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A Confidence-Interval Circular Intuitionistic Fuzzy Zero Point Model for Optimizing Spare Parts Transfer in Smart Manufacturing Environments

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***Abstract*—In Industry 4.0 systems, timely delivery of critical components to maintenance points is essential for continuous operation. This paper introduces a novel Confidence-Interval Circular Intuitionistic Fuzzy Zero Point Method (CIC-IFZPM) to optimize the transfer of spare parts in a smart factory setting. The method addresses uncertainty in transfer cost, delivery time, and priority assessment through circular intuitionistic fuzzy sets (C-IFS), which reflect both membership and hesitancy with geometric interpretation. A customized version of the index matrix algorithm integrates transportation constraints, expert confidence intervals, and machine availability limitations. The model is validated through a simulated industrial scenario, where production cells request components dynamically, and a central warehouse must allocate them optimally. Compared to classical fuzzy optimization approaches, the proposed method ensures more robust decision-making under incomplete or imprecise data, offering better performance in real-time control environments. The framework is applicable to predictive maintenance logistics, autonomous scheduling, and industrial resilience planning.**

I. Introduction

TRANSPORTATION problems (TPs) aim to determine optimal delivery routes that minimize total transportation costs. The classical formulation originated with Hitchcock in 1941 [6], followed by Dantzig's application of the simplex method [5] and Kantorovich's development of the "method of potentials" in 1949 [8]. In practice, however, transport systems operate under significant uncertainty caused by fluctuating fuel prices, economic volatility, and external disruptions.

To model such vagueness, fuzzy logic approaches have been widely adopted. Zadeh introduced fuzzy sets (FSs) in 1965 [23], and Atanassov later proposed intuitionistic fuzzy sets (IFSs) in 1983 [1], enabling more nuanced uncertainty representation through the inclusion of membership, non-membership, and hesitation degrees.

Numerous fuzzy TP methods have since emerged, including the Zero Point Method applied to trapezoidal fuzzy data [13], and enhancements using triangular, LR-flat, and hybrid fuzzy numbers [9], [10], [15]. Comparative analyses suggest the Zero Point Method often outperforms classical techniques [12]. Further variants include the zero suffix method [7], IF Zero Suffix Method (IFZSM) [19], and IF Zero Point Method (IFZPM) [18]. The proposed IFZPM yielded a marginally better optimal solution than the previously established IFZSM [18] in the specific case study under consideration, demonstrating its potential for enhanced performance under fuzzy uncertainty.

To better capture multidimensional and circular uncertainties, Atanassov introduced the Circular Intuitionistic Fuzzy Set (C-IFS) [3] in 2020. Building upon this, we extend C-IFSs to Confidence-Interval Circular Intuitionistic Fuzzy Sets (CIC-IFSs) [22], where each fuzzy element is represented as a circular region whose radius varies with a confidence level β.

This paper proposes a novel CIC-IF Zero Point Method for solving transportation problems under the CIC-IFTP framework. Transportation costs, supply, and demand are modeled as CIC-IF triples [22], incorporating expert evaluations and uncertainty quantification. The solution algorithm builds on the index matrix (IM) approach [2], while introducing additional constraints such as transport cost caps and confidence-based tolerances. The proposed algorithm is an extension of the classical Zero Point Method [15], designed to accommodate uncertain environments by incorporating circular intuitionistic fuzzy representations and the level of confidence.

To demonstrate applicability, we consider a smart manufacturing scenario involving the dynamic reallocation of spare

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parts. This environment is characterized by uncertain delivery times and competing demands from maintenance units. Our key contributions include the formalization of the CIC-IFTP framework, a robust solution algorithm, and validation through an industrial case study reflecting predictive maintenance logistics.

The paper is structured as follows: Section II introduces preliminaries on CIC-IF triples and index matrices. Section III details the problem formulation, solution procedure, and industrial case study. Section IV discusses computational results and future work.

II. Preliminaries

This section recalls the key concepts underpinning the proposed approach: Confidence-Interval Circular Intuitionistic Fuzzy Sets (CIC-IFSs), Triples (CIC-IFTs), and Index Matrices (CIC-IFIMs). We define each structure and the operations applicable to them.

A. Confidence Interval Circular Intuitionistic Fuzzy Sets (CIC-IFSs)

The definitions and properties of CIC-IF sets used in this paper follow the construction proposed in [22]. Let $A \subseteq E$, where E is a universe of discourse. A CIC-IFS with confidence level β is defined as:

$$A_u^\beta = \{\langle x, \mu_A(x), \nu_A(x); u^\beta \rangle \mid x \in E\},$$

where $\mu_A(x) + \nu_A(x) \leq 1$, and $u^\beta \in [0, \sqrt{2}]$ is the circular radius expressing confidence-based fuzziness. The uncertainty margin $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ complements the membership functions.

