

Weak segment visibility in simple polygons: A reflex-vertex scanning algorithm with a guaranteed-area lower bound

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Abstract—We consider weak visibility from a segment inside a simple polygon P . For a segment $s \subseteq P$, its weak visibility region $WVP(s)$ is the set of all points of P visible from at least one point of s . Our objective is to choose a segment with a provably large weak visibility area. This setting is related to watchman-route and mobile-guard models, but here the route is restricted to a single straight segment and the focus is on guaranteed visible area rather than on full coverage or route-length minimization. Such a model is relevant to line-constrained sensing platforms, including rail-guided inspection devices and fixed-wing or wind-constrained aerial platforms operating along approximately straight sensing paths.

Let $n_r > 0$ denote the number of reflex vertices of P . We prove that there exists a segment $s \subseteq P$ such that $\text{area}(WVP(s)) \geq \frac{\text{area}(P)}{n_r}$. Moreover, if a reflex vertex with maximum visibility area does not already see the entire polygon, then a shortest-path-based segment construction yields the strictly stronger bound $\text{area}(WVP(s)) > \frac{\text{area}(P)}{n_r}$. The proof combines a reflex-vertex covering property, an averaging argument over individual visibility areas, and a segment construction that preserves or strictly improves the visible area. This yields a constructive, triangulation-free reflex-vertex scanning algorithm. In its basic form, the method scans all reflex vertices, evaluates their visibility areas, selects the best one, and constructs a guaranteed segment; in the strengthened case, the segment is chosen using the first bend of a shortest path from the maximizing reflex vertex to a point outside its visibility region. We also discuss a pairwise refinement over reflex-vertex pairs and state the running time so as to distinguish the combinatorial scan from the geometric cost of evaluating a single visibility candidate.

I. INTRODUCTION

VISIBILITY in simple polygons is a classical topic in computational geometry, with links to surveillance, inspection, navigation, and geometric optimization. In this paper

we study *weak visibility from a segment*: for a segment $s \subseteq P$ inside a simple polygon P , the weak visibility region $WVP(s)$ consists of all points of P visible from at least one point of s . Weak visibility from a given segment is a standard geometric primitive, and efficient algorithms and data structures for segment weak-visibility queries in simple polygons have been studied extensively [1]. Our viewpoint is different: instead of treating the segment as a query object, we ask how to choose a segment so as to guarantee a large visible area.

This places the problem close in spirit to mobile-guard and watchman-route models, where visibility is accumulated along a route traveled by an observer. The classical watchman route problem asks for a route from which the entire polygon is visible; in simple polygons, exact polynomial-time algorithms are known [2], [3], [4]. More recent work has also considered bicriteria and quota variants optimizing route length and visible area [5], as well as reflex-vertex-sensitive bounds and structural decompositions [6]. In contrast, here the route is restricted to a *single straight segment*, and the objective is not full coverage or shortest motion, but a guaranteed lower bound on the visible area achieved by such a segment. The model is also relevant to line-constrained sensing platforms, such as rail-guided inspection devices, linear scanning mechanisms, and fixed-wing or wind-constrained aerial platforms whose sensing pass is effectively close to a straight segment.

The present work is also related to point visibility in simple polygons. Visibility polygons of interior viewpoints can be computed in linear time by classical algorithms [7], and point-based visibility optimization problems have also been studied

[8], [9]. A related line of work concerns lower bounds on the proportion of polygon area visible from a suitably chosen viewpoint; recent results include guaranteed-area estimates for general and orthogonal polygons [10], [11]. Our contribution differs in that the observer moves along a segment rather than remaining at a point, and the guarantee is expressed in terms of the number n_r of reflex vertices. To the best of our knowledge, this explicit guaranteed-area viewpoint for a single interior segment has not been isolated in this form.

Let $n_r > 0$ denote the number of reflex vertices of P . Our main result shows that there always exists a segment $s \subseteq P$ whose weak visibility region covers at least a $\frac{1}{n_r}$ fraction of the polygon area. Moreover, if a reflex vertex with maximum visibility area does not already see the entire polygon, then a shortest-path-based segment construction yields a strictly stronger bound. The proof is constructive and yields a triangulation-free reflex-vertex scanning algorithm. In its guaranteed form, the method scans all reflex vertices, evaluates their visibility areas, selects the best one, and constructs a segment accordingly; an optional pairwise refinement may then be used heuristically. We also distinguish explicitly between the combinatorial scan over candidates and the geometric cost of evaluating a single visibility candidate, leading to an honest complexity statement. The remainder of the paper introduces the notation, proves the lower bounds, presents the algorithmic framework, and concludes with numerical remarks and open directions.

