

A declarative decision support framework for supply chain problems

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Abstract—The author presents a novel declarative approach to modeling, solving and decision support for supply chain problems as a declarative decision support framework. The proposed framework makes it possible to ask different types of questions (general, specific, logical etc.). The implementation of the framework was performed in the CLP (Constraint Logic Programming) environment.

To increase the efficiency of the framework, particularly in the area of optimization made its integration with MP (Mathematical Programming) environment. The paper also presents the implementation of illustrative model, using the proposed framework. In addition, an efficiency analysis of the presented solution in relation to the application of mathematical programming have been conducted.

I. INTRODUCTION

THE supply chain (SC) is commonly seen as a collection of various types of companies (raw materials, production, trade, logistics, transport, etc.) working together to improve the flow of products, information and finance. As the words in the term indicate, the supply chain is a combination of its individual links in the process of supplying products (material/products and services) to the market.

There is a considerable literature on the supply chain management problems [1,2].

The major difficulties that appear in supply chain management (SCM) include large amount of information and multiple constraints relating to each participant in the management process. These participants share many of those constraints and information items [3], which further complicates the management. The constraints have various characters and structures. The most common are the constraints related to resources, time, finance, transportation, environment, business, law and safety. They can be linear constraints, non-linear, binary integer and logical. Managers/Decision makers are typically interested in feasibility and/or optimality of the decisions they make in the environment with many constraints. The most natural way to support the decision makers is to enable them to ask questions and obtain answers within acceptable time.

Good environments for the modeling of constraints, questions and logical conditions include declarative

environments, CLP (Constraint Logic Programming) in particular.

Our motivation was to develop a framework for the modeling and decision support for supply chain management problems. The use of this framework would help obtain quick answers to key questions (Is it possible...?, What If...?, What is the minimum/maximum..?) asked by managers/decision makers.

This paper proposes the concept of a declarative decision support framework for supply chain problems and presents its implementation in the CLP environment. The illustrative example shows the potential of the framework.

The remainder of the article is organized as follows. Section 2 presents problem statement, research methodology, contribution etc. The concept and implementation aspects of a declarative decision support framework are provided in Section 3. Computational examples, tests of the implementation platform and discussion are presented in Section 4. Possible extensions of the proposed approach as well as the conclusions are included in Section 5.

II. PROBLEM STATEMENT AND METHODOLOGY

Most of the SC decision and optimization problems are modeled and solved by operations research (OR) methods. The vast majority of the literature reviewed [1,2,3,4], have formulated SC models as linear programming (LP), integer programming (IP) and mixed integer linear programming (MILP) problems. Declarative environments such as CLP facilitate problem modeling and introduction of logical and symbolic constraints [5,6]. Unfortunately, high complexity and the multiple types of constraints of decision-making models as well as combinatorial nature contribute to poor efficiency of modeling in OR methods and inefficient optimization in CLP. Therefore, a new approach to modeling and solving such problems was developed [7,8,9,10]. A declarative environment was chosen as the best structure for this approach especially in modeling [5]. Mathematical programming environment was used for problem optimization [11]. This integrated approach is the basis for the creation of the implementation environment to support managers.

A. Problem description –illustrative example

The problem of supply chain management considered here refers to the supply chain in which:

- the supply chain consists of factories, distribution centers and customers (Fig. 1);
- customer orders are executed by deliveries from distribution centers;
- distribution centers are supplied by the factory;
- transport is multimodal (several modes of transport, a limited number of means of transport for each mode);
- the environmental aspects of use of transport modes are taken into account;
- different products are combined in one batch of transport;
- the cost of supplies is presented in the form of a function (in this approach, linear function of fixed and variable costs).