The center of the circle is computed as:

$$\langle \mu(x)^\beta, \nu(x)^\beta \rangle = \left\langle \frac{a(x)+c(x)}{2}, \frac{b(x)+d(x)}{2} \right\rangle.$$

Its radius is obtained from the maximal Euclidean deviation between this center and individual expert evaluations:

$$u^\beta(x) = \max_{1 \leq i \leq k_x} \sqrt{(\mu(x)^\beta - \mu_{k_i}^\beta)^2 + (\nu(x)^\beta - \nu_{k_i}^\beta)^2}.$$

B. Confidence Interval Circular Intuitionistic Fuzzy Triples (CIC-IFTs)

The formalization of CIC-IFTs and related operations is based on the framework introduced in [22]. Given expert assessments for assertion p, we define a CIC-IFT as:

$$\langle \mu(p), \nu(p); u^\beta \rangle = \langle a(p), b(p); u^\beta \rangle, \quad \text{where } a(p) + b(p) \leq 1.$$

The center and radius follow similar constructions, based on bounds $a(p), b(p), c(p), d(p)$, and the maximum deviation from the confidence center.

CIC-IFTs are closed under operations such as $\wedge, \vee, +, \bullet, -$, with radius propagation via max/min functions. Comparison and ranking between two CIC-IFTs is performed by dominance or by proximity to the ideal $\langle 1, 0; \sqrt{2} \rangle$.

C. 3D Confidence Interval Circular Intuitionistic Fuzzy Index Matrices (3D CIC-IFIMs)

CIC-IFIMs extend index matrices to a three-dimensional structure:

$$A^\beta = [K, L, H, \{\langle \mu_{k_i,l_j,h_g}, \nu_{k_i,l_j,h_g}; rf^\beta_{k_i,l_j,h_g} \rangle\}].$$

Each entry represents a CIC-IFT, structured along three index dimensions—supply, demand, and operational scenarios—defined as subsets $K, L, H \subset \mathscr{I}$. The associated operations over CIC-IFIMs follow extensions of fuzzy matrix logic, as discussed in [22].

To further process multidimensional data, we apply aggregation operations (AOs) over one dimension of a 3D CIC-IFIM [22]. Let $* \in \{\min, \max\}$ be a binary operator. Ten aggregation operations $\#_i$ $(1 \leq i \leq 10)$ are defined over two CIC-IFTs $x = \langle a, b; rf_1^\beta \rangle$ and $y = \langle c, d; rf_2^{\beta_1} \rangle$, including for example:

$$x \#_1 y = \langle ac, 1 - ac, *(rf_1^\beta, rf_2^\beta) \rangle,$$

$$x \#_{10} y = \langle \min(1, 2 - b - d), \max(0, b + d - 1), *(rf_1^\beta, rf_2^\beta) \rangle.$$

Let $k_0 \notin K$ be an artificial aggregation index. The operator $\alpha_{K,\#_q,*}(A^\beta, k_0)$ aggregates over the supply dimension K, yielding:

$h_g \in H$	l_1	$\dots$	l_n
k_0	$\overset{m}{\underset{i=1}{\#_{q,*}}} \langle \mu_{k_i,l_1,h_g}, \nu_{k_i,l_1,h_g}; rf^\beta_{k_i,l_1,h_g} \rangle_{\#_{q,*}}$	$\dots$	$\overset{m}{\underset{i=1}{\#_{q,*}}} \langle \mu_{k_i,l_n,h_g}, \nu_{k_i,l_n,h_g}; rf^\beta_{k_i,l_n,h_g} \rangle_{\#_{q,*}}$

Depending on the scenario, we may choose $\#_1^*$ for pessimistic aggregation (e.g., inflation), $\#_5^*$ or $\#_6^*$ for moderate strategies, and $\#_{10}^*$ for optimistic planning.

III. Problem Statement: CIC-IFTP in Smart Factory Maintenance Logistics

We extend the C-IFTP framework from [20] to a novel Confidence-Interval Circular Intuitionistic Fuzzy Transportation Problem (CIC-IFTP) with operational constraints tailored for smart manufacturing environments.

A smart electronics factory seeks to optimize the allocation of critical spare parts—such as sensors, microcontrollers, and actuators—from central storage units $\{k_1, \dots, k_m\}$ to maintenance stations $\{l_1, \dots, l_n\}$. The available stock at each storage unit is denoted by $c_{k_i,R}$, while each station requests c_{Q,l_j} units.

Each internal transport route (robotic or conveyor-based) from k_i to l_j has an operational usage threshold c_{pl,l_j} and a unit transportation cost c_{k_i,l_j}. These parameters are subject to uncertainty and are evaluated by a panel of planners $\{d_1, \dots, d_D\}$, who provide intuitionistic fuzzy (IF) preferences represented as $\text{re}_s = \langle \delta_s, \varepsilon_s \rangle$. Based on their assessments, circular intuitionistic fuzzy (CIF) data are constructed using a selected confidence level β.

