

Fuzzy logic in the multi-agent financial decision support system

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Abstract—The article presents the application of a fuzzy logic in building the trading agents of the a-Trader system. The system supports investment decisions on the FOREX market. The first part of the article contains a discussion related to the use of fuzzy logic as an agents' knowledge representation. Next, the algorithms of the selected fuzzy logic buy-sell decision agents are presented. In the last part of the article the agent performance is evaluated on real FOREX data.

I. INTRODUCTION

FINANCIAL decisions are made under conditions of risk and uncertainty that influence their level of performance. These decisions are usually supported by decision support systems and various computational models. Among them, there are multi-agent systems [2] which use various methods based on mathematics, statistics, finance or artificial intelligence [3, 4, 6, 10, 13, 14, 18, 20, 21,25, 28, 48]. A-Trader [22] supporting investment decisions on the FOREX market (Foreign Exchange Market) may serve as the example of such a system. FOREX is one of the biggest financial foreign exchange markets in the world. Currencies are traded against one another in pairs, for instance EUR/USD, USD/PLN. Trading on FOREX relies on opening/closing long/short positions. A long position is a situation in which one purchases a currency pair at a certain price and hopes to sell it later at a higher price. This is also referred to as the notion of "buy low, sell high" in other trading markets. On FOREX, when one currency in a pair is rising in value, the other currency is declining, and vice versa. If a trader thinks a currency pair will fall, he will sell it and hope to buy it back later at a lower price. This is considered a short position, which is the opposite of a long position. The A-Trader receives tick data which are grouped to minute aggregates (M1, M5, M15, M30), hourly aggregates (H1, H4), daily aggregates (D1), weekly aggregates (W1) and monthly aggregates (MN1). The A-Trader supports a High Frequency Trading (HFT) and puts strong emphasis on price formation processes, short-term positions, fast computing, and efficient and robust indicators.

High frequency traders are constantly taking advantage of very small quote changes with a high rate of recurrence to

generate important profit rates. As many HFT experts underline, the traders seek profits from the market's liquidity imbalances and short-term pricing inefficiencies. Hence, the minimization of time from the access to quote information, through the entry of an order until its execution, is vital. Generally speaking, to support traders, the systems must provide as soon as possible advice as to which position should be taken: buy, sell or do nothing. Time series forecasting is more difficult while online trading has to be served.

The architecture of a-Trader and the description of the different groups of agents have already been detailed [23, 24]. In general, the agents possess their own knowledge, they can continuously learn and change their knowledge in order to improve their performance.

Different methods of agents' knowledge representation can be applied in a-Trader. In our previous work [23, 24] we were focused on three-valued knowledge representation of this group of agents. Value "1" denoted „buy" decision, value "-1" denoted „sell" decision, value "0" denoted „leave unchanged". Agents are implemented using the C# environment and MQL4 language.

The key part of the system is the Supervisor agent. Its task is, among others, to coordinate the work of agents on trading strategy and it presents the final strategy (suggestions of open/close positions) to the trader. The Supervisor uses various strategies and evaluates their performance.

The a-Trader allows also for making arbitrarily independent decisions by traders (experts) on the basis of their knowledge and experience. The traders' decisions can be stored in a database, evaluated and compared with strategies provided automatically by agents.

The purpose of this paper is to present a manner of applying a fuzzy logic as the agents' knowledge representation and evaluating the performance of selected agents in the a-Trader system.

In the first part of the article, the fuzzy logic as agents' knowledge representation is briefly discussed. The algorithms of three selected agents are then described. The final part discusses the results of the performance evaluation of these agents.

II. FUZZY LOGIC AS AGENTS' KNOWLEDGE REPRESENTATION

The literature on the subject presents many different methods for agents' knowledge representation. The main ones include first-order predicate logic, production systems, artificial neural networks, frame representation, ontologies such as semantic web and semantic networks, multi-attributes and multi-values structures, and multi valued logic [12, 17, 32, 33, 39, 40, 42, 43, 44, 46]. Some of these methods are closely related to fuzzy logic.

The first-order predicate logic that is one common knowledge representation is founded on the following general assumptions [11]:

- the knowledge representation is independent of physical media,
- agents' internal states are related to the objects of external environment,
- the knowledge representation consists of symbols forming the structure,
- reasoning is based on the manipulation of these structures to derive other structures.

Often agents' knowledge is represented as multi-attribute and multi-value structures which allow representation the real world environment in wide scope of objects features.