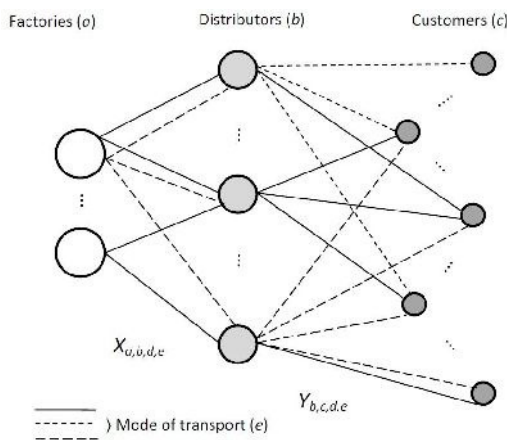


Fig. 1 The structure of supply chain for illustrative example

There are many decision and optimization problems in such supply chain management environment. Specific management problems are presented in the form of questions (possible questions (Q_i) for illustrative example are shown in Table I), which have to meet the reference set of constraints. Indices, and decision variables for the illustrative example have been reported in Table II. The set of reference constraints (1).. (22) for the illustrative example was created and its mathematical/formal notation is included in Appendix A. Brief description of reference set of constraints has been provided below.

Production capacity constraint (1) determines that all deliveries of product *d* produced by the manufacturer *a* and delivered to all distributors *b* using mode of transport *e* do not exceed the manufacturer’s production capacity.

Demand constraint (2) covers all customer *c* demands for product *d* ($Z_{c,d}$) through the implementation of delivery by distributors *b* (the values of decision variables $Y_{b,c,d,e}$). The flow balance of each distributor *b* corresponds to balance constraint (3). The possibility of delivery is dependent on the distributor’s technical capabilities – capacity constraint (4). Time constraint (5) ensures the terms of all deliveries are met. Transport cconstraints (6a), (6b), (7) guarantee

deliveries with available transport taken into account. Constraints (8), (9), (10) set values of decision variables based on binary variables Tc_b , $Xa_{a,b,e}$, $Ya_{b,c,e}$. Dependencies (11) and (12) represent the relationship based on which total costs are calculated. In general, these may be any linear functions. The remaining constraints (13)..(22) arise from the nature of the decision variables.

TABLE I.
THE SET OF QUESTIONS (INCLUDING BUT NOT LIMITED TO)

Question	Description
Q_1	What is the minimum overall cost of timely supply?
Q_2	Can timely supply be realized at the set cost of transportation Kt ?
Q_3	What is the minimum environmental cost Ks of timely supply ?
Q_4	What is the minimum cost of timely supply with the use of no more than N distribution centers?
Q_5	Is timely supply possible without the use of transport means dx ?
Q_6	Is timely supply possible with the following numbers of transport means $d1,d2,d3$?
Q_7	Can timely supply be realized at the set production cost Kp ?
Q_8	What is the minimum cost of supply execution if transport means $d1, d2$ cannot be used simultaneously by distribution center $S1$?

Decision variables of this problem are shown in Table II.

TABLE II.
INDICES AND DECISION VARIABLES

Symbol	Description
Indices	
d	product type (d=1..D)
c	delivery point/customer/city (c=1..C)
a	manufacturer/factory (a=1..A)
b	distributor /distribution center (b=1..B)
e	mode of transport (e=1..e)
A	number of manufacturers/factories
C	number of delivery points/customers
B	number of distributors
D	number of product types
E	number of mode of transport
Decision Variables	
$X_{a,b,d,e}$	delivery quantity of product <i>d</i> from manufacturer <i>a</i> to distributor <i>b</i> using mode of transport <i>e</i>
$Xa_{a,b,e}$	if delivery is from manufacturer <i>a</i> to distributor <i>b</i> using mode of transport <i>e</i> then $Xa_{a,b,e}=1$, otherwise $Xa_{a,b,e}=0$
$Xb_{a,b,e}$	the number of courses from manufacturer <i>a</i> to distributor <i>b</i> using mode of transport <i>e</i>
$Y_{b,c,d,e}$	delivery quantity of product <i>d</i> from distributor <i>b</i> to customer <i>c</i> using mode of transport <i>e</i>
$Ya_{b,c,e}$	if delivery is from distributor <i>b</i> to customer <i>c</i> using mode of transport <i>e</i> then $Ya_{b,c,e}=1$, otherwise $Ya_{b,c,e}=0$
$Yb_{b,c,e}$	the number of courses from distributor <i>b</i> to customer <i>c</i> using mode of transport <i>e</i>
Tc_b	if distributor <i>b</i> participates in deliveries, then $Tc_b=1$, otherwise $Tc_b=0$

III. A DECLARATIVE DECISION SUPPORT FRAMEWORK FOR SUPPLY CHAIN MANAGEMENT PROBLEMS-CONCEPT AND IMPLEMENTATION

The declarative decision support framework was proposed for supply chain management problems. The concept is based on the declarative programming paradigm, which allows high level programming with the use of predicates and facts. Due to the character of problems in the supply chain management, CLP (Constraint Logic Programming) was selected from among many declarative options. The implementation of the framework was performed with the use of ECL^{PS} [5,12].

