

Interactive Granular Computing: Toward Computing Model for Complex Intelligent Systems

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Abstract—We present an approach based on the Interactive Granular Computing (IGrC) model as the basis for developing foundations of Complex Intelligent Systems, *i.e.*, Intelligent Systems dealing with complex phenomena (IS's). The generalization of GrC to IGrC was proposed to support the design of IS's treated in IGrC as examples of complex granules (c-granules) with control. To make such systems successful, it is necessary to enable such systems to have continuous interaction with the physical world. The control of c-granules aims to properly implement the physical semantics of specified transformations of c-granules in the physical world. This implementation is based on the discovery of relevant configurations of physical objects, which provides the basis for perceiving relevant data about these objects and their interactions through the control of c-granules. Additionally, to create high-quality models that serve as the basis for the behavior of IS's, these configurations must be adaptively adjusted by control to allow for the perception of relevant data used to induce those models. Unlike information granules from GrC, the correct implementation of c-granule transformations cannot be restricted to the abstract space. An important property of the IS's discussed here is that they cannot be separated from interactions with the physical world. Hence, they cannot be confined to an abstract space. In particular, the relevance of IGrC in searching for rough computational building blocks for cognition is discussed. These computational building blocks are modeled by complex granules (c-granules) and their networks. It is also proposed to use IGrC as the basis for developing IS's grounded on cognitive computing.

Index Terms—(interactive) granular computing (IGrC); informational-physical complex granule (c-granule); network of c-granules; control of c-granule; reasoning module; rule module; implementational module; decomposition module; rough set; modeling of rough membership function in IGrC.

I. INTRODUCTION

IN THIS paper, we summarize the current status of the IGrC model research based on published papers (see, *e.g.*, [1]–[6] and the developments presented in papers available at <https://dblp.org/pid/s/AndrzejSkowron.html>) as well as unpublished results, paying special attention to the IS's based on IGrC, the structure of c-granules with control and the role of rough sets in IGrC. We also provide an overview of the potential applications of IGrC-based IS's in various fields. Further details regarding IGrC and such applications will be provided in our future publications.

In the rough set approach, reasoning or decision making strategies are developed based on approximation spaces [7]–[10] which depend on perception about a finite set of objects or situations. For example, based on approximation spaces it is possible to estimate the degree of membership of these objects into given concepts (or classification). This is estimated using the degree of inclusion of some neighborhoods of the considered object (granule) into the considered concepts. The neighborhoods (granules), consist of indiscernible or similar objects to the perceived one. Problems related to how such membership functions are modeled are outside of the existing theory of rough sets. However, for IS's dealing with complex phenomena such an approach is not satisfactory because IS's should be able to construct membership functions based on the current perception of the physical world. As envisaged by many researchers (see, *e.g.*, opinion by Frederick Brooks cited later [11]) if models of the vague concepts related to complex phenomena are constructed using traditional mathematical modeling, then they would lack in quality. Rough membership functions are examples of such models. We propose to support modeling of such concepts basing on Interactive Computing Model (IGrC). In a sense, we are following the discussion on necessity to modify the Turing test by putting into sync not only issues of language and reasoning but also perception and action [12].

In IGrC we follow the suggestion of Professor Leslie Valiant, the Turing award winner. According to his opinion the most important problem of AI is the characterization of the computational building blocks for cognition (see <http://people.seas.harvard.edu/~valiant/researchinterests.htm>). IGrC is attempting to model these computational building blocks for cognition using dynamic complex granules (c-granules) generated along interactive granular computations. One should note, that many of such computational building blocks are not pure mathematical objects; for instance, they can be specified in natural language by complex vague concepts with physical semantics containing both abstract and physical objects when aspects of perception and action are considered in realization of semantics.

In IGrC, we deal with complex granules (c-granules) com-

posed out of abstract objects and/or physical objects. Such objects are necessary to enable IS's to deal with perception and action issues. IGrC aims to introduce tools for generating and controlling computations over (networks of) complex granules (c-granules)¹ so that the given requirements can be satisfied. These requirements may concern of the final states of computations (specified by some relevant properties of networks of c-granules) or the whole trajectories of computations (specified, e.g., by some invariants over computations on the (relevant parts) of networks of c-granules). These networks are like networks of physical pointers enabling IS's to continuously interact with the real physical world for better perceiving situations in it. They express realization of the physical semantics expressed by specifications of transformations of networks of c-granules. Information about perceived situations in the physical world is stored in i-layers of network of c-granules and is used by control of c-granule for selection of the relevant transformation of the current network of c-granules. The proposal of the IGrC model combines together many different ideas spread nowadays over different domains such as multi-agent systems, robotics, cognitive science, complex adaptive systems, to name only a few of them (see, e.g., [13]–[22]). In the paper, we restrict discussion about control of c-granules as a compound c-granule to an informal description of its behavior. The discussion on a more formal control specification and its physical semantics will be covered in one of our forthcoming paper.

The IGrC model is created as the basis for the design and analysis of c-granules, in particular IS's. The proposed IGrC model differs from the classical Turing model by corroborating the idea of synchronizing the four components: language, reasoning, perception, and action. In the IGrC model, granular computations form the basis for reasoning that supports problem solving by c-granules.

Problem solving (or decision support) using c-granules (in particular, IS's) requires a proper understanding of real-world situations consisting of configurations of interacting physical objects. Therefore, the control of c-granules must include skills for perceiving situations in the physical world enabling the formation of associations between physical and abstract objects. These skills are supported by reasoning over granular computations performed by the control of c-granules. Consequently, these computations cannot be confined to the abstract space alone. Moreover, they depend on physical laws [23].

The paper is structured as follows. In Sect. II we present motivations and rudiments of the IGrC model. Rough complex granules as computational building blocks for cognition are discussed in Sect. III. In Sec. IV the role of rough sets in problem solving by IS's is outlined. The roadmap for developing a rough cognitive computer based on IGrC is outlined in Sec. V. Finally, we present conclusions.

¹Networks of c-granules can be treated as higher order c-granules.