Multi-valued logic and fuzzy logic are more suitable methods for HFT. Three-valued logic is a very simple language consisting of proposition symbols and logical connectives. It can handle propositions that are known true, known false, or completely unknown. The set of possible models, given a fixed propositional vocabulary, is finite, so entailment can be checked by enumerative models. Inference algorithms for three-valued logic include backtracking and local-search methods and can often solve large problems very quickly [35]. Three-valued logic is reasonably effective for certain tasks, but does not scale to environments of unbounded size because it lacks the expressive power to describe the real world objects.

To reduce this weakness, a fuzzy logic can be applied in HFT. Fuzzy logic is an approach founded on "degrees of truth" rather than the usual "true or false" values (1 or 0). The idea of fuzzy logic was first proposed by Zadeh in the 1960s when he was working on the problem of computer understanding of natural language [47]. Fuzzy logic is a form of multi-valued logic derived from fuzzy set theory [3] to deal with approximate reasoning. In contrast to "crisp logic", where binary sets are processed by binary logic, fuzzy logic variables may have a truth value that ranges between 0 and 1 that allows the user to express imprecision and flexibility in a decision-making system [41], [45], [30]. Fuzzy logic is used, for example, in multi-agent systems for information extraction [34], energy management [26] or robotics [18]. Fuzzy logic was also used for trading on FOREX, for example, in Expert Advisor [31] or technical analysis system [9] or fuzzy time series forecasting [1], [8], [36]. However, in these systems, the probability of decisions

is ranged to [0..1]. In trading systems it is unfavorable, because the trader can buy, sell or leave a currency unchanged. Therefore, in the a-Trader system, the confidence of decisions range is [-1..1], where "-1" level denotes "strong sell" decision, "0" level denotes "strong leave unchanged" decision and "1" level denotes "strong buy" decision. The positions can be open/close with different levels of confidence of decision. For example, the long position can be open, when a level of confidence is 0.6 or short position can be open, when a level of probability is 0.7. Therefore the timeframe for the opening/closing position is wider than in the case of three valued-logic. An example of this difference is presented in Fig 1 and Fig 2 (A_i – denotes the i th agent). In the case of three-valued logic (Fig 1), the green color points denote a "buy" decision, the red color ones denote a "sell" decision, and the black color points denote a "do nothing" decision. There are often agents that generate buy/sell decisions too fast or too late. In the case of fuzzy logic, the ranges of decisions probability often cover the best point for trading. In Fig. 2, the green triangle denotes transition from "do nothing" decision to "buy" decision, and the red inverted triangle denotes transition from "do nothing" decision to "sell" decision, and the black color denotes a "do nothing" decision). Therefore it is possible to place open/close positions closer to the optimal decision than in the case of a three-valued logic. Of course, the level of probability of decision for open/close position plays a vital role. This level can be determined on the basis of trader experience, or by the Supervisor on the basis of, for example, a genetic algorithm.

Using the fuzzy logic as agents' knowledge representation allows the trading decision to be closer to real experts' decisions (made under conditions of risk and uncertainty) that are also taken with a certain level of probability.

Fuzzy logic can be also used by trading advisors for the following tasks:

- forecasting, i.e. the possibility to calculate the output value for input data lies outside the scope initially predicted,
- expressing the agent's knowledge in a flexible, intuitive way.,
- computation of decisions' probability level,
- implementation of different automated learning algorithms,
- validation and consistency measuring that can speed up automated learning and improve user interpretability,
- taking into consideration ambiguity – the "natural" way for expressing uncertain knowledge.

The next part of the article describes selected fuzzy logic buy-sell decision agents implemented in a-Trader.



Fig. 1 Three-valued logic agents' decisions
Source: Own work.



Fig. 2 Fuzzy logic agents' decisions
Source: Own work.

III. DESCRIPTION OF THE FUZZY LOGIC BUY-SELL DECISION AGENTS

A-Trader contains approximately 1400 agents, including about 800 processing data agents (they calculate different indicators on the FOREX market, for instance trend indicators, oscillators) and 300 agents (running in all time periods) setting the buy-sell decision, including: 200 three valued logic agents and 100 fuzzy logic agents, also 200 agents providing the strategies. In order to illustrate the performance analysis, four agents were chosen: *BollingerFuzzy*, *WilliamsFuzzy*, *TrendLinear-RegFuzzy* and *ConsensusFuzzy*.