The following general assumptions were applied:

- possibility of modeling constraints of any type;
- automatic generation of implementation models in the form of MILP models;
- data recorded as facts.

Figure 2 presents the general concept of the framework. The framework comprises several phases: modeling, presolving, generating and solving. It has two inputs and uses the set of facts. Inputs are the set of questions and the set of constraints to the reference model of a given problem. Based on them, the primary model of the problem is generated as a CLP model, which is then presolved. The built-in CLP method (constraint propagation [5,6]) and the method of problem transformation designed by the authors [8,9] (Section 3A) are used for this purpose. Presolving procedure results on the transformed model CLP^T . This model is the basis for the automatic generation of the MILP (Mixed Integer Linear Programming) model, which is solved in MP (with the use of an external solver or as a library of CLP). The general concept of the framework consists in modeling and presolving of a problem in the CLP environment with the final solution (including optimization) found in the MP environment. This approach is the result of experience as well as extensive research devoted to both environments and their integration [10,13,14,15,16,17]. In all its phases, the framework uses the set of facts having the structure appropriate for the problem being modeled and solved (Fig. 2). The set of facts is the informational layer of the framework, which can be implemented as a relational database, XML database, etc.

The functional layer comprises adequate sets of predicates: P_1 (CLP model generation), P_2 (CLP model presolving through constraint propagation and transformation, post-transformation generation of CLP^T model), and P_3 (generation of the final $MILP^T$ model in the format of the MP solver).

The presolving phase is an important element of the framework as it makes it possible to simplify the model for the problem being solved and to reduce the combinatorial search space.

For the presolving phase to be effective, unfeasible combinations of model dimensions have to occur. In practice, unfeasible combinations of the index of decision

variables and/or facts occur. The proposed framework uses constraint propagation and transformation for the presolving procedure. Constraint propagation is a concept and method that appears in constrained-based environments. Constraint propagation embeds any reasoning which consists in explicitly forbidding values from some variable domain of a problem, because all constraints can not be satisfied otherwise [6].

Transformation transforms decision variables of the problem along with constraints and facts. The transformation of facts for the illustrative example is shown in Fig. 3, and the post-transformation variables are compiled in Table AII.

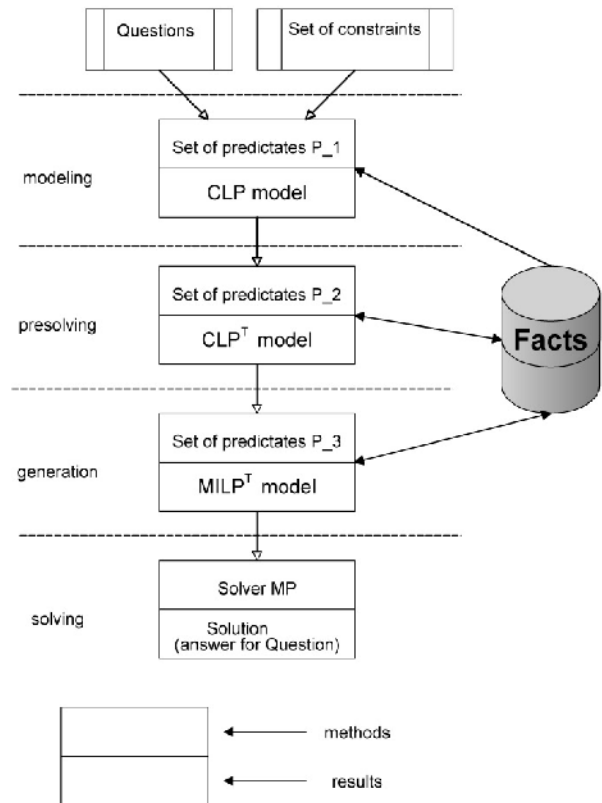


Fig. 2 A concept of a declarative decision support framework

A. Transformation of the problem-presolving phase

In the case of the problem presented, the transformation consisted in changing the problem representation from graph to routing. Instead of analyzing all possible transport connections from the factory to the distribution center and then from the center to the customer, only the feasible connections (factory-center-customer) were generated and named routes. This resulted in the removal of certain indices and in the aggregation of other indices for decision variables, parameters, etc., which eventually led to the reduction in the number of decision variables and constraints [7,8]. The new set of decision variable, constraints and facts was the basis for creating the CLP^T model.