II. RUDIMENTS OF THE IGrC MODEL

A. Motivation

There is a huge literature about Granular Computing (GrC) (see, e.g., [1], [24]–[37] and references from these works) encompassing many different issues of GrC concerning, e.g., philosophy, methodology, frameworks, tools, methods, models. However, it is also emphasised in [29]:

The current studies of decision-making with GrC lack a theoretical foundation and normative research paradigm.

The IGrC model is a substantial extension of the GrC model. Granules from GrC can be treated as special cases of c-granules from IGrC². In Table I GrC and IGrC are compared based on several important features.

The IGrC model (see, e.g., [1]–[6] and <https://dblp.org/pid/s/AndrzejSkowron.html>) is the computing model on which we propose to develop such theoretical foundations for the design and analysis of IS's. Here, one should take opinions of many researchers which emphasizes that in case complex phenomenon classical mathematical modeling is not satisfactory. For example, according to Frederick Brooks [11]:

Mathematics and the physical sciences made great strides for three centuries by constructing simplified models of complex phenomena, deriving, properties from the models, and verifying those properties experimentally. This worked because the complexities ignored in the models were not the essential properties of the phenomena. It does not work when the complexities are the essence.

Hence, one can not expect that humans are able to derive models of the vague concepts related to complex phenomena using traditional mathematical modeling resulting in construction of the high quality models of these concepts. The current discussion on Turing test [12] is also worthwhile mentioning here:

The Turing test, as originally conceived, focused on language and reasoning; problems of perception and action were conspicuously absent. The proposed tests will provide an opportunity to bring four important areas of AI research (language, reasoning, perception, and action) back into sync after each has regrettably diverged into a fairly independent area of research.

According to the above opinions and many other evidences in different areas (see e.g., [6]) construction of models for complex vague concepts related to complex phenomena in the physical world should be based on an unconventional computing model able to deal with abstract and physical objects for providing modeling interactions of IS's with the

²Any information granule *inf* from GrC can be identified as a c-granule *g_{inf}* consisting of an i-layer with *inf* and two transformation specifications: *store* and *read* (labeled by the addresses pointing out to the physical memory). These specifications may be used by control of the c-granule to either store *inf* in the p-layer of *g_{inf}* or read *inf* from it into the i-layer, without information loss.

TABLE I
COMPARISON OF GrC AND IGrC

	GrC	IGrC
GENERAL FEATURES OF COMPUTING MODEL		
based on the abstract Turing computation model	YES	NO
computations are pure mathematical objects	YES	NO
issues of language, reasoning, perception and action are brought into sync	NO	YES
advanced reasoning tools based on the computing model that facilitate the control of computations involving both abstract and physical objects are being developed and utilized in the computing model	NO	YES
MAIN FEATURES OF GRANULES		
abstract semantics of (information) granules	YES	YES
physical semantics of (complex) granules	NO	YES
features (attributes) of granules defined using the abstract space only	YES	NO
features (attributes) of granules dependent on interaction with physical objects	NO	YES
granules are equipped with control	NO	YES
the dynamics of granules are defined apriori in the abstract space only	YES	NO
the dynamics of granules depends on their control and interactions with physical objects	NO	YES
associations between abstract and physical objects related to granules are being constructed and used in the computing model	NO	YES
skills for encoding information into physical objects provided in the computing model	NO	YES
skills for decoding information from physical objects provided in the computing model	NO	YES
changes of granules are being made based on the abstract space only	YES	NO
changes of granules are restricted to their abstract parts only	YES	NO
changes of the information represented in granules are being made based on the abstract space only	YES	NO
changes of the information represented in granules are also being made based on interaction with physical objects	NO	YES
STATES, TRANSITION, COMPUTATIONS IN COMPUTING MODEL		
states: granules interacting with physical objects	NO	YES
transition relation (association) defined based on information in the abstract space only	YES	NO
transition relation (association) dependent on interactions with physical objects	NO	YES
computations consist of abstract states only	YES	NO
computations depend on physical laws	NO	YES
adaptation of steering of granular computations provided by control of granules dependent on interaction with physical objects	NO	YES

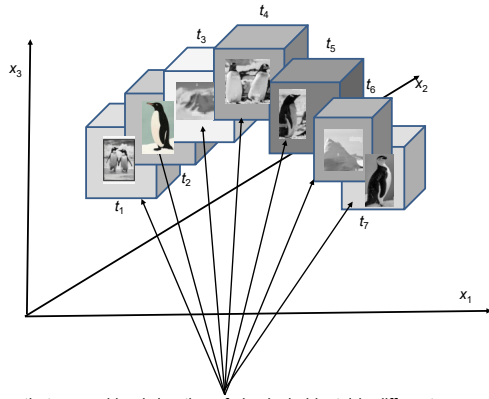
physical world in the process of perceiving of situations. We propose to base processes of construction of models for such concepts on IGrC.

Novelty of this paper is in (i) further clarifying the role of physical semantics for specifications of transformations of networks of c-granules and issues concerning its generation in the physical world by control of c-granules, (ii) adding discussion on searching for rough computational building blocks (c-granules) over computations of IGrC based on information (decision) systems as well as (iii) emphasizing usefulness of IGrC for developing foundations of cognitive computing by IS's. Our claim is that the foundations of IS's should be based on the unconventional IGrC model.

B. Basic postulates

We assume that IS's are equipped with c-granules which are able to perceive fragments of the physical world (hunks of matter [38]). In this process two layers of c-granules are important, namely the informational layer (i-layer) and the

physical layer (p-layer). In the i-layer, spatiotemporal windows (addresses) (see Fig. 1) are specified in a given language; say w , addresses in a sense parts of the physical world included in the regions $\|w\| \subseteq \mathbb{R}^3$ of the physical space defined by w , where $\mathbb{R}$ is the set of reals. Complex granules perceive these parts of the physical world using their control, in particular an implementational module (IM) of control is responsible for defining so called physical semantics of specifications labeling spatio-temporal windows. Roughly speaking, on the basis of a given specification labeling a given spatio-temporal window, the IM constructs a configuration of physical objects and initiates interaction of the c-granule control with and in this configuration. The control of c-granule perceives the properties of this configuration and stores them (as the results of the implementation) in the i-layer. One should note that these results may be different from the expected ones which are given in the specification. In the process of perceiving relevant information about the situation, the control of c-granule may activate or generate a family of c-granules linking the i-layer



Links to a spatio-temporal hunk (portion of physical objects) in different moments (or periods) of time $t_1, \dots, t_7$ specified by the c-granule control using specifications of spatio-temporal windows pointing to different fragments (portions of matter) of the 3-dimensional physical world.