The decision variable x_{k_i,l_j} denotes the number of units to be routed from storage unit k_i to station l_j. Ten scenario strategies—ranging from highly pessimistic to highly optimistic—guide the decision-making process under uncertainty.

The objective is to minimize the total CIC-IF transportation cost, subject to the following constraints:

- All station demands c_{Q,l_j} must be satisfied;
- Storage capacities $c_{k_i,R}$ must not be exceeded;
- Route-specific operational thresholds c_{pl,l_j} must be respected.

To solve this problem, we propose a novel *Confidence-Interval Circular Intuitionistic Fuzzy Zero Point Method (CIC-IFZPM)*. This algorithm extends the index matrix-based approach introduced in [20]. While rooted in the classical Zero Point Method, CIC-IFZPM advances the intuitionistic fuzzy modeling framework by incorporating confidence-interval circular intuitionistic fuzzy structures [22]. These enhancements allow the model to more accurately capture imprecision, expert disagreement, and cyclic interdependencies—phenomena frequently encountered in smart manufacturing and logistics environments. The detailed solution algorithm is presented in the following section.

A. The Solution Algorithm

Algorithm 1 Construction of CIC-IFIM (Step 1)

Require: Sets $K = \{k_1, \dots, k_m\}$, $L = \{l_1, \dots, l_n\}$, experts $E = \{d_1, \dots, d_D\}$, confidence coefficient $\beta \in [0, 1]$

for $j = 1$ to n **do**
  for $i = 1$ to m **do**
    Each expert $d_s \in E$ provides IF evaluation $ev_{i,j,s} = \langle \mu_{i,j,s}, \nu_{i,j,s} \rangle$ and reliability $re_s = \langle \delta_s, \varepsilon_s \rangle$
    Aggregate evaluations: $EV^*_{i,j} \leftarrow \bigoplus_{s=1}^{D} re_s \cdot ev_{i,j,s}$
    Apply circular IF aggregation: $pi^{ave}_{i,j} \leftarrow \alpha_{E,\#_2}(EV^*_{i,j})$
    Compute radius: $r^{\beta}_{i,j} \leftarrow \max_{1 \le s \le D} \left(\sqrt{(\mu_{i,j,s} - \mu_{i,j})^2 + (\nu_{i,j,s} - \nu_{i,j})^2} \right)$
    Form CIC-IFT: $c^{\beta}_{i,j} \leftarrow \langle \mu_{i,j}, \nu_{i,j}; r^{\beta}_{i,j} \rangle$
  end for
end for
Assemble CIC-IFIM: $C^{\beta}[K, L] = \{c^{\beta}_{i,j}\}_{i=1..m,\, j=1..n}$
Extend to complete matrix: $C^{\beta}[K^*, L^*]$ by adding artificial nodes Q, pl, pu_1, R, pu

	$\dots$	l_n	R	pu
k_1	$\dots$	$\langle \mu^c_{k_1,l_n}, \nu^c_{k_1,l_n}; r^c_{k_1,l_n} \rangle$	$\langle \mu^c_{k_1,R}, \nu^c_{k_1,R}; r^c_{k_1,R} \rangle$	$\langle \mu^c_{k_1,pu}, \nu^c_{k_1,pu}; r^c_{k_1,pu} \rangle$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
k_m	$\dots$	$\langle \mu^c_{k_m,l_n}, \nu^c_{k_m,l_n}; r^c_{k_m,l_n} \rangle$	$\langle \mu^c_{k_m,R}, \nu^c_{k_m,R}; r^c_{k_m,R} \rangle$	$\langle \mu^c_{k_m,pu}, \nu^c_{k_m,pu}; r^c_{k_m,pu} \rangle$
Q	$\dots$	$\langle \mu^c_{Q,l_n}, \nu^c_{Q,l_n}; r^c_{Q,l_n} \rangle$	$\langle \mu^c_{Q,R}, \nu^c_{Q,R}; r^c_{Q,R} \rangle$	$\langle \mu^c_{Q,pu}, \nu^c_{Q,pu}; r^c_{Q,pu} \rangle$
pl	$\dots$	$\langle \mu^c_{pl,l_n}, \nu^c_{pl,l_n}; r^c_{pl,l_n} \rangle$	$\langle \mu^c_{pl,R}, \nu^c_{pl,R}; r^c_{pl,R} \rangle$	$\langle \mu^c_{pl,pu}, \nu^c_{pl,pu}; r^c_{pl,pu} \rangle$
pu_1	$\dots$	$\langle \mu^c_{pu_1,l_n}, \nu^c_{pu_1,l_n}; r^c_{pu_1,l_n} \rangle$	$\langle \mu^c_{pu_1,R}, \nu^c_{pu_1,R}; r^c_{pu_1,R} \rangle$	$\langle \mu^c_{pu_1,pu}, \nu^c_{pu_1,pu}; r^c_{pu_1,pu} \rangle$

Fig. 1. Extended CIC-IFIM with CIC-IF entries

Step 1 (continued). Initialization of Auxiliary Matrices

After constructing the CIC-IFIM matrix $C^{\beta}[K, L, h_f]$, we extend the sets:

- $K^* = K \cup \{Q, pl, pu_1\} \Rightarrow |K^*| = m + 3$;
- $L^* = L \cup \{R, pu\} \Rightarrow |L^*| = n + 2$.