II. PRELIMINARIES AND PROBLEM STATEMENT

Let P be a simple polygon in the plane with n vertices and positive area. We write $\text{area}(P)$ for its area. A vertex of P is called *reflex* if its interior angle is strictly greater than π , and $n_r > 0$ denotes the number of reflex vertices of P .

For a point $p \in P$, its *visibility region* is

$$V(p) = \{x \in P : \overline{px} \subseteq P\}.$$

For a segment $s \subseteq P$, its *weak visibility region* is

$$WVP(s) = \{x \in P : \exists y \in s \text{ such that } \overline{xy} \subseteq P\} = \bigcup_{y \in s} V(y).$$

Throughout the paper, a segment $s \subseteq P$ means a closed straight-line segment fully contained in P ; its endpoints need not be vertices of P .

A natural optimization problem is to choose a segment $s \subseteq P$ so as to maximize $\text{area}(WVP(s))$. Our goal is not to solve this optimization problem exactly, but to prove guaranteed lower bounds on the visible area of a suitably chosen segment and to derive a simple constructive algorithm achieving them.

III. A GUARANTEED-AREA LOWER BOUND

We now prove the main theoretical result underlying the proposed method. The argument has three basic steps. First, every point of a nonconvex simple polygon is visible from at least one reflex vertex. Second, an averaging argument implies that at least one reflex vertex sees a $\frac{1}{n_r}$ fraction of

the polygon area. Third, such a reflex vertex can be embedded in an interior segment, and the weak visibility region of that segment contains the visibility region of the vertex. We then show that, if the maximizing reflex vertex does not already see the entire polygon, a shortest-path-based segment construction yields a strictly stronger bound.

For the first step, we use a standard property of shortest paths in simple polygons.

Lemma 1 (Bends of shortest paths occur at reflex vertices). *Let P be a simple polygon, and let $\pi(a, b)$ be a shortest path between two points $a, b \in P$. Then every interior bend of $\pi(a, b)$ occurs at a reflex vertex of P .*

Proof. Let v be an interior bend of $\pi(a, b)$, and let u and w be the two vertices immediately preceding and following v on the path. Thus the path contains the two consecutive segments $\overline{uv}$ and $\overline{vw}$.

Suppose, for contradiction, that v is not a reflex vertex of P . Then, since v is also a bend, the interior angle of P at v is lower than π . Hence, in a sufficiently small neighborhood of v , the polygon P is locally convex: in particular, for points $u' \in \overline{uv}$ and $w' \in \overline{vw}$ chosen sufficiently close to v , the segment $\overline{u'w'}$ is contained in P , and $u'vw'$ is a triangle.

Now replace the subpath $\overline{u'v} \cup \overline{vw'}$ by the segment $\overline{u'w'}$. Since u', v, w' are not collinear at a bend, the triangle inequality is strict, and therefore

$$|u'w'| < |u'v| + |vw'|.$$

So this replacement yields a shorter path from a to b contained in P , contradicting the assumption that $\pi(a, b)$ is a shortest path. Therefore, it is not true that the interior angle of P at v is lower than or equal to π ; thus, v must be a reflex vertex of P . $\square$

This fact yields the required reflex-vertex covering property, that follows.

Proposition 1 (Every point is seen by a reflex vertex). *Let P be a nonconvex simple polygon. Then every point $p \in P$ is visible from at least one reflex vertex of P .*

Proof. Fix an arbitrary point $p \in P$. If p sees all of P , then in particular it sees every reflex vertex of P . Since a nonconvex simple polygon has at least one reflex vertex, the claim follows.

Assume now that p does not see all of P . Choose a point $q \in P \setminus V(p)$, and let $\pi(p, q)$ be a shortest path from p to q inside P . Since $q \notin V(p)$, the path $\pi(p, q)$ is not the segment $\overline{pq}$, so it has at least one bend. By Lemma 1, every interior bend of $\pi(p, q)$ occurs at a reflex vertex of P . Let r be the first bend of $\pi(p, q)$ after p . Then the first link of the path is the segment $\overline{pr} \subseteq P$, so r is visible from p . Hence p is visible from a reflex vertex of P . $\square$

Once this covering property is available, the desired lower bound follows from a direct averaging argument over all reflex vertices.