Fig. 1. Example of spatio-temporal window.

with the p-layer, creating a network of c-granules, *i.e.*, a higher order c-granule. The c-granules of the network should create a robust configuration with respect to interactions with the environment. The c-granules of such a configuration should be like *niches* considered by John Holland [15]. Better understanding of niches from formal (mathematical) point of view is still a challenge. There are several important issues to be resolved by control of c-granules in construction of networks of c-granules for perceiving situations. Among them a few are as follows (see Fig. 2):

- How to perceive properties of the objects and their interactions in a concerned configuration? (Where, how, when to do this).
- How to generate the relevant configurations of physical objects and modify interactions between them?
- How can cooperation or competition be organized in societies of c-granules to support problem solving?
- How to ensure that the perceived properties are robust (to a high degree) with respect to unpredictable interactions from the environment?

Interaction is a critical issue in the understanding of IS's dealing with complex phenomena. For example, in [39] the following opinion is presented:

Interaction is a critical issue in the understanding of complex systems of any sorts: as such, it has emerged in several well-established scientific areas other than computer science, like biology, physics, social and organizational sciences.

Hence, one of the central issue concerns about interactions which should be treated properly by the computing model as on the basis of this IS's need to be designed and analyzed.

In IGrC model, we assume that interactions take place between configurations of physical objects and they can be partially perceived by c-granules. Three types of interactions of objects are distinguished:

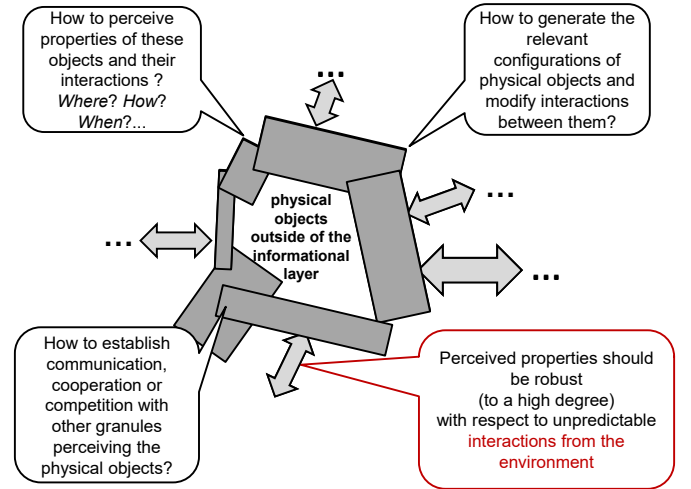


Fig. 2. Queries about a network of physical objects [6].

- 1: interactions between distinguished abstract objects from the i-layer; elaborated in GrC, *e.g.*, for granulation or aggregation of granules (see, *e.g.*, [26], [40]);
- 2: interactions between some distinguished physical objects from the p-layer; studied in physics and represented in domain knowledge data bases and/or physical laws by the relevant (information) granules in GrC;
- 3: interactions between some abstract objects from the i-layer and some physical objects from the p-layer; for perceiving situations in the physical world a special interfaces between abstract and p-layers is realized by networks of so called informational-physical complex granules (c-granules, for short).

Example 1: Association in abstract i-layer of c-granule g .

Let us assume that $f : X \rightarrow Y$, where X, Y are sets and f is a function from X into Y computed by an algorithm A . Computation of values of f are realized by interaction in i-layer of an abstract granule g_{arg} representing arguments of f and the granule g_A representing A . g_A is computing values of f on arguments sent by g_{arg} .

Example 2: Association between abstract i-layer and p-layer of c-granule g

$$f : X \xrightarrow{g} Y,$$

denoting a specification of the association for IM of the control of c-granule g with necessary details for its realization in the physical world and information about the expected results from Y .

Association f is realized using the physical semantics induced by the control of c-granule g responsible for implementation and perception:

- *Implementation step* : For given granules representing $x \in X$ and a specification of f , IM of the control of g extends (by some relevant new c-granules) the current c-granule g

to a new c-granule. This new granule provides ‘physical pointers’ from the object in which x has been directly encoded to a physical object in o_x , associated to x (by f). The object o_x is pointed out in the p-layer of this new c-granule by a spatio-temporal window w .

- *Perception step*: Some properties of parts of o_x , and other objects from the new c-granule as well as properties of interactions between them (and with the environment) are perceived by IM of the control of g and used for computing the value of f on x . This is done, in particular by decoding information from some parts of directly accessible physical objects of the new constructed granule into i-layer of g or inferring information on the basis of the:
 - i. already perceived information and/or
 - ii. available knowledge bases or physical laws.

Note that encoding and decoding mentioned above are the basic associations between abstract objects and physical objects ‘directly accessible’ by the control of g . Associations of abstract objects with more compound physical objects are realized as it is described above using physical objects which are not necessarily ‘directly accessible’. Information about associations of abstract and ‘not directly accessible’ physical objects is inferred by the control of c-granule on the basis of already perceived information stored in the information layer as well as domain knowledge and/or physical laws about the perceived physical objects and their interactions. Hence, computations realized by the control of c-granules depend on domain knowledge and/or physical objects [23]. Moreover, one can see that information about the realized associations between abstract and physical objects is, in general, only partial and relative to the perception skills offered by the control of c-granule.