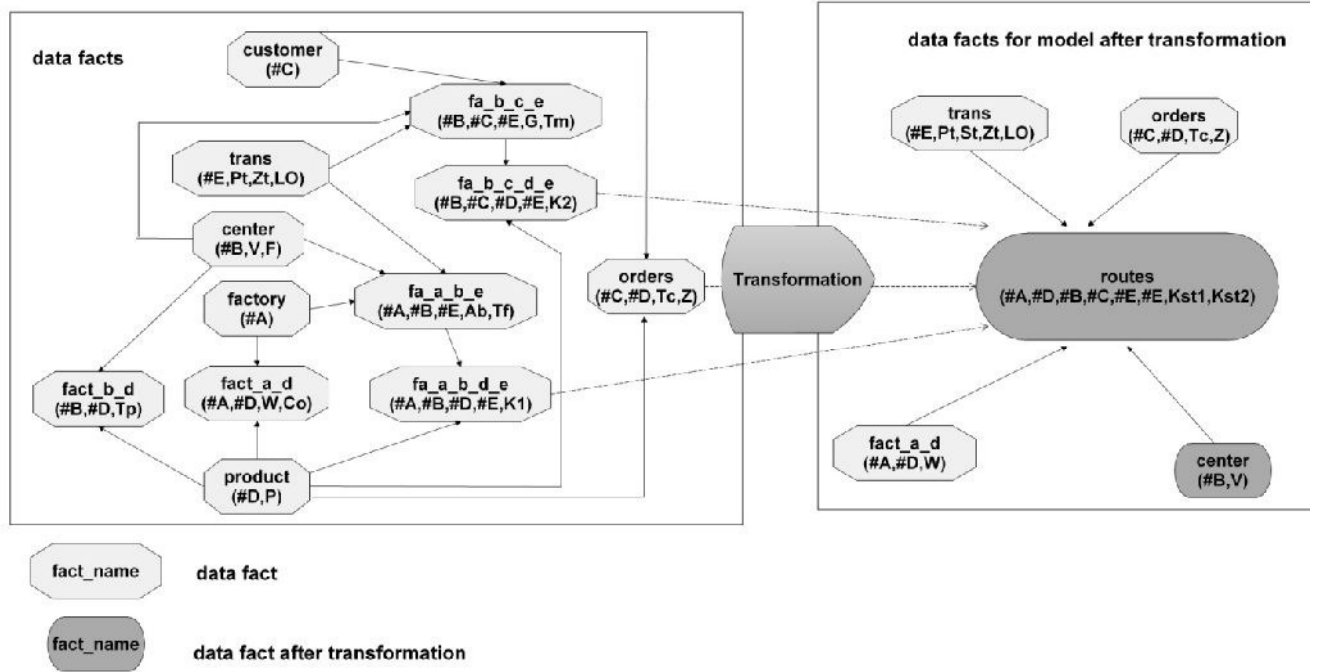


Fig. 3 Structure of facts for illustrative example before and after transformation

IV. COMPUTATION EXAMPLES FOR ILLUSTRATIVE MODEL

In order to verify and evaluate the proposed approach, many numerical experiments were performed. All the examples relate to the supply chain with five factories ($a=1..5$), four distributor centers ($b=1..4$), fifteen customers ($c=1..15$), four modes of transport ($e=1..4$), and fifteen types of products ($d=1..15$) and fifty orders (No).

All data instances for these experiments were recorded in the form of facts and included Appendix B. The structure of facts and their description has been shown in Fig. 3 and Table V.

Computational experiments consisted in asking questions Q_1..Q_8 to illustrative example. For each question was generated and solved suitable implementation model using declarative decision support framework. The answers to these questions are shown in Table III.