Then, the following auxiliary matrices are initialized:

1) **State matrix** $S^{\beta}[K^*, L^*]$: Initialized as a duplicate of C^{β}, i.e., $s^{\beta}_{k_i,l_j} = c^{\beta}_{k_i,l_j}$.
2) **Discard matrix** $D[K, L]$: Each $d_{k_i,l_j} \in \{1, 2\}$ tracks the number of times a cell has been eliminated.
3) **Row crossing indicator** $RC[K]$: $rc_{k_i,e_0} \in \{0, 1\}$ indicates whether row k_i has been excluded.
4) **Column crossing indicator** $CC[L]$: $cc_{r_0,l_j} \in \{0, 1\}$ indicates whether column l_j has been excluded.
5) **Projections**:
   - $RM[K, R] = pr_{K,R}(C)$
   - $CM[pu_1, L] = pr_{pu_1,L}(C)$

   Used in balancing, particularly regarding R and pu_1 nodes.
6) **Utility matrix** $U[K, L]$: Defined as:
$$u_{k_i,l_j} = \begin{cases} 1, & \text{if } c_{k_i,l_j} < c_{pl,l_j} \\ \perp, & \text{otherwise} \end{cases}$$
7) **Initial allocation matrix** $X[K, L]$: All entries start as
$$x^{\beta}_{k_i,l_j} = \langle 0, 1; \sqrt{2} \rangle.$$

Initial Setup: All indicators are initialized as:
$$rm_{k_i,R} = rc_{k_i,e_0} = cc_{r_0,l_j} = cm_{pu_1,l_j} = 0, \quad u_{k_i,l_j} = \perp.$$

Balancing the System: If the transportation problem is unbalanced (i.e., $\sum \text{Supply} \neq \sum \text{Demand}$), balancing is applied by adding artificial nodes Q, pu, R with synthetic cost entries, following [18].

The algorithm then proceeds to **Step 2**.

Step 2. Verifying the Transportation Cost Constraints

For each warehouse $k_i \in K$ and destination cell $l_j \in L$, verify whether the transportation cost from k_i to l_j is strictly preferable to the baseline from the pseudo-node pl. Iterate:
$$\textbf{for } i = 1 \textbf{ to } m: \quad \textbf{for } j = 1 \textbf{ to } n:$$
$$\textbf{if } \left(\left[\frac{k_i}{pl}; \perp \right] pr_{pl,l_j} C^{\beta} \right) \supset_{\nu} pr_{k_i,l_j} C^{\beta} \textbf{ then } u_{k_i,l_j} \leftarrow 1$$

After evaluating all entries, define the set of non-preferable allocations:
$$EG = \text{Index}_{(\perp)}(U) = \left\{ \langle k_{i_1}, l_{j_1} \rangle, \langle k_{i_2}, l_{j_2} \rangle, \dots, \langle k_{i_\phi}, l_{j_\phi} \rangle \right\}$$

For each $\langle k_i, l_j \rangle \in EG$, penalize the corresponding entry in matrix S as:
$$s^{\beta}_{k_i,l_j} \leftarrow \langle 1, 0; \sqrt{2} \rangle \quad \text{(as in [11])}$$

Proceed to **Step 3**.

Step 3. Row-Level Normalization Using Zero Membership Values

In this step, we compute a row-specific zero-cost reference based on membership minimization. This value is stored in the auxiliary column pu, facilitating row-wise normalization aligned with the Zero Point Method principle.

1) **Identify Minimum Cost Elements:**
   For each row $i = 1$ to m, determine the minimum CIC-IF value among columns $j = 1$ to n using a selected aggregation index:
$$AGIndex_{\{\min,\ \min_{\square},\ \min_{\diamond},\ \min^{circ}_{R}\}}(pr_{k_i,L} S^{\beta}) = \langle k_i, l_{v_j} \rangle$$
2) **Compare to Baseline:**
   If the minimal cell is still preferable (or at least non-worse) than the baseline pseudo-node pl:
$$pr_{k_i,l_{v_j}} S^{\beta} \subseteq_{\nu} \left(\left[\frac{k_i}{pl}; \perp \right] pr_{pl,l_{v_j}} S^{\beta} \right),$$
   then define:
$$S^{\beta}_6[k_i, l_{v_j}] := pr_{k_i,l_{v_j}} S^{\beta}$$
$$S^{\beta}_7 := \left[\perp; \frac{pu}{l_{v_j}} \right] S^{\beta}_6$$
$$S^{\beta} := S^{\beta} \oplus_{(\circ_1,\circ_2,*)} S^{\beta}_7$$
3) **Perform Row-Wise Normalization:**
   For every $i = 1, \dots, m$ and $j = 1, \dots, n$, apply:

$$IO_{-(\circ_1,\circ_2,*)}\left(\langle k_i,l_j,S^\beta\rangle,\ \langle k_i,pu,pr_K S^\beta\rangle\right)$$

This operation ensures that each row in S^β has at least one cell with minimal (zero-like) cost, allowing a valid zero-point to be selected in the next stage.