C. Comments on structure of complex granules (c-granules)

Any c-granule is a dynamic object consisting of, at any moment of local time of c-granule, the following components (see Fig. 3):

- information granules (i-granules) stored in its i-layer: some distinguished (families, collections of) objects from (parts of) the i-layer (considered in GrC); in general any i-granule consists a family of specifications of spatio-temporal windows labeled by perceived or stored information; this information may contain different components, *e.g.*, related to specification of the transformation of the current network of c-granules being under the control of the original c-granule (see Fig 4) or information perceived so far through a particular window;
- physical granules (p-granules) located in its p-layer: some distinguished (families, collections of) objects from (parts of) the p-layer;
- associations between i-granules and p-granules; these associations are specified by formal specifications of transformations and realized as the physical semantics by IM of the control of c-granule through implementation in

the physical world; the physical semantics in the form of a network of c-granules under the control of the original c-granule (in a more general case the c-granules from the network may have their own control too) makes it possible to perceive partially the current situation in the physical world; these associations (it should be noted that these associations are not purely mathematical (abstract) objects) satisfy the following conditions:

- 1) if p-granule g_p (collection of physical objects) is associated to i-granule g_i then g_i specifies a part of the physical space where g_p is located;
- 2) g_i contains information so far perceived about g_p by the control of the original c-granule;
- 3) g_i also contains a specification of the goal for realization of which g_p was created in the physical space.

Objects from g_p may be directly accessible by the control of the original c-granule, *i.e.*, information from g_i may be directly encoded into objects from g_p or some features of objects from g_p may be decoded into g_i . Sometimes an object from g_p may not be directly accessible by the control of the original c-granule. Then three distinguished three parts, namely *soft_suit*, *link_suit*, and *hard_suit* of the original c-granule take part in perceiving information (see Fig 3).

In Fig 3 a general structure of c-granule is presented. In *soft_suit* there are physical objects that are directly accessible, *i.e.*, objects whose properties can be decoded into the i-layer by measurements or objects into which some relevant information from the i-layer can be encoded. The physical objects from the *link_suit* are used for transmission of interactions from *soft_suit* to *hard_suit* and *hard_suit* contains physical objects to be perceived according to the specifications of spatio-temporal windows represented in the i-layer. The scope of c-granule is a specification of spatio-temporal window in which other spatio-temporal windows are included. For a given specification of spatio-temporal window w , by $\|w\|$ we denote a region of the physical space corresponding to w . In a more general case w may represent a nested list of spatio-temporal specifications, *i.e.*, its components may represent new lists of specifications of spatio-temporal windows (Fig 3).

C-granules may have own control (c-granules with control) or can be without control (with empty control). Compound granules such as networks of c-granules or societies of c-granules are constructed from basic (atomic) c-granules. Details of construction of societies will be discussed elsewhere.

It should be noted that

- through interaction of i-granules from g_i with p-granules directly accessible by the control of the considered c-granule from g_p , information represented in i-granules may be encoded into p-granules;
- through interaction of i-granules from g_i with p-granules from g_p directly accessible by the control of the considered c-granule, the encoded information in these p-granules may be transmitted to some target p-granules from g_p ;

C-GRANULE: INTUITION

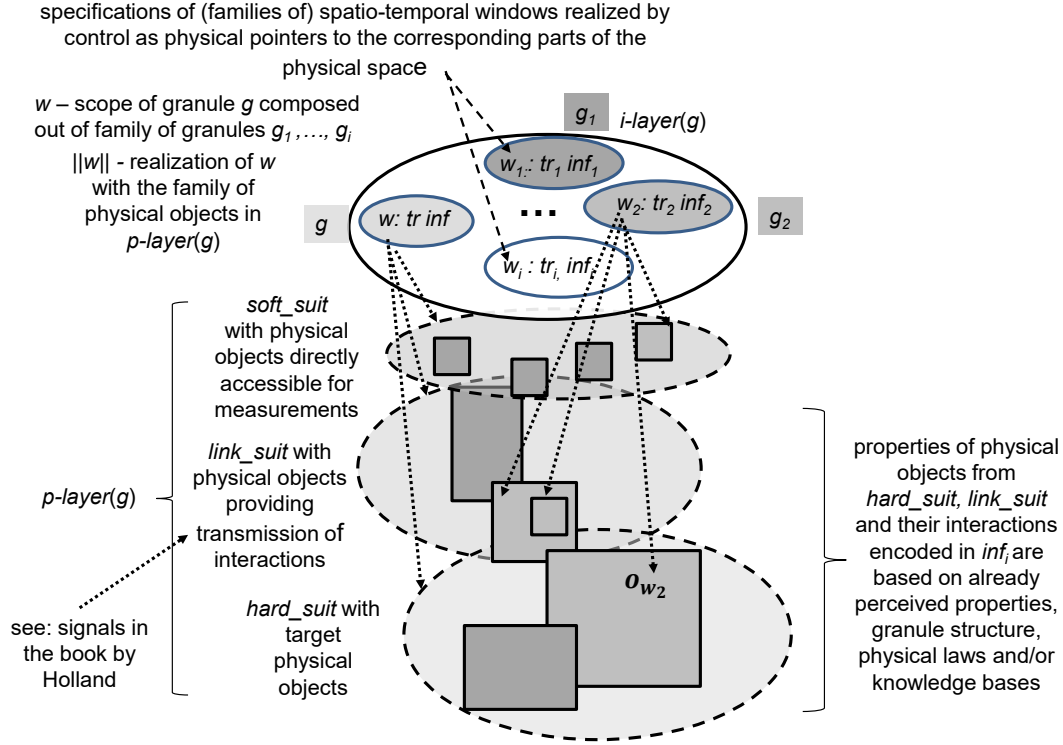


Fig. 3. C-granule intuition.