A. . The *BollingerFuzzy* agent

The *BollingerFuzzy* agent is created on the basis of the Bollinger Bands indicator [5]. These bands are volatility constraints placed above and below a moving average. Volatility is expressed by the standard deviation, which changes as volatility increases and decreases.

The bands automatically widen when volatility increases and narrow when volatility decreases. The buy decision's probability level is calculated when the price is close to the upper Bollinger Band or breaks above it, and the sell decision is calculated when the price is close to the lower Bollinger Band or falls below it. The algorithm of this agent is the following:

Algorithm 1

Input: q //a value of quotation,
bbandup // value determination by processing data agent named BBANDUP, which calculates the upper band,
bbandlo // value determination by processing data agent named BBANDLO, which calculates the lower band.
sma // value determination by processing data agent named SMA, which calculates the simple moving average of quotation.

Output: The fuzzy logic decision D (value range [-1..1]).

BEGIN

Let $D:=0$, $calcBands:=0$; //counter for fuzzification.
 $maxcount:=0$. //maximum counter limit for fuzzification.
 $\Delta = \text{Abs}((sma - ((bbandlo + bbandup) / 2)) / 10)$.

If $q < (bbandlo + \Delta)$ **then**
If ($calcBands > 0$) **then** $calcBands=0$, $calcBands:=calcBands-1$.
If ($calcBands < -maxcount$) **then** $calcBands=-maxcount$,
 $D=calcBands/maxcount$;

If $q > (bbandup - \Delta)$ **then**
If ($calcBands < 0$) **then** $calcBands=0$, $calcBands:=calcBands+1$.
If ($calcBands > maxcount$) **then** $calcBands=maxcount$;
 $D=calcBands/maxcount$;

END

In a trading system the fuzzification is understood as a process of conversion of an input variable (i.e. signals determined by processing agents) to fuzzy set.

B. . The *WilliamsFuzzy* agent

The *WilliamsFuzzy* agent is created on the basis of Williams %R indicator [19]. Williams %R is a momentum indicator that is the inverse of the Fast Stochastic Oscillator. Also referred to as %R, the indicator reflects the level of the

close relative to the highest high for the look-back period. In contrast, the Stochastic Oscillator reflects the level of the close relative to the lowest low. %R corrects for the inversion by multiplying the raw value by -100. As a result, the Fast Stochastic Oscillator and Williams %R produce exactly the same lines, only the scaling is different. Williams %R oscillates from 0 to -100. The buy decision's probability level is calculated when Williams %R value falls below -80 and the sell decision is calculated when Williams %R value rises above -20. The algorithm of this agent is as follows:

Algorithm 2

Input: q //a value of quotation,
williams // value determining by processing data agent named WILLIAMS, which calculates the Williams %R indicator.
 Δ – an external parameter denotes range of williams %R less than -80 or above -20 (it is assumed that the maximum value of williams %R does not have to be 0 and the minimum value of williams %R does not have to be -100. This parameter range is [1..20] and it is calculated by the genetic algorithm or determined by user.

Output: The fuzzy logic decision D (value range [-1..1]).

BEGIN

Let $D:=0$.

If $williams \leq -80$ **then** $D=(williams - (-80 - \Delta)) / \Delta$.
If $williams \geq -20$ **then** $D=- (williams - (-20)) / \Delta$.

END

C. . The *TrendLinearRegFuzzy* agent

The agent operates on the basis of the assumption that the trend of a certain number of M quotations is approximated with the straight line by the equation: $y = ax + b$. The straight line inclination depends on the value of the "a" parameter or the tangent value of the inclination angle with the use of linear regression [20], [27], [38]. The agent computes the probability level of a buy decision when the coefficient value changes from positive to negative and the probability level of a buy decision is calculated when the coefficient changes value from negative to positive. The change in the agent's decision is made with the use of hysteresis, the level of which is defined by means of the coefficient Δ , the value of which should be higher than the transaction costs.

The algorithm can be described as follows:

Algorithm 3

Input: $w = \langle w_1, w_2, \dots, w_M \rangle$ //The vector of quotation value of the quotations consisting of M quotations (current quotation and $M-1$ previous quotations – the M is calculated by the genetic algorithm or determined by the user),
preva // the previous value the a coefficient.

Output: The fuzzy logic decision D (value range [-1..1]) with respect to w and *preva* value.