After this normalization step, continue to **Step 4**.

Step 4. Column-Level Zero Membership Normalization
In this step, we determine the minimum cost for each column of the matrix S^β and normalize the elements accordingly.

1) **Identify minimum cost per column:**
$$\text{for } j=1 \text{ to } n:$$
$$AGIndex_{\{\min,\ \min_\square,\ \min_\diamond,\ \min_{R^\circ}\}}\left(pr_{K,l_j}S^\beta\right)=\langle k_{w_i},l_j\rangle$$
2) **Construct intermediate matrices and adjust:**
$$S_6^\beta[k_{w_i},l_j]:=pr_{k_{w_i},l_j}S^\beta,\quad S_7^\beta:=\left[\frac{pu_1}{k_{w_i}};\perp\right]S_6^\beta$$
$$S^\beta:=S^\beta\oplus_{(\circ_1,\circ_2,*)}S_7^\beta$$
3) **Normalize column-wise:**
$$\text{for } j=1 \text{ to } n,\quad \text{for } i=1 \text{ to } m:$$
$$IO_{-(\circ_1,\circ_2,*)}\left(\langle k_i,l_j,S^\beta\rangle,\ \langle pu_1,l_j,pr_{pu_1,L}S^\beta\rangle\right)$$

Proceed to **Step 5.**

Step 5. Optimality Criteria Evaluation
Step 5.1. For each warehouse $k_i\in K$, verify whether the total offered quantity is less than or equal to the sum of allocations with zero membership degree:

$$\text{for } i=1 \text{ to } m:\quad Index_{(\min\mu),k_i}(C^\beta)=\{\langle k_i,l_{v_1}\rangle,\ldots,\langle k_i,l_{v_V}\rangle\}$$
$$G_{v_r}^\beta[k_i,l_{v_r}]:=pr_{k_i,l_{v_r}}C^\beta,\quad G^\beta[k_i,R]:=pr_{k_i,R}C^\beta$$

If:
$$G^\beta[k_i,R]\subseteq_v\bigoplus_{r=1}^{V}G_{v_r}^\beta,\quad \text{then go to Step 5.2;}$$
$$\text{else set } RM[k_i,R]:=1 \text{ and go to Step 6.}$$

Step 5.2. For each region $l_j\in L$, verify whether the required quantity does not exceed the sum of allocated zero-membership values:

$$\text{for } j=1 \text{ to } n:\quad Index_{(\min\mu),l_j}(C^\beta)=\left\{\langle k_{w_1},l_j\rangle,\ldots,\langle k_{w_W},l_j\rangle\right\}$$
$$G_{w_r}^\beta[k_{w_r},l_j]:=pr_{k_{w_r},l_j}C^\beta,\quad G^\beta[pu_1,l_j]:=pr_{pu_1,l_j}C^\beta$$

If:
$$G^\beta[pu_1,l_j]\subseteq_v\bigoplus_{r=1}^{W}G_{w_r}^\beta,\quad \text{then go to Step 8;}$$
$$\text{else set } CM[pu_1,l_j]:=1 \text{ and go to Step 6.}$$

Step 6. Update the Cost CIC-IF Index Matrix
In this step, the matrix S^β (initially identical to C^β) is refined to improve cost allocation feasibility. All elements of the form $\langle 0,1;\ r_{k_i,l_j}^{\beta,c}\rangle$ for $i=1,\ldots,m$ and $j=1,\ldots,n$ are considered "zero-membership" and marked for elimination through the minimal number of horizontal and vertical lines.

- If a row or column contains no such zero-membership entry, the element with the smallest membership degree is crossed out.
- The auxiliary matrix $D[K,L]$ tracks eliminations: $d_{k_i,l_j}=1$ for one line, $d_{k_i,l_j}=2$ for both.
- Two matrices, $RC[K]$ and $CC[L]$, indicate whether a row or column is crossed: $rc_{k_i,e_0}\in\{0,1\}$, $cc_{r_0,l_j}\in\{0,1\}$.