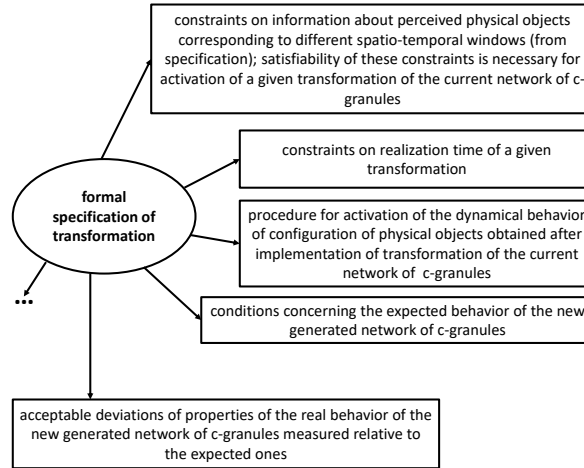


Fig. 4. Information stored in i-layer about transformation of network of c-granules.

- through interaction of i-granules from g_i with p-granules from g_p it may be decoded information from p-granules and stored in i-granules;
- information in the information layer of c-granule may be updated not only through interaction with the directly accessible objects; especially information about objects

from g_p may be updated on the basis of reasoning using already collected information and also domain knowledge bases and/or physical laws.

D. Dynamics of complex granules (c-granules)

Dynamics of c-granules is defined by control of c-granule. Formally, the control may be understood as a distinguished c-

granule with the i-layer, which describes the control behavior determined by the p-layer. An intuition of control is presented in Fig. 5.

Control organizes (and initiates) communications (transmissions, interactions) between the i-layer and p-layer using network (configuration) of relevant c-granules (generated by its IM). The considered network collects in the i-layers the properties of perceived physical objects and their interactions.

The control of C-granules includes different modules that act as sub-granules of the control granule. These sub-granules interact with each other and with the environment to carry out target tasks (needs). Among them the basic ones are the following:

- cycle module (CM);
- rule module (RM);
- implementational module (IM);
- decomposition module (DM);
- reasoning module (ResM).

More advanced control may contain many other modules such as a learning module (LM), adaptation module (AM), or domain knowledge module (DKM).

The CM is responsible for the realization of the basic cycle of control of the original c-granule.

The RM consists a set of rules of the form

$$\alpha \Rightarrow_{tr} \beta,$$

where

- α is a condition of the rule verified on information about the status of perception of the current situation,
- tr is a specification of transformation of the current network of c-granules; this specification describes which c-granules in the network should be eliminated, suspended, modified, not changed etc³ and
- β is the property of the expected result of realization.

Thus, the set of rules should be designed by the designer or learned from data.

The IM is a module responsible for generation of the physical semantics for a given specification of transformation (see Fig. 6). IM first tests the possibility of direct realization of a given specification in the physical world. If this is not possible, DM requires decomposition (see Fig. 7) of this specification, otherwise IM creates the relevant configuration of physical objects and initiates interactions in it aiming to realize the specification in the physical world.

More formal description of the DM is provided in [41]. It should be noted that the development of the DM module requires discovery of several decomposition languages. This discovery process can be supported by chatbots [42] helping to learn the strategy of divide-and-conquer of complex vague concepts (expressed in natural language) in human problem-solving. The key role in this process is played by information granulation [43], [44].

³More details on specification of transformations will be presented in our next paper.

The ResM is an important module supporting reasoning of the control of c-granules. For example, it allows control to decide if the current status of perception of the situation is satisfactory for selection of a rule from a given set of rules or to decide if the deviation of the expected results expressed in the rule from the real results obtained after implementation of a given transformation is acceptable. When such deviations are not acceptable, ResM may support adaptation of the current set of rules.

In Figures 8 - 9 is presented an illustrative example related to the ResM structure in one of our recent projects.

Control of c-granule aims to provide the most relevant decision systems for approximation of concepts. Information (decision) systems are represented in i-layer of c-granule; they are not isolated and not a priori given but they are modified by control of c-granule and (indirectly by) through interactions with the environment. Objects in information (decision) systems are (fragments of) multiple and/or multi-variate time series represented in i-layer of c-granule obtained as a result of perceiving the situation in the physical world along computations of c-granule over networks of c-granules. Attributes represent properties of such objects. There is a necessity of providing possibility to change the currently used attributes during computation for proper identification of perceived situation (making it possible to select the relevant transformations for realization). Decisions are elements of complex game, *i.e.*, pairs consisting of complex vague concepts labeled by specification of transformation (*e.g.*, decision (plan)) related to this concept. Complex games are discovered from such decision tables. Development of strategies for discovery of rough membership functions for different complex vague concepts are necessary.

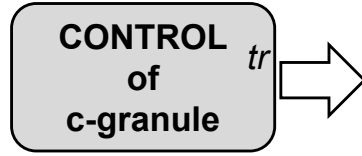
Hence here is a necessity to develop different reasoning (judgment) methods supporting, *e.g.*,

- discovery of complex games and their adaptation (reasoning about changes - rough calculus);
- identification of the relevant properties of situations in the physical world;
- control of computations over networks of c-granules toward generating computations satisfying a given specification; this may be related to the whole computation or to its final state;
- estimation of rough membership degrees of the perceived in the physical world situations to the considered concepts;
- resolving conflicts between rules specifying transformations to be performed;
- discovery of new sources of the relevant for the considered problem data (data governance [45]);
- discovery of compound sensors and/or actuators, robots.

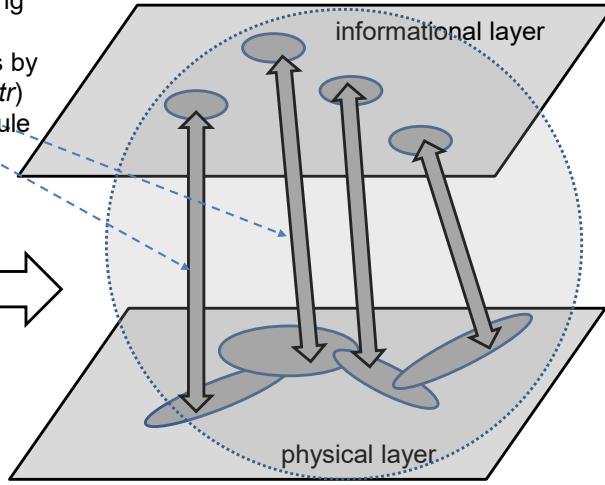
Let us also emphasize the important role of reasoning methods in supporting discovery of relevant rough computational building blocks for cognition (rc-granules in IGrC) or aggregation of decisions (*e.g.*, in synthesis of complex sensors, materials or robots). One should note that there are several 'white spots' in formal reasoning methods waiting to

C-GRANULE WITH CONTROL: INTUITION

Control is involved in establishing associations between the informational and physical layers by implementing transformations (tr) through its implementation module (IM) (physical semantics).