BEGIN

Let $sumy:=0$; $sumx:=0.0$; $sumxy:=0$; $sumx2:=0$.
// where: *sumy* means the sum of the value of M quotations, *sumx* means the sum of the particular quotations' number in vector (suma numerów poszczególnych notowań np. jeśli $M=5$ to $sumx = 1+2+3+4+5$ czyli 15) in the vector, *sumxy* means the sum of the products of the quotation value and particular

quotation number in the vector, and $sumx2$ means the sum of the squares of quotation numbers in the vector.

$D:=0$, $countTRL:=0$; //counter for fuzzification,
 $maxcount:=0$. //maximum counter limit for fuzzification.

```

For (j=1;j<=M;i++)
  sumy:= sumy+wi; sumxy:= sumxy+wi*i; sumx:= sumx +i;
  sumx2:= sumx2+i2; i:=i+1;
c:= sumx2*M-sumx*sumx.
If c=0 then c:=0,1.
a:=(sumxy*M-sumx*sumy)/c.
If (a=preva=0) or (a<0 and preva<0) or (a>0 and preva>0) then D:=0.
If (a>0 and preva<0) then
  If (countTRL>0) then countTRL=0.
  countTRL:=countTRL-1.
  If (countTRL<-maxcount) then countTRL=-maxcount.
  D=countTRL/maxcount;
If (a<0 and preva>0) then
  If (countTRL<0) then countTRL=0.
  countTRL:=countTRL+1.
  If (countTRL>maxcount) then countTRL=maxcount.
  D=countTRL/maxcount;
preva:=a.
END

```

D. The ConsensusFuzzy agent

The *ConsensusFuzzy* agent (detailed in [22, 23]) is founded on the consensus theory [15, 16, 29] and determines the decisions on the basis of the set of decisions generated by other fuzzy logic agents in the system.

The algorithm is as follows:

Algorithm 4

Input: $A = \{D^{(1)}, D^{(2)}, \dots, D^{(M)}\}$ //The profile consists of M fuzzy logic agents' decisions, where M – number of fuzzy logic agents in the system, $D^{(1)}, D^{(2)}, \dots, D^{(M)}$ – decisions of particular agents

Output: The Fuzzy logic consensus CON (value range [-1..1]) according to A .

BEGIN

Let $CON:=0$.

Determine a sequence B by sorting elements of A profile in an increasing order.

$k_1=(M+1)/2$ the element of B .

$k_2=(M+2)/2$ the element of B .

Set CON as any value from interval $[k_1, k_2]$.

END.

It should be noted that currently in the system there are 100 agents using fuzzy logic representation. This set of trading agents may be easily extended if required. The evaluation of the performance of presented fuzzy logic agents will be shown further in the article.

IV. EXPERIMENTS

The agents performance analysis is performed for data within the M1 period of quotations from the FOREX market. For the purpose of this analysis, a test was performed in which the following assumptions were made:

1. EUR/USD quotes were selected from randomly chosen periods, notably:
 - 17-04-2015, 9:40 am to 17-04-2015, 9:50 pm, (710 quotations)

- 20-04-2015, 0:00 am to 20-04-2015, 7:00 pm (1140 quotations),
 - 22-04-2015, 0:00 am to 22-04-2015, 7:00 pm (1140 quotations),
2. At the verification, the strategies (signals for open long/close short position-equals to 1, close long/open short position-equals to -1) of the Supervisor are based on different decisions' probability levels calculated by fuzzy logic agents described in section III (the example of strategy is presented in Figure 3, where the green line means the "long position" and the red one the "short position").
 3. It was assumed that decisions' probability levels for open/close position are determined by the genetic algorithm (on the basis of earlier periods).
 4. It was assumed that the unit of performance analysis ratios (absolute ratios) is pips (a change in price of one "point" in Forex trading is referred to as a pip, and it is equivalent to the final number in a currency pair's price).
 5. The transaction costs are directly proportional to the number of transactions.
 6. The capital management - it was assumed that in each transaction the investor engages 100% of the capital held at the leverage 1:1. It should be pointed out that the investor may define another capital management strategy.
 7. The performance analysis was performed with the use of the following measures (ratios):
 - rate of return (ratio x_1),
 - the number of the transaction,
 - gross profit (ratio x_2),
 - gross loss (ratio x_3),
 - the number of profitable transactions (ratio x_4),
 - the number of profitable transactions in a row (ratio x_5),
 - the number of unprofitable transactions in a row (ratio x_6),
 - Sharpe ratio (ratio x_7)
$$S = \frac{E(r) - E(f)}{|O(r)|} \cdot 100\% \quad (1)$$

where:

 - $E(r)$ – arithmetic average of the rate of return,
 - $E(f)$ – arithmetic average of the risk-free rate of return,
 - $O(r)$ – standard deviation of rates of return.
 - the average coefficient of volatility (ratio x_8) is the ratio of the average deviation of the arithmetic average multiplied by 100% and is expressed:

$$V = \frac{s}{|E(r)|} \cdot 100\% . \quad (2)$$

where:

V – average coefficient of variation,

s – average deviation of the rates of return,

$E(r)$ – arithmetic average of the rates of return.

- Value at Risk (ratio x_9) – the measure known as value exposed to the risk - that is the maximum loss of the market value of the financial instrument possible to bear in a specific timeframe and at a given confidence level [7].

$$VaR = P * O * k \quad (3)$$

where:

P – the initial capital,

O – volatility - standard deviation of rates of return during the period,

k – the inverse of the standard normal cumulative distribution (assumed confidence level 95%, the value of k is 1,65),

- the average rate of return per transaction (ratio x_{10}), counted as the quotient of the rate of return and the number of transactions.
8. For the purpose of the comparison of the agents' performance, the following evaluation function was elaborated:

$$y = (a_1 x_1 + a_2 x_2 + a_3 (1 - x_3) + a_4 x_4 + a_5 x_5 + \dots$$

$$+ a_6 (1 - x_6) + a_7 x_7 + a_8 (1 - x_8) + a_9 (1 - x_9) + a_{10} x_{10}) \quad (4)$$

where x_i denotes the normalized values of ratios mentioned in item 6 from x_1 to x_{10} . It was adopted in the test that coefficients a_1 to $a_{10} = 1/10$. It should be mentioned that these coefficients may be modified with the use of, for instance, an evolution/genetic method or determined by the user (investor) in accordance with his/her preference (for instance the user may determine whether he/she is interested in the higher rate of return with a simultaneous higher risk level or lower risk level, but simultaneously agrees to a lower rate of return).

The function is given the values from the range [0..1], and the agent's efficiency is directly proportional to the function value.

9. The results obtained by the tested agents were compared with the results of the Buy-and-Hold benchmark (a trader buys a currency at the beginning and sell a currency at the end of an investment period) and the EMA benchmark (Exponential Moving Average -- a type of moving average that is similar to a simple moving average, except that more weight is given to the latest data).

Table 1 presents the results obtained in the particular periods.