Lemma 2 (Averaging over reflex vertices). *Let R be the set of reflex vertices of a nonconvex simple polygon P , with $|R| = n_r$. Then there exists a reflex vertex $r^* \in R$ such that*

$$\text{area}(V(r^*)) \geq \frac{\text{area}(P)}{n_r}.$$

Proof. By Proposition 1, every point of P is visible from at least one reflex vertex. Therefore,

$$P \subseteq \bigcup_{r \in R} V(r).$$

Taking areas and using subadditivity, we obtain

$$\text{area}(P) \leq \sum_{r \in R} \text{area}(V(r)). \quad (1)$$

If every $r \in R$ satisfied $\text{area}(V(r)) < \frac{\text{area}(P)}{n_r}$, then summing over R would give

$$\sum_{r \in R} \text{area}(V(r)) < \sum_{r \in R} \frac{\text{area}(P)}{n_r} = n_r \cdot \frac{\text{area}(P)}{n_r} = \text{area}(P),$$

contradicting (1). Therefore, at least one reflex vertex $r^* \in R$ satisfies

$$\text{area}(V(r^*)) \geq \frac{\text{area}(P)}{n_r}.$$

□

The remaining step is to pass from a single reflex vertex to a valid segment contained in the polygon.

Lemma 3 (Segment through a reflex vertex). *For every reflex vertex r of a simple polygon P , there exists a nondegenerate segment $s \subseteq P$ such that $r \in s$.*

Proof. Since r is a reflex vertex, its interior angle is strictly greater than π . Therefore there exists a line through r whose direction lies inside the interior wedge at r . For sufficiently small $\varepsilon > 0$, the intersection of this line with a neighborhood of r contains a nondegenerate segment centered at r and contained in P . Taking the maximal connected subsegment of the line inside P that contains r , we obtain a nondegenerate segment $s \subseteq P$ with $r \in s$. □

We can now combine the three ingredients. The construction principle is illustrated in Fig. 1.

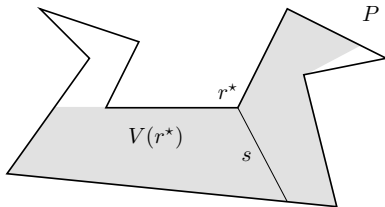


Fig. 1. Illustration of the segment construction. A reflex vertex r^* with large visibility region $V(r^*)$ is embedded in an interior segment $s \subseteq P$. Since $r^* \in s$, every point visible from r^* is weakly visible from s , and hence $V(r^*) \subseteq WVP(s)$.

Theorem 1 (Guaranteed-area segment theorem). *Let P be a simple polygon with $n_r > 0$ reflex vertices. Then there exists a segment $s \subseteq P$ such that*

$$\text{area}(WVP(s)) \geq \frac{\text{area}(P)}{n_r}.$$

Proof. By Lemma 2, there exists a reflex vertex r^* such that

$$\text{area}(V(r^*)) \geq \frac{\text{area}(P)}{n_r}.$$

By Lemma 3, there exists a segment $s \subseteq P$ with $r^* \in s$. Since weak visibility from a segment contains the visibility region of each of its points, we have

$$V(r^*) \subseteq WVP(s).$$

Therefore,

$$\text{area}(WVP(s)) \geq \text{area}(V(r^*)) \geq \frac{\text{area}(P)}{n_r}.$$

□

Theorem 1 only uses the fact that the chosen segment contains the maximizing reflex vertex r^* . If r^* does not already see the entire polygon, then a more careful choice of the segment yields a strictly larger weak visibility region.

Lemma 4 (Shortest-path segments add positive area). *Let P be a simple polygon, let r be a reflex vertex of P , and suppose that $V(r) \neq P$. Then there exists a reflex vertex v visible from r such that, for the segment $s := \overline{rv}$ one has*

$$\text{area}(WVP(s)) > \text{area}(V(r)).$$

Proof. Choose a point $q \in P \setminus V(r)$, and let $\pi(r, q)$ be a shortest path from r to q inside P . Since $q \notin V(r)$, this path is not the segment $\overline{rq}$, so it has at least one bend. Let v be the first bend of $\pi(r, q)$ after r . By Lemma 1, v is a reflex vertex of P , and $\overline{rv} \subseteq P$, so $s = \overline{rv} \subseteq P$ is a feasible segment.

Let w be the next point of the path after v , so that the path contains the two consecutive links $\overline{rv}$ and $\overline{vw}$. Choose a point u in the relative interior of $\overline{vw}$, sufficiently close to v . Since every subpath of a shortest path is again shortest between its endpoints, the subpath from r to u is a shortest path from r to u . If u were visible from r , then the segment $\overline{ru} \subseteq P$ would give a shorter path from r to u than the broken path through v , a contradiction. Hence $u \notin V(r)$.