Control is initiating communications between the informational layer and the physical layer using relevant c-granules (generated by its IM), allowing the collection of properties of perceived physical objects and their interactions within the informational layer.



NETWORK OF C-GRANULES INTERACTING WITH ABSTRACT AND PHYSICAL OBJECTS

Fig. 5. C-granule with control: intuition.

PHYSICAL SEMANTICS: INTUITION

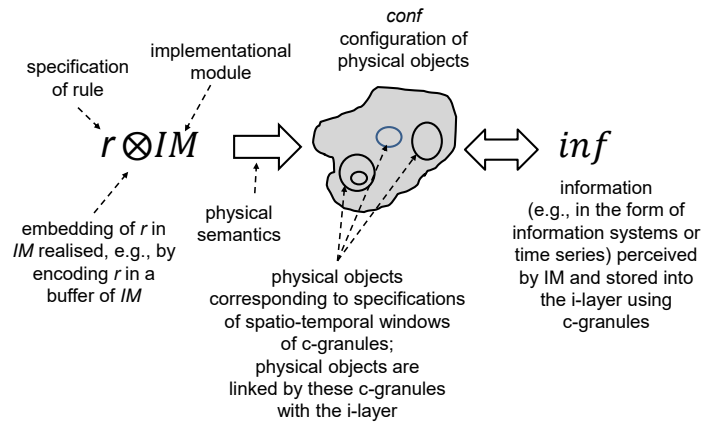


Fig. 6. Physical semantics: intuition.

be filled by the relevant methods. One example of such 'white spot' concerns experience based reasoning (see *e.g.*, [46], [47]). Hence, it is necessary to develop reasoning methods for supporting dialogues of IS's with experts to help them in performing of experience based reasoning [48], [49].

Control of c-granule in interaction with the physical world

generates computations over networks of c-granules. The aim of the control is to assure whether the generated computations satisfy given requirements or target goals. They may concern the final states of such computations or the whole trajectories of computations (*e.g.*, in case of required invariants to be preserved in computations). One should note that these

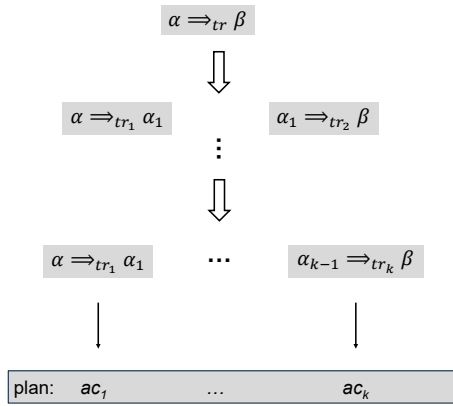


Fig. 7. Decomposition.

computations are not pure mathematical objects, they are unconventional because they are constructed over abstract and physical objects. In particular, they may depend on domain knowledge or physical laws [23].

III. DISCOVERY OF ROUGH COMPLEX GRANULES (RC-GRANULES) AS COMPUTATIONAL BUILDING BLOCKS FOR COGNITION AND THE LANGUAGE OF GAME OF WITTGENSTEIN

As it was observed in Sect. I, nowadays rough membership functions are defined as pure mathematical objects. However, according to the cited opinion by Frederick Brooks [11] models of these functions, being often models of vague concepts related to complex phenomena, can't be constructed (induced) using traditional mathematical modeling. Hence, existing approaches to modeling of rough membership functions should be enriched by mechanisms for continuous interaction with the physical real world for perception of the current situation making it possible to estimate adaptively membership values. In the perceptual approach to rough sets in IGrC, rough membership value for the currently perceived situation in the real physical world is estimated on the basis of perceived data by (networks of) c-granules dynamically interacting with the real physical world up to the moment when understanding of the perceived situation is satisfactory for making this estimation by control of c-granule.

One should note that the approach based on IGrC is consistent with the concept of language-game by Ludwig Wittgenstein [50] who argued that a word or even a sentence has meaning in the language only as a result of the *rule of the game* being played. In the case of IGrC, the meaning as use (<https://plato.stanford.edu/entries/wittgenstein/#MeanUse>) is realized as physical semantics *i.e.*, is realized in the physical world and its results are partially perceived by c-granules.

In the discussed IGrC-based approach, computational building blocks for concept cognition (see <http://people.seas.harvard.edu/~valiant/researchinterests.htm>) are constructed alongside computations over granular

networks generated by the control of c-granules interacting with the environment. The control of c-granules is responsible for discovering computational building blocks via sensors or actuators, as well as aggregating pre-existing blocks. Searching for the relevant computational building blocks on different levels of hierarchical modeling is supported by strategies with backtracking, in case the currently searching 'path' occurs to be not promising. Control of c-granules aims to search for the relevant computational building blocks, *e.g.*, patterns, classifiers or clusters from which the high quality membership functions of the target complex vague concepts can be discovered. These target concepts are responsible for triggering the actions or plans on the highest level of the hierarchical modeling. Hierarchical learning of computational building blocks is based on aggregation of information (decision) systems: the Cartesian product of object descriptions from aggregated information systems [6], [51], [52] is filtered by constraints defined with the use of discovered relational systems. Relations from these systems are defined over products of value sets of attributes from the considered systems.

C-granule control also employs searching strategies for approximate reasoning schemes that link different levels of the concept hierarchy [40].

One should also note an important role of reasoning strategies for conflict resolution between different computational building blocks represented by rough membership functions matching the current situation in the physical world and voting for different target decisions.

By changing the language in which computational building blocks are represented one can obtain the approach based on rough sets or on combination of rough and fuzzy sets for discovery of relevant computational building blocks.