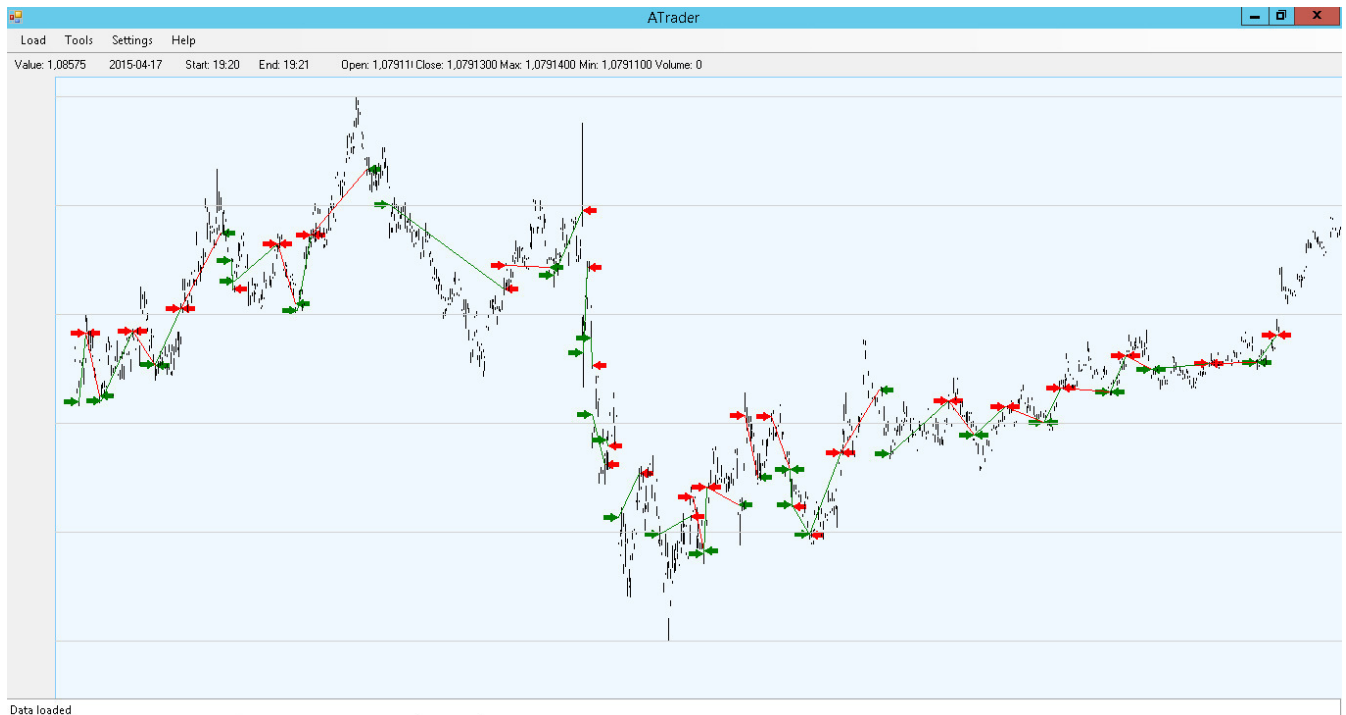


Fig. 3 The example of strategy visualization.
Source: Own work.

TABLE I.
PERFORMANCE ANALYSIS RESULTS

Ratio	BollingerFuzzy			WilliamsFuzzy			TrendLinearRegFuzzy			ConsensusFuzzy			EMA			B & H		
	Period 1	Period 2	Period 3	Period 1	Period 2	Period 3	Period 1	Period 2	Period 3	Period 1	Period 2	Period 3	Period 1	Period 2	Period 3	Period 1	Period 2	Period 3
Rate of return [Pips]	185	67	69	-41	68	36	56	51	-82	141	53	-1	231	-183	-189	26	-55	-4
The number of transactions	34	38	32	9	11	11	41	126	63	42	51	43	134	201	164	1	1	1
Gross profit [Pips]	48	16	30	50	28	39	33	20	16	48	20	36	41	32	3	26	0	0
Gross loss [Pips]	54	43	53	27	27	28	37	21	17	36	32	40	35	28	23	0	-55	-4
The number of profitable transactions	25	24	22	3	7	6	18	58	19	30	29	26	63	79	67	1	0	0
The number of profitable consecutive transactions	13	8	6	1	5	4	4	6	3	9	6	7	7	5	6	1	0	0
The number of unprofitable consecutive transactions	2	2	2	2	2	2	4	9	12	3	3	3	5	9	4	0	1	1
Sharpe ratio	0.84	0.93	0.91	1.01	0.97	1.00	0.94	0.84	1.20	0.84	0.92	1.0	0.3	0.02	0.7	0	0	0
The average coefficient of volatility [%]	1.12	9.13	1.12	3.34	1.95	4.70	9.85	3.20	4.11	1.01	0.74	1.29	3.33	1.95	2.56	0	0	0
The average rate of return per transaction	5.44	1.76	2.15	4.56	6.18	3.27	1.36	0.40	-1.30	3.36	1.04	-0.02	1.72	-0.91	-1.15	26	-55	-4
Value of evaluation function (y)	0.26	0.42	0.43	0.02	0.12	0.61	0.07	0.31	0.01	0.54	0.38	0.43	0.20	0.16	0.21	0.03	0.01	0.02