The answer to question Q_1 is the minimum entire cost of timely delivery all orders (Table III). This cost is the aggregate costs of the entire chain and consists of five elements (23). The first element comprises the fixed costs associated with the operation of the distributor involved in the delivery (e.g. distribution centre, warehouse, etc.). The second element corresponds to environmental costs of using various means of transport. Those costs are dependent on the number of courses of the given means of transport, and on the other hand, on the environmental levy, which in turn may depend on the use of fossil fuels and carbon-dioxide emissions. The third and fourth element determined by the cost of transportation. The last element is the cost of production. The answer to question Q_2 defines the feasibility of timely supply/delivery at the set (closed)

transportation cost (25). Table II shows two versions of the question with different parameters. The answer to question Q_3 determines minimum environmental costs (26) of timely delivery (Table I).

Two versions of the answer to question Q_4 are shown in Table III. These versions define the minimum overall cost of timely delivery when two distribution centers are used (Q_4A) and when one distribution center is used (Q_4B).

The possibility of executing timely delivery without the use of selected transportation means is defined in the answer to question Q_5 (three versions of the question are shown in Table III). The answer to question Q_6 specifies the possibility of timely delivery at the set number of each transportation means (Table III).

The possibility of executing timely delivery at the established, closed production cost (24) is included in the answer to question Q_7. Table III shows three versions of the answer. The answer to question Q_8 specifies the minimum cost of timely delivery at the logical condition met, ruling out the possibility of that the selected center can be used concurrently by two different transportation means.

In the second phase of the experiments, a comparative analysis was performed for questions Q_1 and Q_8 (most compute-intensive of all) for different numbers of orders (No) in two environments (declarative decision support framework (MILP^T) and MP (MILP)) to evaluate the effectiveness and efficiency of the proposed framework relative to the classical MP environment.

Obtained more than 50-fold reduction of time searching for solutions (Table IV). This is due to the fact that the application of the framework has allowed the up to 50-fold

reduction of integer decision variables up to 20-fold reduction of constraints (Table IV).

TABLE III.
ANSWERS TO QUESTION FOR ILLUSTRATIVE EXAMPLE

Question	Parameters	Answer
Q_1	----	17805
Q_2A	$K_t \leq 1520, T \leq 15$	YES
Q_2B	$K_t \leq 1400, T \leq 20$	NO
Q_3	----	6250
Q_4A	$N=2$	17945
Q_4B	$N=1$	18126
Q_5A	$S_1=0, T=12$	YES
Q_5B	$S_2=0, T=12$	YES
Q_5C	$S_3=0, T=12$	NO
Q_6A	$S_1=5, S_2=5, S_3=5, S_4=5, T=9$	YES
Q_6B	$S_1=5, S_2=5, S_3=5, S_4=5, T=9$	NO
Q_6C	$S_1=5, S_2=5, S_3=5, S_4=5, T=12$	YES
Q_7A	$K_p \leq 9820, T \leq 8$	NO
Q_7B	$K_p \leq 9820, T \leq 12$	YES
Q_7C	$K_p \leq 39200, T \leq 8$	YES
Q_8	----	17856

TABLE IV.
NUMERICAL EXPERIMENTS ON THE EFFICIENCY

No	Model	Vint	C	Answer	T
Q_1					
10	MILP	45286	29510	5500	26
	MILP ^T	860	1590	5500	2
25	MILP	45286	33990	9100	101
	MILP ^T	1268	1596	9100	2
50	MILP	45286	41990	17805	545
	MILP ^T	2078	1603	17805	13
75	MILP	45286	49990	27695*	600**
	MILP ^T	2996	1612	27175	56
Q_8					
10	MILP	45290	29518	5610	45
	MILP ^T	864	1590	5610	3
25	MILP	45290	33998	9210	112
	MILP ^T	1272	1604	9210	2
50	MILP	45290	41998	17856	588
	MILP ^T	2082	1611	17856	14
75	MILP	45290	49998	28102	600**
	MILP ^T	3000	1620	27450	68

No-the number of orders; Vint-the number of integer decision variables, C-the number of constraints, T-time of finding solution (in seconds)