Because u lies in the interior of the link $\overline{vw}$, there exists a sufficiently small open neighborhood $U \subseteq P$ of u such that every point of U is visible from v . Since $u \notin V(r)$ and $V(r)$ is closed in P , U may be chosen so small that $U \cap V(r) = \emptyset$. Therefore

$$U \subseteq V(v) \subseteq WVP(s) \quad \text{and} \quad U \cap V(r) = \emptyset.$$

Hence $U \subseteq WVP(s) \setminus V(r)$. Since U has positive area,

$$\text{area}(WVP(s)) > \text{area}(V(r)).$$

□

Theorem 2 (Strict improvement from a shortest-path segment). *Let P be a simple polygon with $n_r > 0$ reflex vertices, and let r^* be a reflex vertex maximizing $\text{area}(V(r))$ over all reflex vertices of P . If $V(r^*) \neq P$, then there exists a segment $s \subseteq P$ such that*

$$\text{area}(WVP(s)) > \text{area}(V(r^*)) \geq \frac{\text{area}(P)}{n_r}.$$

In particular,

$$\text{area}(WVP(s)) > \frac{\text{area}(P)}{n_r}.$$

Proof. By Lemma 4, there exists a segment $s \subseteq P$ such that

$$\text{area}(WVP(s)) > \text{area}(V(r^*)).$$

By Lemma 2,

$$\text{area}(V(r^*)) \geq \frac{\text{area}(P)}{n_r}.$$

Combining the two inequalities gives the claim. $\square$

Theorem 1 provides a direct constructive principle: it is sufficient to identify a reflex vertex with maximum individual visibility area and then choose an interior segment containing that vertex. Theorem 2 shows that, whenever this maximizing reflex vertex does not already see the entire polygon, a shortest-path-based segment construction yields a strictly stronger area guarantee. These observations form the basis of the reflex-vertex scanning algorithm described in the next section.

IV. A REFLEX-VERTEX SCANNING ALGORITHM

Theorems 1 and 2 are constructive and lead directly to a simple candidate-scanning procedure. The guaranteed part of the method requires only a scan over reflex vertices. If the maximizing reflex vertex does not already see the entire polygon, then a shortest-path-based segment construction yields a strictly stronger area guarantee. An optional refinement may additionally scan candidate segments associated with reflex-vertex pairs, but this second stage is heuristic and is not needed for either the basic or the strengthened guarantee.

A. Guaranteed construction

Let R be the set of reflex vertices of a simple polygon P , with $|R| = n_r$. By Theorem 1, it is sufficient to identify a reflex vertex with maximum individual visibility area and then choose an interior segment containing that vertex. By Theorem 2, if that maximizing reflex vertex does not already see the entire polygon, then one may choose the segment using the first bend of a shortest path and obtain a strictly stronger bound.

Algorithm 1 always returns a segment s satisfying

$$\text{area}(WVP(s)) \geq \frac{\text{area}(P)}{n_r},$$

and if $V(r^*) \neq P$, then the shortest-path construction yields the stricter bound

$$\text{area}(WVP(s)) > \frac{\text{area}(P)}{n_r}.$$

Algorithm 1: Guaranteed reflex-vertex scan with shortest-path enhancement

Input: A simple polygon P with $n_r > 0$ reflex vertices

Output: A segment $s \subseteq P$ such that
 $\text{area}(WVP(s)) \geq \frac{\text{area}(P)}{n_r}$, and strictly
 $> \frac{\text{area}(P)}{n_r}$ whenever $V(r^*) \neq P$