For each $i=1$ to m, $j=1$ to n:

- If $s_{k_i,l_j}^\beta=\langle 0,1;r_{k_i,l_j}^{\beta,c}\rangle$ and $rm_{k_i,R}=0$ and $d_{k_i,l_j}=0$, then:
$$rc[k_i,e_0]:=1;\quad d_{k_i,l_j}:=1\quad \forall l_j,\ \text{in row } S_{(k_i,\perp)}^\beta$$
- If $s_{k_i,l_j}^\beta=\langle 0,1;r_{k_i,l_j}^{\beta,c}\rangle$, $cm_{pu_1,l_j}=0$ and $d_{k_i,l_j}=1$, then:
$$d_{k_i,l_j}:=2;\quad cc_{r_0,l_j}:=1;$$
$$d_{k_i,l_j}:=1\quad \forall k_i,\ \text{in column } S_{(\perp,l_j)}^\beta$$

Step 7. Refinement of the Revised Cost Matrix
Identify the smallest non-crossed cost element in S^β and subtract it from all uncovered entries. Then, add the same value to each entry covered by two lines.

1) **Identify minimal non-covered cost:**
$$\langle k_x,l_y\rangle:=AGIndex_{(\min,\max)}(S^\beta)$$
2) **Adjust uncovered entries:**
$$IO_{-(\circ_1,\circ_2,*)}(S^\beta,\langle k_x,l_y,S^\beta\rangle)$$
3) **Adjust double-covered entries:**
   - For $d_{k_i,l_j}=2$:
$$S_1^\beta:=pr_{k_x,l_y}C^\beta,$$
$$S_2^\beta:=pr_{k_i,l_j}C^\beta\oplus_{(\circ_1,\circ_2,*)}\left[\frac{k_i}{k_x};\frac{l_j}{l_y}\right]S_1^\beta$$
$$S^\beta:=S^\beta\oplus_{(\circ_1,\circ_2,*)}S_2^\beta$$
   - For $d_{k_i,l_j}=1$:
$$S^\beta:=S^\beta\oplus_{(\circ_1,\circ_2,*)}pr_{k_i,l_j}C^\beta$$

Proceed to **Step 8**.

Step 8. Allocation of Maximum Feasible Quantity

1) Find the cell with the largest cost value in S^β using:
$$AGIndex_{(\max,\min,*)}(S^\beta)=\langle k_{x^*},l_{y^*}\rangle$$
2) Assign the maximum feasible quantity to this cell and reduce either the row or column:
   - Compare:
$$s_{Index_{(\min\mu),k_{x^*}}}(C^\beta)\quad\text{and}\quad s_{Index_{(\min\mu),l_{y^*}}}(C^\beta)$$
   - Let s_{k_e,l_g} be the lesser of the two. Assign it and reduce S^β accordingly:
     – If $s_{k_e,R}^\beta<s_{Q,l_g}^\beta$:
$$X^\beta:=X^\beta\oplus_{(\circ_1,\circ_2,*)}\left[\perp;\frac{l_g}{R}\right]S_8^\beta$$
     (reduce row)
     – Else:
$$X^\beta:=X^\beta\oplus_{(\circ_1,\circ_2,*)}\left[\frac{k_e}{Q};\perp\right]S_9^\beta$$
     (reduce column)

Repeat Step 8 until $|S^\beta|=6$. Then proceed to **Step 9**.

Step 9. Degeneracy Resolution in IF Solution
If $|D|<m+n-1$, introduce a new basic variable $x_{k_\alpha,l_\beta}^\beta$ at the minimum cost among unassigned cells:
$$AGIndex_{\{(\min/\max),\not\perp,\notin D\}}(C^\beta)=\langle k_\alpha,l_\beta\rangle$$
Assign:
$$x_{k_\alpha,l_\beta}^\beta:=\langle 0,1;0\rangle$$

Step 10. Finalizing the IF Transportation Plan

1) If for any $x^{\beta}_{k_i,l_j} \neq \langle \perp, \perp \rangle$ and $\langle k_i, l_j \rangle \in EG$, the problem is infeasible. Stop.
2) Otherwise, define the final optimal IF transport plan:
$$X^{\beta}_{opt}[K, L, \{x^{\beta}_{k_i,l_j}\}]$$
3) Assign defaults for unallocated cells:
$$x^{\beta}_{k_i,l_j} = \langle 0, 1; \sqrt{2} \rangle \quad \text{if } x^{\beta}_{k_i,l_j} = \langle \perp, \perp \rangle$$
4) Compute the total aggregated transportation cost:
$$AGIO^{1}_{\oplus_{(\max,\min,*)}} \left(C^{\beta}_{\{Q,pl,pu_1\},\{R,pu\}} \otimes_{(\min,\max,*)} X^{\beta}_{opt} \right)$$
5) Finally, de-fuzzify each circular IF value to obtain a crisp fuzzy pair (as in [4]):
$$\left\langle \frac{a}{a+b}, \frac{b}{a+b} \right\rangle$$

B. Illustrative Example: Smart Factory Spare Parts Distribution

We demonstrate the application of the proposed Confidence-Interval Circular Intuitionistic Fuzzy Transportation Problem (CIC-IFTP) model to a smart manufacturing scenario, where spare parts must be dynamically dispatched from warehouse units to robotic production cells across a cyber-physical shopfloor.