TABLE V.
STRUCTURE OF THE FACTS FOR ILLUSTRATIVE EXAMPLE

Fact	Description
product(#D,P).	products , D-product ID, P-the capacity occupied by product.
factory(#A).	factories , A-factory ID.
fact_a_d(#A,#D,W,Co)	products in factories, A-factory ID, D-product ID, W-production capacity at factory A for product D, Co-the cost of product D in factory A.
center(#B,V,F)	distribution centers, B-center ID, V-maximum capacity, F-the fixed cost of distribution center.
fact_b_d(#B,#D,Tp)	products in distribution centers, B-center ID, D-product ID, Tp-the time needed for distributor B to prepare the shipment of product D.
trans(#E,Pt,Zt,Od)	transportation, E-number of mode of transportation, Pt-the capacity of transport unit, St-payload ,Zt-the number of transport units using mode of transportation E, Od-the environmental cost
customer(#C)	customers, C-customer ID
fa_a_b_e(#A,#B,#E,Ab,Tf)	delivery from factory to distribution center using mode of transportation E, A-factory ID, B-center ID, E-number of mode of transportation, Ab-the fixed cost of delivery, Tf-the time of delivery
fa_b_c_e(#B,#C,#E,G,Tm)	delivery from distribution center to customer using mode of transportation E, B-center ID, C-customer ID, E-number of mode of transportation, G-the fixed cost of delivery, Tm-the time of delivery
fa_a_b_d_e(#A,#B,#D,#E,K1)	the variable costs of delivery of product, A-factory ID, B-center ID, D-product ID, E-number of mode of transportation, Ks3-the variable cost of delivery of product from factory to distribution center (optional)
fa_b_c_d_e(#B,#C,#D,#E,K2)	the variable costs of delivery of product, B-center ID, C-customer ID, D-product ID, E-number of mode of transportation, Ks4-the variable cost of delivery of product from distribution center to customer (optional)
orders(#C,#D,Tc,Z)	orders, C-customer ID, D-product ID, the cut-off time of delivery to customer of product, Z-customer demand for product

V. CONCLUSIONS

Two types of questions can be asked in the proposed declarative decision support framework.

General questions may require domain solution, which in practice determines the availability of capacity, the number of transport units, timely supply etc. The specific wh-questions will in practice define the best, fastest, cheapest, or the most expensive of the possible solutions. To obtain answers to these questions, optimization is necessary. Both

question types can contain logical conditions relating, for example, to the disjoint use of transport units, distributors, etc. The illustrative example shows only part of potential of the framework designed to increase both the speed and the size of the problems solved (Table IV).

This is particularly evident if we compare the possibilities of the framework in relation to the classical approach based on mathematical programming (Table IV).

Further work will consist in the implementation of more complex models [18], uncertainty, fuzzy logic etc. [19], and as a cloud internet application [20]. New questions will be implemented to broaden the scope of decision support.

It is also considering development of models to take account product demand interdependencies [21]. In the future it is planned to integrate framework with ERP and APS systems [22]. It is planned to also use a hybrid approach to optimize the use of graphs, for example, to image retrieval [23].

APPENDIX A

TABLE A1
SUMMARY PARAMETERS

Symbol	Description
<i>Input parameters</i>	
F_b	the fixed cost of distributor/distribution center b
P_d	the area/volume occupied by product d
V_b	distributor b maximum capacity/volume
$W_{a,d}$	production capacity at factory a for product d
$Co_{a,d}$	the cost of product d at factory a
$R_{b,d}$	if distributor b can deliver product d then $R_{b,d}=1$, otherwise $R_{b,d}=0$
$Tp_{b,d}$	the time needed for distributor b to prepare the shipment of product d
$Tc_{c,d}$	the cut-off time of delivery to the delivery point/customer c of product d
$Z_{c,d}$	customer demand/order c for product d
Zt_e	the number of transport units using mode of transport e
Pt_e	the capacity of transport unit using mode of transport e
$Tf_{a,b,e}$	the time of delivery from manufacturer a to distributor b using mode of transport e
St_e	the payload of transport unit using mode of transport e
$K1_{a,b,d,e}$	the variable cost of delivery of product d from manufacturer a to distributor b using mode of transport e
$R1_{a,b,e}$	if manufacturer a can deliver to distributor b using mode of transport e then $R1_{a,b,e}=1$, otherwise $R1_{a,b,e}=0$
$A_{a,b,e}$	the fixed cost of delivery from manufacturer a to distributor b using mode of transport e
$Koa_{b,c,e}$	the total cost of delivery from distributor b to customer c using mode of transport e
$Tm_{b,c,e}$	the time of delivery from distributor b to customer c using mode of transport e
$K2_{b,c,d,e}$	the variable cost of delivery of product d from distributor b to customer c using mode of transport e
$R2_{b,c,e}$	if distributor b can deliver to customer c using mode of transport e then $R2_{b,c,e}=1$, otherwise $R2_{b,c,e}=0$
$G_{b,c,e}$	the fixed cost of delivery from distributor b to customer c using mode of transport e
$Kog_{b,c,e}$	the total cost of delivery from distributor b to customer c using mode of transport e
Od_e	the environmental cost of using mode of transport e
CW	Arbitrarily large constant