Source: Experiment results

In general, it may be noticed that the fuzzy logic agents generated not only profitable decisions. In the performance analysis not only the rate of return was taken into consideration but also other ratios, including the level of risk involved in the investment. It may be noticed that the values of efficiency ratios of particular agents differ in each period: for instance the estimated values of such ratios as *Gross Profit* and the *Number of Profitable Consecutive Transactions*. The values of *Rate of Return*, *Sharpe Ratio* and *Average Rate of Return per Transaction* show significant dispersal among particular agents. It may also be noticed that in the case of the agents *WilliamsFuzzy*, *TrendLinearRegFuzzy* and *EMA*, the values of these ratios have shown variability in particular periods. The evaluation function provides the immediate choice of the best agent. It may be noticed that the values of the evaluation function oscillate in the range from 0.01 to 0.61. Thus, the use of this function reduces the deviation of the values of the ratios. The results of the experiment allow us to state that the ranking of agents' evaluation differs in particular periods. In the first period, the *ConsensusFuzzy* was the best agent, the *BollingerFuzzy* agent was ranked higher than *WilliamsFuzzy*, and *TrendLinearRegFuzzy* agents were ranked lower than the *EMA*. In the second period, the *BollingerFuzzy* was the best agent, and the *ConsensusFuzzy* and *TrendLinearReg* agents were ranked higher and *WilliamsFuzzy* was ranked lower than the *EMA*. Considering the third period, it may be noticed that the *WilliamsFuzzy* was the best agent and *BollingerFuzzy* and *ConsensusFuzzy* agents were ranked higher and the *TrendLinearRegFuzzy* was ranked lower than *EMA*. The *B&H* benchmark was ranked lowest in all the periods, and in second and third periods it generated the losses. It should be noticed that in the first period, the upward trend was observed, therefore *B&H's Rate of Return* was positive. The second and the third periods shown a downward trend, and therefore the *B&H's Rate of Return* is negative.

Taking into consideration all the periods, it may be stated that there is no agent ranked highest most often. Also, agents achieving the highest Rate of Return were not always ranked in the highest positions. The low level of risk was influenced by the ranks of the *ConsensusFuzzy* and *WilliamsFuzzy* agents. And, on the other hand, the *EMA* was often ranked low because of a high risk level (low value of *Sharpe Ratio*). Moreover, it generated a high number of transactions, so transactions costs are very high.

In the case of fuzzy logic agents, the value of buy-sell decision agents' evaluation is most often higher than the value of *EMA* and *B&H* benchmarks (see last row of Table 1). In the case of three-valued logic agents, instead, there are many cases where the value of buy-sell decision agents' evaluation is lower than the value of *EMA* and *B&H* (see [Korczak et al. 2013, 2014]). Also, the values of such ratios as *Rate of Return* and *Number of profitable transactions* were about several percent higher in the case of fuzzy logic.

The risk measuring ratios (*Sharpe ratio*, *the average coefficient of volatility*) values were similar using fuzzy and three-valued logic.

The fuzzy logic has also demonstrated the better performance of the Supervisor strategies, because the opening/closing positions were generated closer to the optimal point determined by the genetic algorithm. In order to analyse the fuzzy logic agents' decisions efficiency it is also necessary to take into consideration the thresholds for open/close positions determined by the genetic algorithm (Table 2).

The optimal thresholds for opening/closing long/short positions differ in the case of particular agents. However, these levels often do not equal 1, 0 or -1 (as in the case of three-valued logic). In addition, the levels for the open long position are different to levels for the close short position, and levels for the open short position are different to levels for the close long position.

TABLE II.
THRESHOLDS FOR OPEN/CLOSE POSITION

Agent	Open Long	Close Long	Open Short	Close Short
BollingerFuzzy	0.38	-0.72	-0.98	0.37
WilliamsFuzzy	1.00	-1.00	-0.73	1.00
TrendLinearRegFuzzy	1.00	-0.88	-0.89	0.88
ConsensusFuzzy	0.18	-1.00	-0.94	0.27

Source: Experiment results.

Therefore, the fuzzy logic agents may suggest the "out of market" status - in a period of uncertainty on the market - in a broader scope than three-valued logic agents.

V. CONCLUSION

The fuzzy logic agents in the a-Trader system take independent buy-sell decisions with a certain level of probability. The analysis results presented in this article allow us to draw the conclusion that the application of fuzzy logic as an agents' knowledge representation allows for opening/closing long/short positions closer to the optimal level than the agents based on the three-valued logic.

In consequence the prediction performed by a-Trader were more precise, in periods with both upward and downward trends.

The implementation of fuzzy logic entailed the development of new agents and new trading strategies. The computational complexity of fuzzy logic algorithms is not higher than in the case of three-valued logic, so the computing time of trading positions was almost the same.

It can be also concluded that depending on the current situation on the FOREX market, the level of performance of a particular agent changes. There is no one agent which definitely dominates over the others. The automatic setting of the best agent in time close to real time is performed by

the use of the performance evaluation function. It has, in turn, a positive influence on investment effectiveness.

Currently tests are being performed on the implementation of the fuzzy logic agents using fundamental analysis and the analysis of experts' sentiments. It is also planned to evaluate the a-Trader system on more periods and other quotations pairs.

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