Let:

$K = \{k_1, k_2, k_3\}$: three autonomous warehouse stations; $L = \{l_1, l_2, l_3, l_4\}$: four production cells requiring critical spare components; $E = \{d_1, d_2, d_3\}$: expert panel assessing fuzzy transport costs under uncertainty; $\beta = 0.85$: confidence level chosen by the system operator.

Each expert $d_s \in E$ provides a confidence-weighted intuitionistic fuzzy evaluation $ev_{k_i,l_j,d_s} = \langle \mu_{k_i,l_j,d_s}, \nu_{k_i,l_j,d_s} \rangle$, with individual ratings $re_s = \langle \delta_s, \varepsilon_s \rangle$ as follows:

$\{re_1, re_2, re_3\} = \{\langle 0.65, 0.10 \rangle, \langle 0.55, 0.08 \rangle, \langle 0.75, 0.07 \rangle\}$.

Using the aggregation operator $\alpha_{E,\#_2}$, we compute the adjusted cost matrix PI^{ave}, followed by transformation into the Confidence-Interval Circular Intuitionistic Fuzzy Cost Matrix C^{β} at decision moment h_f. Each entry $c^{\beta}_{k_i,l_j} = \langle \mu^{\beta}, \nu^{\beta}; r^{\beta} \rangle$ includes a radius r^{β} that geometrically captures expert disagreement at confidence level β.

The resulting CIC-IFIM for the smart transportation problem appears as:

	l_1	l_2	l_3	l_4	R	pu
k_1	$\langle 0.52, 0.28; 0.2 \rangle$	$\langle 0.62, 0.18; 0.3 \rangle$	$\langle 0.22, 0.18; 0.2 \rangle$	$\langle 0.72, 0.18; 0.2 \rangle$	$\langle 0.42, 0.28; 0.3 \rangle$	$\langle \perp, \perp \rangle$
k_2	$\langle 0.42, 0.35; 0.2 \rangle$	$\langle 0.32, 0.18; 0.3 \rangle$	$\langle 0.42, 0.18; 0.3 \rangle$	$\langle 0.22, 0.28; 0.3 \rangle$	$\langle 0.62, 0.18; 0.3 \rangle$	$\langle \perp, \perp \rangle$
k_3	$\langle 0.32, 0.28; 0.3 \rangle$	$\langle 0.22, 0.28; 0.2 \rangle$	$\langle 0.52, 0.18; 0.3 \rangle$	$\langle 0.62, 0.28; 0.2 \rangle$	$\langle 0.32, 0.58; 0.3 \rangle$	$\langle \perp, \perp \rangle$
Q	$\langle 0.32, 0.28; 0.3 \rangle$	$\langle 0.42, 0.38; 0.3 \rangle$	$\langle 0.52, 0.28; 0.3 \rangle$	$\langle 0.02, 0.10; 0.3 \rangle$	$\langle \perp, \perp \rangle$	$\langle \perp, \perp \rangle$
pl	$\langle 0.47, 0.38; 0.3 \rangle$	$\langle 0.52, 0.48; 0.3 \rangle$	$\langle 0.67, 0.28; 0.3 \rangle$	$\langle 0.7, 0.33; 0.2 \rangle$	$\langle \perp, \perp \rangle$	$\langle \perp, \perp \rangle$
pu_1	$\langle \perp, \perp \rangle$	$\langle \perp, \perp \rangle$	$\langle \perp, \perp \rangle$	$\langle \perp, \perp \rangle$	$\langle \perp, \perp \rangle$	$\langle \perp, \perp \rangle$

The total transport cost is computed using:
$$AGIO^{1}_{\oplus_{(\max,\min,*)}} \left(C_{(\{Q,pl,pu_1\},\{R,pu\})} \otimes_{(\min,\max,*)} X_{\text{opt}} \right).$$

For the scenario with minimal uncertainty and pessimistic valuation, the final cost is represented by the CIC-IF triple: $\langle 0.42, 0.28; 0.2 \rangle \Rightarrow$
Fuzzy projection: $\langle 0.6, 0.4 \rangle \Rightarrow$ Crisp cost: 5344.

In comparative analysis: Optimistic scenario yields: $\langle 0.08, 0.42; 0.2 \rangle \rightarrow$ 1869; Realistic scenario: $\langle 0.25, 0.56; 0.2 \rangle \rightarrow$ 2769.62.

The ranking function $R^{\beta,\text{circ}}$ assists in determining scenario preferences and strategic planning under expert-informed fuzziness in the smart factory.

C. Method Validation and Comparative Advantage

The Improved Zero Point Method (IZPM) has been demonstrated to consistently outperform well-known heuristics such as VAM, SVAM, GVAM, BVAM, and RVAM for both crisp and fuzzy transportation problems [15]. Unlike these methods, which often lead to suboptimal or infeasible solutions, IZPM ensures optimality through a structured and robust process, even in unbalanced settings.

Building upon this strong foundation, the proposed CIC-IFZPM algorithm extends IZPM by introducing two key enhancements: (1) modeling circular intuitionistic fuzzy data, which allows capturing cyclic uncertainty in dynamic environments, and (2) applying confidence intervals to represent varying degrees of reliability in expert estimates.

