the main benefit of using IBA-based aggregation/decision-making models lies in their ability to perform a fine gradation of instances using the [0,1] approach, which remains consistent with the Boolean frame. From a practical standpoint, IBA-based models are fully interpretable and transparent. The process of IBA expert-based modeling starts with formulating a clear verbal model that comprehensively addresses the needs and preferences of decision-makers. This model can be easily articulated and explained to executives lacking significant mathematical knowledge or familiarity with IBA. Furthermore, the verbal model can be easily interpreted as a logical/mathematical model, i.e., a single logical aggregation. Subsequently, the logical model is transformed into a suitable GPB, either manually or utilizing existing software solutions [26].

$$score = GBP(A, L, C, I) \quad (22)$$

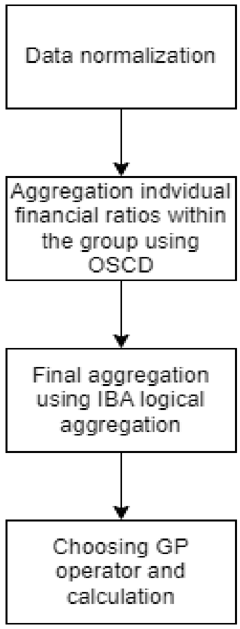

Fig 3. The proposed meta-model

As a final step, the GP operator is chosen based on data correlation, and the resulting value for each instance is calculated. Finally, the proposed metamodel is illustrated in Fig. 1.

D. Model realizations

In order to calculate scores at the group level, three different OSCD operators are used: 1) OSCD based on product t-norm, probabilistic sum, and arithmetic mean; 2) OSCD based on product t-norm, probabilistic sum, and geometric mean; 3) Lukasiewicz t-norm and t-conorm. These OSCD operators are presented in Figures 4-6.

The first two OSCD operators differ only in the mean operator, while the third one is based on a different t-norm. All three operators are thoroughly discussed and elaborated from a theoretical point of view in the literature [23]. Still, this may be seen as an attempt to investigate their practical value from the perspective of the presented problem. In other words, the experiment will show which of these operators is the best choice for score calculation at the group level. The value of the parameter a was chosen according to the literature [23].

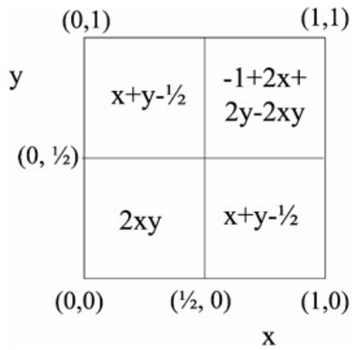

Fig 4. The graphical interpretation of OSCD with standard product t-norm, probabilistic sum and arithmetic mean **[23]**

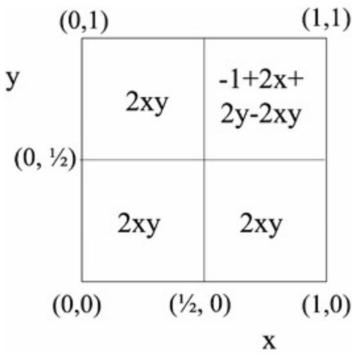

Fig 5. The graphical interpretation of OSCD with standard product t-norm, probabilistic sum and geometric mean **[23]**

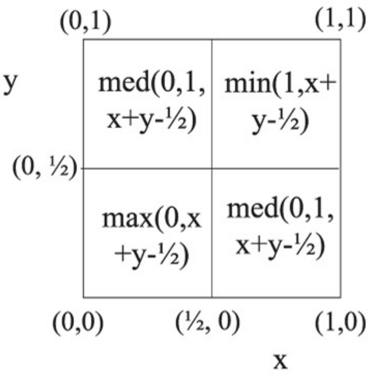

Fig 6. The graphical interpretation of OSCD with Lukasiewicz t-norm and t-conorm **[23]**

The verbal model may be easily translated into the following logical aggregation model:

$$LA(A,L,C,I) = \big(\neg I \wedge C \wedge (A \vee L)\big) \vee (I \wedge \neg C \wedge L) \quad (23)$$

Afterwards, the logical aggregation model is treated within the IBA framework and transformed into the corresponding GBP.

$$score = GBP(A,L,C,I) = A \otimes C + L \otimes C + C \otimes I - A \otimes L \otimes C - A \otimes C \otimes I - 2L \otimes C \otimes I + A \otimes L \otimes C \otimes I \quad (24)$$

The final step in the proposed approach involves choosing the GP operator. Considering that various groups of financial ratios are to be aggregated, the standard product appears to be a natural choice. This choice is supported by correlation analysis. Indeed, correlations between groups of aggregated indicators are illustrated in Figures 7-9. Since correlation coefficients are not high, the product operator is selected for the GP [39]. Therefore, the final aggregation score is calculated using the following expression:

$$score = A \cdot C + L \cdot C + C \cdot I - A \cdot L \cdot C - A \cdot C \cdot I - 2 \cdot L \cdot C \cdot I + A \cdot L \cdot C \cdot I \quad (25)$$

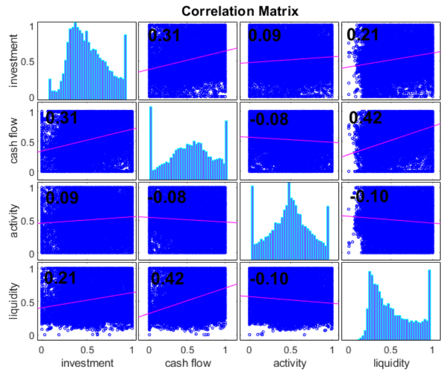

Fig 7. Correlation Matrix of scores at the level of the group calculated using OSCD with standard product t-norm, probabilistic sum and arithmetic mean