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1 Compute the set  $R$  of reflex vertices of  $P$ ;
2 foreach  $r \in R$  do
3   | compute  $V(r)$  and  $A(r) := \text{area}(V(r))$ ;
4 end
5 Choose  $r^* \in \arg \max_{r \in R} A(r)$ ;
6 if  $V(r^*) = P$  then
7   | construct any nondegenerate segment  $s \subseteq P$  such
   |   that  $r^* \in s$ ;
8 else
9   | choose a point  $q \in P \setminus V(r^*)$ ;
10  | compute a shortest path  $\pi(r^*, q)$  inside  $P$ ;
11  | let  $v$  be the first bend of  $\pi(r^*, q)$  after  $r^*$ ;
12  | set  $s := r^*v$ ;
13 end
14 return  $s$ ;
```

Algorithm 2: Pairwise reflex-vertex refinement

Input: A simple polygon P , its set R of reflex vertices, and an initial segment s^* returned by Algorithm 1

Output: A segment $\hat{s} \subseteq P$ with the largest weak visibility area among all tested candidates

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1 Set  $\hat{s} \leftarrow s^*$ ;
2 Set  $A^* \leftarrow \text{area}(WVP(s^*))$ ;
3 foreach unordered pair  $\{r_i, r_j\} \subseteq R$  do
4   | construct a candidate segment  $s_{ij}$ , whenever the
   |   chosen construction rule yields a feasible segment
   |   contained in  $P$ ;
5   | if  $s_{ij}$  is feasible then
6     | compute  $A_{ij} := \text{area}(WVP(s_{ij}))$ ;
7     | if  $A_{ij} > A^*$  then
8       |    $\hat{s} \leftarrow s_{ij}$ ;
9       |    $A^* \leftarrow A_{ij}$ ;
10    | end
11  | end
12 end
13 return  $\hat{s}$ ;
```

B. Optional pairwise refinement

The guaranteed construction uses only one reflex vertex and, in the strengthened case, one specifically chosen shortest-path bend. In practice, one may additionally test candidate segments associated with pairs of reflex vertices and keep the best feasible segment encountered.

Algorithm 2 is heuristic in the present paper. It may improve the observed coverage, but it is not required for the $\frac{1}{n_r}$ guarantee or for the strict improvement guaranteed by Theorem 2.

In particular, we do not claim that scanning reflex-vertex pairs yields a stronger universal lower bound.

Remark 1. *A second reflex endpoint does not automatically improve the weak visibility area. The strict improvement proved in Theorem 2 applies only when the maximizing reflex vertex r^* does not already see the entire polygon and the second endpoint is chosen by a shortest-path-based construction. In contrast, even if two reflex vertices can be joined by a feasible interior segment, the weak visibility region of that segment need not be strictly larger than the visibility region of the better endpoint alone. For example, in a butterfly-shaped simple polygon with two reflex vertices, both reflex vertices may already see the entire polygon. In that case, joining them yields no gain in visible area. This is why the pairwise refinement in the present paper is treated only as a heuristic enhancement. A butterfly-shaped example of this kind is shown in Fig. 2.*

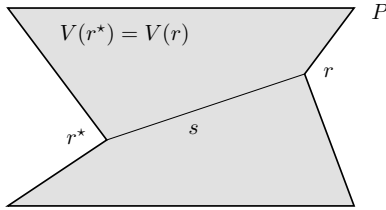


Fig. 2. A butterfly-shaped counterexample illustrating Remark 1. The two reflex vertices r^* and r can be joined by a feasible interior segment $s \subseteq P$, but this does not increase the weak visibility area: both reflex vertices already see the entire polygon, so $V(r^*) = V(r) = P$ and hence $WVP(s) = P$.

C. Complexity discussion

It is important to distinguish between the combinatorial scan over candidates and the geometric cost of evaluating one candidate.

Let $T_{\text{pvis}}(n)$ denote the cost of computing the visibility region of one point in an n -vertex simple polygon together with its area, and let $T_{\text{spath}}(n)$ denote the cost of computing one shortest path inside an n -vertex simple polygon, or at least extracting the first bend of such a path. Then Algorithm 1 runs in

$$O(n_r T_{\text{pvis}}(n) + T_{\text{spath}}(n)),$$

up to lower-order terms for identifying reflex vertices and selecting a point $q \in P \setminus V(r^*)$.

In many implementations, the point q may be chosen directly from geometric information already available after computing $V(r^*)$, and only the first bend of the shortest path is needed. In that case, the additional shortest-path step is typically lower-order relative to the visibility computations, and the guaranteed construction may be summarized as

$$O(n_r T_{\text{pvis}}(n) + T_{\text{spath}}(n)),$$

or simply as $O(n_r T_{\text{pvis}}(n))$ when shortest-path extraction is treated as a secondary primitive.

Similarly, let $T_{\text{vis}}(n)$ denote the cost of computing the weak visibility region of one candidate segment together with its area. If Algorithm 2 evaluates all reflex-vertex pairs, then its running time is

$$O(n_r^2 T_{\text{vis}}(n)),$$

again up to lower-order terms for candidate generation and feasibility checks.

Thus the combinatorial part of the procedure is linear in n_r for the guaranteed construction and quadratic in n_r for the optional refinement, while the total running time depends on the underlying visibility-evaluation primitive.