IV. NOTE ON ROUGH SETS IN APPROXIMATE PROBLEM SOLVING BY IS'S

In this section, we outline the role of rough sets in approximate problem solving by IS's. The presented discussion on IGrC model makes it possible to generalize the existing approaches to approximation of concepts to a more general case of approximation of granules (*e.g.*, construction of complex abstract or physical objects satisfying a given specification to a satisfactory degree or construction of the high quality of classifiers from training sets). The key idea of approximation of granules can be intuitively expressed in the following way:

For a given specification of problem to be solved, the control of a given c-granule (or society of granule) is aiming to generate, in interaction with the environment, a computation on granular networks satisfying this specification to a satisfactory degree.

The formalization concerning this understanding of approximation will be discussed in more detail in our next paper. Here, we would like to mention that this formalizations would clarify in particular, the following issues:

- problem specification is encoded into the initial granular network of computation,

REASONING MODULE OF C-GRANULE CONTROL: EXAMPLE

Upon receiving a task (question) from the user, the control of the c-granule initiates reasoning in the information layer of its sub-granule called the reasoning module.

This reasoning is realized by granular computations involving interactions (communications) with databases, experts, or other users, and aims to provide an answer to the task.

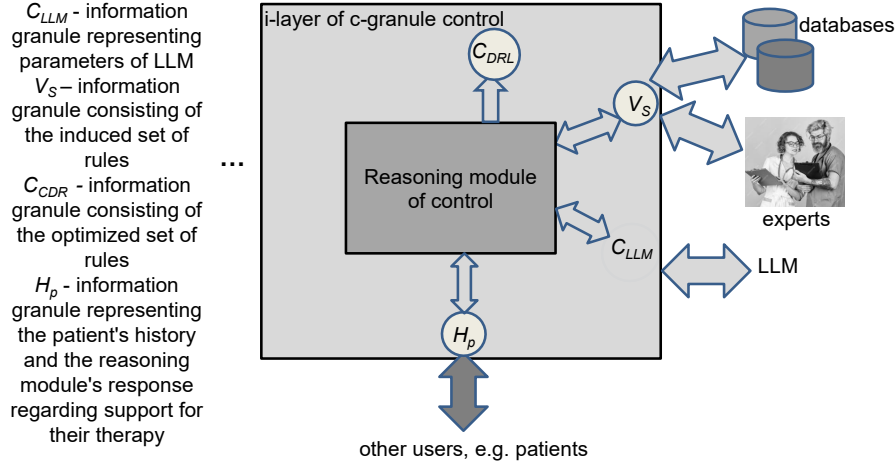


Fig. 8. Example concerning the c-granule representing the ResM of c-granule control.

C-GRANULE REPRESENTING REASONING MODULE OF C-GRANULE CONTROL: EXAMPLE

The reasoning module includes, in particular, a link, realized by a special c-granule, to the knowledge base of reasoning patterns and strategies for detecting new reasoning patterns.

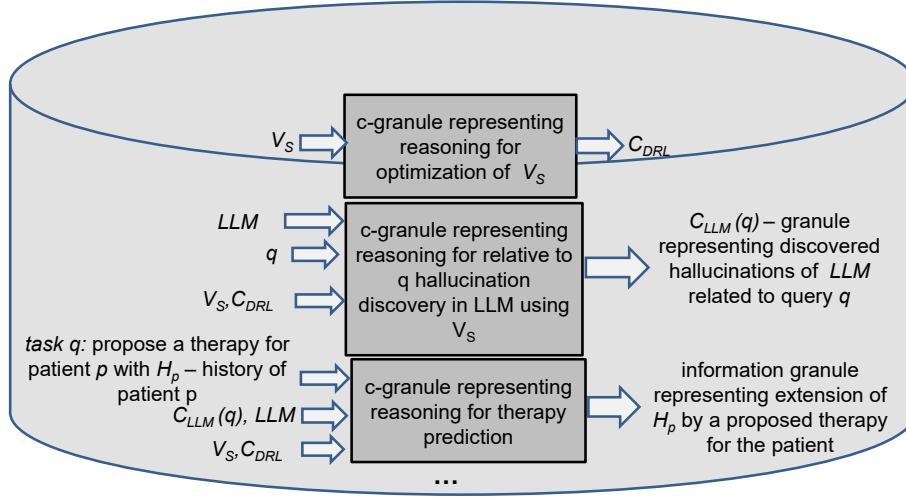


Fig. 9. Example concerning the c-granule in reasoning module that represents a knowledge base with reasoning patterns.

- solutions of the considered problem are expressed by granules (e.g., when the task is to learn classifiers or compound physical or chemical objects),
- different fragments of granular networks are expressed using different languages and linked by interfaces (e.g., based on aggregation operations between granules),
- successive steps of computation are obtained by control of c-granule interacting with the physical world (e.g.,

by generating more compound granular networks from existing ones or extending the existing ones by granules resulting from the relevant interactions with the physical world or domain knowledge bases),

- selection of rule for transformation of the current granular network is performed,
- the rule set for transformation of granular networks is adaptively learned,
- control can search for new data sets in the physical world for updating granular networks (by reasoning, *e.g.*, when, where to search for such data),
- different components of approximation spaces are distinguished to make it possible to define the quality of approximation of granules,
- selection of optimization procedures for approximation is performed among many possible ones,
- control of c-granule is designed to make it possible to generate computation of the required quality (*e.g.*, by optimization and adaptation of its behavior),
- quality of computation is evaluated by the relevant quality measures (*e.g.*, some of them may use the whole computation for evaluation when the task is to preserve some invariants during computation and some other ones depend only on the final granular network in the computation),
- quality of generated approximate solution is tested in the physical world.

In this generalized approach to the rough set-based approximation of granules, reasoning performed by the control of c-granules over the generated granular computations plays a crucial role. As we discussed above, reasoning supporting approximation of granules is performed along computations by control of c-granule.