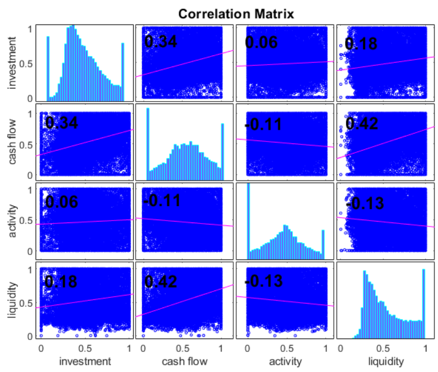

Fig 8. Correlation Matrix of scores at the level of the group calculated using OSCD with standard product t-norm, probabilistic sum and geometric mean

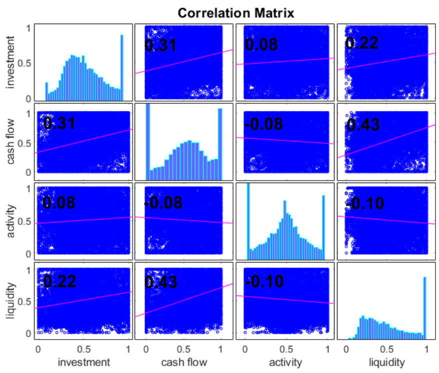

Fig 9. Correlation Matrix of scores at the level of the group calculated using OSCD with Lukasiewicz t-norm and t-conorm

V. Experimental Results

The proposed approach was validated by comparing the predicted values with the actual class of stock trend. The acquired dataset is balanced, consisting of 101 instances (50.25%) with a negative trend, assigned class 0, while 100 instances (49.75%) exhibit a positive trend, assigned class 1.

The performance of classification models was evaluated using AUC/ROC. These metrics provide a comprehensive measure of model performance in terms of binary classification. The model with the group calculated using OSCD with Lukasiewicz t-norm and t-conorm (M3) achieved an AUC of 0.7223, indicating a good ability to distinguish between positive and negative stock trends. The model with the group calculated using OSCD with standard product t-norm, probabilistic sum, and arithmetic mean (M1) followed with 0.7206, and the model with the group calculated using OSCD with standard product t-norm, probabilistic sum, and geometric mean (M2) with 0.7036. The ROC curves for each model were plotted to visualize their performance. In Fig. 10, it can be seen that the curves for M1 and M3 overlap, while the ROC for M2 is clearly worse.

Therefore, we may conclude that the choice of OSCD functions has a significant influence on the results of classification. It seems that the geometric mean operator has a negative influence on the results. On the other hand, the t-norm/t-conorm operator did not have a strong influence, since M1 and M2 utilize different operators.

In order to perform more detailed analysis, confusion matrices for all three models are calculated for a chosen threshold, while classification performance metrics are presented in Table II. The best values are marked in bold font.

For the chosen threshold, the model with the group calculated using OSCD with Lukasiewicz t-norm and t-conorm outperformed the remaining two models. Bearing in mind that we do not rely on ML approaches and the difficulty of the problem, AUC of 0.7223 is satisfactory. High precision in this domain is crucial because false positives can lead to significant costs. Together with recall and F1 score, the models suggest their reliability and effectiveness in real-world applications.

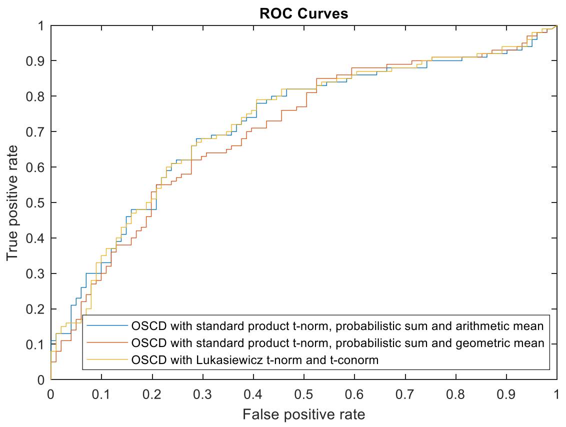

Fig 10. ROC curves for proposed models

TABLE III.
PERFORMANCE OF CLASSIFICATION

	M1	M2	M3
AUC	0.7206	0.7036	**0.7223**
Precision	0.6923	0.6737	**0.7021**
Recall	0.6300	0.6400	**0.6600**
F1 score	0.6597	0.6564	**0.6804**

VI. CONCLUSION

In this research, a combination of two logic-based aggregation methods is proposed for stock trend prediction. To the best of our knowledge, this is the first time such an approach with IBA and OSCD has been proposed. Although these approaches share a similar background, they have different mathematical properties and the potential to model different real-world situations. In this paper, on one hand, the financial indicators within the groups are aggregated by OSCD to perform the maximization/minimization according to the input. On the other hand, IBA, as a consistent real-valued generalization of classical Boolean algebra, is used as a framework for modeling and the logical aggregation of groups of financial indicators.

In this study, we have collected and utilized financial data from 23 companies across various industries to test the proposed approach. The overall results provide evidence that the proposed approach can be used as a tool for investment decision-making in any industry sector. Moreover, based on the obtained results, companies can be analyzed and ranked.

The synergy of interpolative Boolean algebra and ordinal sums of conjunctive and disjunctive functions is explored for the first time in this research. The initial results are optimistic, so in future work, we can consider parametric classes of these functions as suggested in [40] to fine-tune data evaluation. For this task, we need a domain expert to express the desired inclination and a higher amount of data to learn parameters, for example, using genetic algorithms. Furthermore, future work will be oriented towards the development of a more complex system that includes a broader range of financial indicators covering different aspects of a company's financial performance. Additionally, in this paper, two different aggregation methods are used together.

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