The proposed algorithm is triangulation-free in the sense that the candidate selection logic does not depend on a triangulation of the polygon. From a practical viewpoint, the guaranteed scan provides a simple baseline with a provable area guarantee, the shortest-path enhancement sharpens this baseline whenever $V(r^*) \neq P$, and the optional pairwise refinement may further improve observed coverage in practice.

V. NUMERICAL ASPECTS AND PRACTICAL IMPLICATIONS

Although the guaranteed-area bound is geometric in nature, an implementation of the proposed method must handle several low-level computational issues carefully. The main tasks are reflex-vertex detection, visibility computation, segment construction, shortest-path extraction, and area evaluation. In particular, the computation of point visibility polygons and weak segment visibility regions is treated here as an underlying geometric primitive; the present contribution concerns candidate selection and guaranteed area rather than a new visibility-subroutine design.

Reflex vertices can be identified by standard orientation tests on consecutive triples of polygon vertices, assuming a fixed orientation of the polygon boundary. In the guaranteed scan, each reflex vertex r is used as a viewpoint for computing the visibility region $V(r)$ and its area. Once the maximizing reflex vertex r^* has been selected, one either chooses any line through r^* inside the interior wedge at that vertex, or, in the strengthened construction, chooses a point outside $V(r^*)$, computes a shortest path from r^* to that point inside P , and uses the first bend of that path to define the segment. In both cases, the resulting line segment is clipped or selected so as to remain fully contained in P .

After computing a visibility polygon or a weak visibility polygon, its area can be obtained by the standard shoelace formula, see [12] for more details. The numerical reliability of this step depends primarily on the correctness of the preceding visibility computation. Degenerate cases such as collinear boundary points, rays passing through polygon vertices, and nearly coincident intersection points should therefore be treated using a clear tie-breaking rule. Robust orientation and segment-intersection predicates are especially important, since they affect both reflex-vertex classification and visibility clipping. In practical implementations, filtered predicates or exact arithmetic for geometric tests may be preferable when reproducibility is important.

From a computational viewpoint, the proposed method is intentionally lightweight. Its purpose is not to introduce a new visibility primitive, but to organize existing point- and segment-visibility computations into a simple reflex-vertex scanning framework with a provable area guarantee.

Although the present paper is primarily theoretical, the proposed model is also relevant to line-constrained sensing scenarios in which an observer or sensor is restricted to a straight or nearly straight motion path. Examples include rail-guided inspection devices, linear scanning mechanisms, and fixed-wing or wind-constrained aerial platforms whose sensing pass is effectively close to a straight segment. In such settings, weak visibility from a segment provides a natural geometric abstraction of the area observable during one sensing pass.

VI. CONCLUSIONS

We studied weak visibility from a segment inside a simple polygon and considered the problem of selecting a segment with large visible area. The main result shows that if P is a simple polygon with $n_r > 0$ reflex vertices, then there exists a segment $s \subseteq P$ such that $\text{area}(WVP(s)) \geq \frac{\text{area}(P)}{n_r}$.

The proof is constructive and leads to a simple reflex-vertex scanning algorithm: evaluate the visibility area of each reflex vertex, choose the best one, and embed it in an interior segment. Moreover, if a reflex vertex with maximum visibility area does not already see the entire polygon, then a shortest-path-based segment construction yields the strictly stronger bound, $\text{area}(WVP(s)) > \frac{\text{area}(P)}{n_r}$.

This yields a triangulation-free guaranteed construction, with a clean separation between candidate enumeration and the geometric cost of evaluating visibility.

The optional pairwise refinement discussed in the paper is computationally natural and may improve the observed coverage in practice, although such an improvement is not automatic and depends on the geometry of the tested candidates. This distinction is important both theoretically and algorithmically: the present proceeding contribution focuses on guaranteed-area results and a simple computational framework, rather than on the exact optimization of weak segment visibility.

A natural direction for future work is to determine how far the bound $\frac{\text{area}(P)}{n_r}$ can be improved in general. To the best of our knowledge, the question of whether a stronger universal lower bound holds for the present single-segment guaranteed-area setting remains unresolved. The discussion in this paper shows that arbitrary reflex-vertex pairings do not automatically yield a stronger bound, whereas shortest-path-based constructions do provide a strict local improvement whenever the maximizing reflex vertex does not already see the entire polygon. At the same time, highly nonconvex comb-

like examples suggest that the present $\frac{1}{n_r}$ guarantee may be non-tight.

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