In the considered case study, CIC-IFZPM produced a slightly improved transportation cost compared to the previously proposed IFZSM [18], demonstrating its enhanced capability to yield robust solutions in uncertain environments. Moreover, the method preserved the optimality structure of the classical ZPM while effectively generalizing it to a broader fuzzy framework.

The time complexity of the CIC-IFZPM algorithm is $\mathcal{O}(2Dmn + 13mn + m + n)$, where D is the number of experts, and m, n are the numbers of supply and demand nodes, respectively. The dominant computational cost arises from the expert-based construction of the CIC-IFIM matrix and the structured zero-point allocation. However, since D is typically a small constant (e.g., 3–7 experts), the overall complexity scales linearly with the number of experts and quadratically with the transportation matrix size. Thus, the method retains the same computational class as the classical Improved Zero Point Method [16].

Although only one illustrative example is currently provided, a dedicated software implementation of the CIC-IFZPM algorithm is under development. This implementation will enable large-scale computational experiments on real and synthetic datasets, including stress tests in highly uncertain circular fuzzy environments. The objective is to rigorously examine the algorithm's efficiency, scalability, and stability across various industrial configurations. These experiments are planned for a follow-up study, which will also include a public release of the source code.

Table I provides a concise comparative overview of the main methods used for solving transportation problems, highlighting their scope and distinguishing features.

IV. Conclusion

his paper introduces an advanced CIC-IF Zero-Point Method tailored for Transportation Problems under uncertainty, within the CIC-IFTP framework. The proposed methodology leverages Confidence-Interval Circular Intuitionistic Fuzzy Numbers (CIC-IFNs) and the concept of index matrices to enhance decision-making in environments characterized by imprecise cost evaluations, dynamic supply availability, and fluctuating demand. Applied to a smart factory logistics scenario, the algorithm demonstrates its capability to model and resolve

TABLE I
OVERVIEW OF MAIN METHODS FOR SOLVING TRANSPORTATION PROBLEMS

Abbr.	Full Name	Description
VAM	Vogel's Approximation Method	Classical heuristic for initial feasible solution based on penalty costs; does not guarantee optimality.
SVAM	Shimshak's Vogel's Approximation Method	Modification of VAM that ignores penalties involving dummy rows/columns.
GVAM	Goyal's Vogel's Approximation Method	Assigns maximum transportation cost to dummy cells instead of zero.
BVAM	Balakrishnan's Vogel's Approximation Method	Enhances SVAM with additional allocation rules.
RVAM	Ramakrishnan's Vogel's Approximation Method	Uses four-step reduction and VAM for better approximation.
MODI	Modified Distribution Method	Optimizes allocations after an initial feasible solution.
FMDM	Fuzzy Modified Distribution Method	Fuzzy version of MODI that handles trapezoidal fuzzy numbers.
FVAM	Fuzzy Vogel's Approximation Method	Fuzzy adaptation of VAM using fuzzy numbers for cost, supply, and demand.
FZPM	Fuzzy Zero Point Method	One-stage method using fuzzy arithmetic; often gives optimal results directly.
IZPM	Improved Zero Point Method	Enhanced ZPM proven to outperform many heuristics; guarantees optimality.
CI-CIFZPM	Confidence-Interval Circular Intuitionistic Fuzzy Zero Point Method	Our proposed extension of IZPM that integrates circular intuitionistic fuzzy data and confidence intervals for robust decision-making under uncertainty.

the allocation of spare parts across cyber-physical production cells. It robustly accounts for expert uncertainty through adjustable confidence levels and scenario analysis (pessimistic, realistic, optimistic). The model ensures feasible and non-degenerate solutions, even in the presence of incomplete or ambiguous information.

The approach notably mitigates degeneracy through a structured mechanism of IF-index tracking and scenario-based refinement. It proves especially effective in applications requiring adaptive, multi-expert-driven decision processes, such as Industry 4.0 systems, smart supply chains, and decentralized logistics.

From a methodological standpoint, the proposed CIC-IF ZPM provides several key contributions: it integrates intuitionistic fuzzy logic with circular preference structures and confidence intervals; it generalizes the classic Zero-Point Method by embedding it in a multi-scenario fuzzy setting; it introduces a novel mechanism for degeneracy resolution via circular ranking indices.

These advances position the method as a competitive alternative to traditional fuzzy or crisp TP solvers, especially in complex environments where expert evaluation under uncertainty is essential. Related concepts for modeling uncertainty in logistics using granular computing can be found in [17], while multi-criteria optimization frameworks for transport tasks are also discussed in [14].

Future developments will focus on extending the model to handle confidence-interval elliptic intuitionistic fuzzy structures [21], with applications in elliptic IF multi-criteria decision-making. Additionally, dedicated software tools will be developed to support practical deployment in intelligent logistics platforms.

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