Fig. 10 presents a challenge for rough sets in IGrC related to discovery of approximate solutions of problems. Note that in this case, there are no provided examples belonging to the lower approximation of the concept *granular computation with satisfactory quality relative to the specified task*⁴. However, such examples should be discovered. There are numerous domains for which this challenge is important. Among them (i) discovery of learning algorithms and construction of classifiers [13], [53], (ii) automatic design of robots [54], (iii) drug discovery [55], (iv) algorithmic trading [56], (v) generative AI [57] are a few. One could request the development of common foundations based on IGrC and RS for designing and analyzing discovery systems from these different domains [58].

V. ROADMAP FOR DEVELOPING A ROUGH COGNITIVE COMPUTER BASED ON IGrC

Human-inspired computing is defined as the intelligence computing model enlightened by human brain intelligence and biological processes. We propose to take (networks of) c-granules as the basic ingredients of such a model. They seem to be the relevant in modeling basic and higher order neurons.

⁴The lower approximation may be defined, *e.g.*, by specifying ‘satisfactory quality’ by the relevant threshold.

They have richer structure than traditional artificial neurons and provide a more suitable objects for modeling of cognitive computer based on IGrC than the traditional artificial neurons. For example, on different levels of hierarchical modeling c-granules with different languages for expressing perceived properties may be used. Certainly, here arises a challenge how to discover such relevant languages. Moreover, they may have sophisticated control mechanisms. These control mechanisms should make it possible to adaptively change the networks toward satisfying the target goals (or needs of granules or agents). Hence, in this way we may obtain the approach for modeling cognitive computers based on generalization of neural networks to adaptive granular networks. In the future, we will investigate dynamical structures related to granulation of c-granules (*e.g.*, into networks) and their degranulation processes aiming at searching for other c-granules understood as computational building blocks for cognition, using the Valiant formulation. These computational building blocks can be used, *e.g.*, to approximation of complex vague concepts triggering realization of transformations’ specifications (*e.g.*, in the form actions or plans) by control of c-granules. For modeling of cognitive computer societies of networks of c-granules will be used following the idea of distributed control experience presented in the literature (see, *e.g.*, [58]–[64]). Further cooperation with neuroscientists [65] may substantially enhance the development strategies for modeling such dynamical structures.

Issues of multiscale reasoning inspired by biological processes are of great importance for further developing IGrC. Here, it is worthwhile to cite the following statement from [66]:

[...] *One of the fascinating goals of natural computing is to understand, in terms of information processing, the functioning of a living cell. An important step in this direction is understanding of interactions between biochemical reactions. [...] the functioning of a living cell is determined by interactions of a huge number of biochemical reactions that take place in living cells.*

VI. CONCLUSIONS

In this paper we have presented the IGrC model as the basis for the design and analysis of Complex Intelligent Systems (IS’s). In this way, we have taken a step toward developing the theoretical foundation for this task. The IGrC is not purely mathematical and it substantially depends on continuous interaction with the physical world aiming to generate the right physical semantics. The control module of c-granules steers computations over granular networks toward achieving target goals or needs. This process involves interacting with numerous other c-granules based on physical objects, experts, users, knowledge bases, and physical laws. The presented approach can be used as foundations for developing IS’s in different domains, including AI’s discovering learning algorithms, specific robots, new drugs, strategies in algorithmic trading or models of cognitive computers.

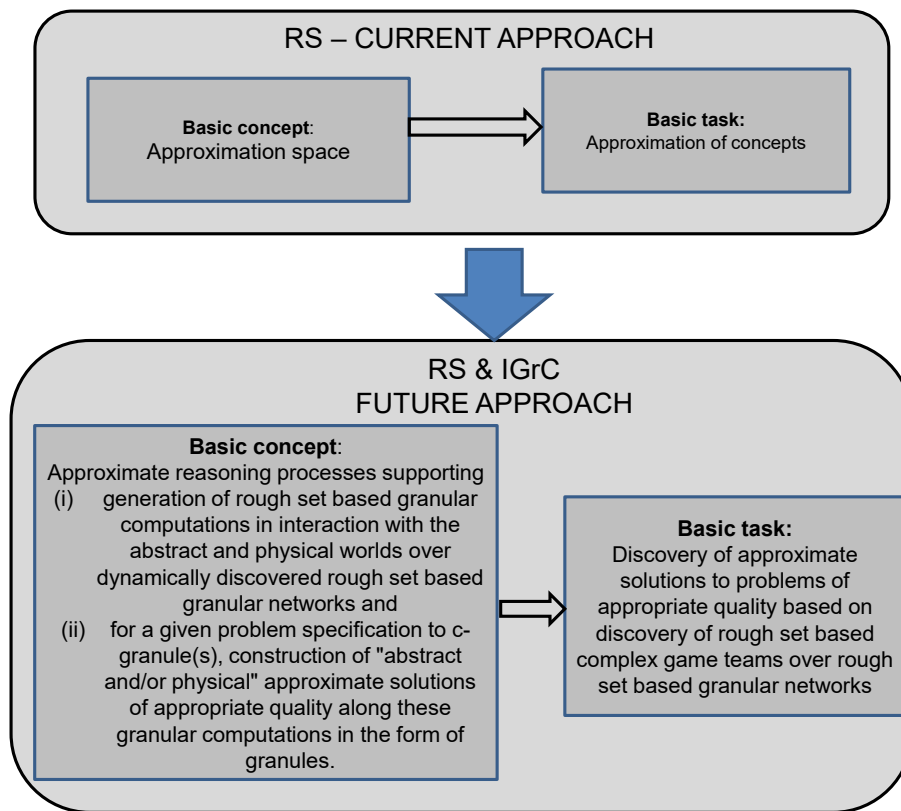


Fig. 10. A challenge for RS: discovery of approximate solutions of problems.

Our aim is to build foundations for IS's realizing the following dream [67]:

Tomorrow, I believe, we will use

[INTELLIGENT SYSTEMS]

to support our decisions in defining our research strategy and specific aims, in managing our experiments, in collecting our results, interpreting our data, in incorporating the findings of others, in disseminating our observations, in extending (generalizing) our experimental observations through exploratory discovery and modeling - in directions completely unanticipated.

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