$$\sum_{b=1}^B \sum_{e=1}^E X_{a,b,d,e} \cdot R_{b,d} \leq W_{a,d} \forall a = 1..A, d = 1..D \quad (1)$$

$$\sum_{b=1}^B \sum_{c=1}^C (Y_{b,c,d,e} \cdot R_{b,d}) \geq Z_{c,d} \forall j = c..C, d = 1..D \quad (2)$$

$$\sum_{a=1}^A \sum_{c=1}^C X_{a,b,d,e} = \sum_{c=1}^C \sum_{e=1}^E Y_{b,c,d,e} \forall b = 1..B, d = 1..D \quad (3)$$

$$\sum_{d=1}^D (P_d \cdot \sum_{a=1}^A \sum_{c=1}^C X_{a,b,d,e}) \leq Tc_b \cdot V_b \forall b = 1..B \quad (4)$$

$$X_{a,b,e} \cdot Tf_{a,b,e} + X_{a,b,e} \cdot Tp_{b,d} + Y_{a,b,c,e} \cdot Tm_{b,c,e} \leq Tc_{b,d} \quad (5)$$

$$\forall a = 1..A, b = 1..B, C = 1..C, d = 1..D, e = 1..E$$

$$R1_{a,b,e} \cdot X_{a,b,e} \cdot Pt_e \geq X_{a,b,d,e} \cdot P_c \quad (6a)$$

$$\forall a = 1..A, b = 1..B, d = 1..D, e = 1..E$$

$$R2_{b,c,e} \cdot Y_{b,c,d,e} \cdot Pt_e \geq Y_{b,c,d,e} \cdot P_c \quad (6b)$$

$$\forall b = 1..B, c = 1..C, d = 1..D, e = 1..E$$

$$\sum_{a=1}^A \sum_{b=1}^B X_{a,b,e} + \sum_{b=1}^B \sum_{c=1}^C Y_{b,c,e} \leq Zt_e \forall e = 1..E \quad (7)$$

$$\sum_{a=1}^A \sum_{e=1}^E X_{a,b,e} \leq CW \cdot Tc_e \forall e = 1..E \quad (8)$$

$$X_{b,c,e} \leq CW \cdot X_{a,b,e} \forall a = 1..A, b = 1..B, e = 1..E \quad (9)$$

$$Y_{b,c,e} \leq CW \cdot Y_{a,b,c,e} \forall b = 1..B, c = 1..C, e = 1..E \quad (10)$$

$$Koa_{a,b,e} = A_{a,b,e} \cdot X_{a,b,e} + \sum_{d=1}^D K1_{a,b,d,e} \cdot X_{a,b,d,e} \quad (11)$$

$$\forall a = 1..A, b = 1..B, e = 1..E$$

$$Kog_{b,c,e} = G_{b,c,e} \cdot Y_{b,c,e} + \sum_{d=1}^D K2_{b,c,d,e} \cdot Y_{b,c,d,e} \quad (12)$$

$$\forall b = 1..B, c = 1..C, e = 1..E$$

$$X_{a,b,d,e} \geq 0 \forall a = 1..A, b = 1..B, d = 1..D, e = 1..E \quad (13)$$

$$X_{b,c,e} \geq 0 \forall a = 1..A, b = 1..B, e = 1..E \quad (14)$$

$$Y_{b,c,e} \geq 0 \forall b = 1..B, c = 1..C, e = 1..E \quad (15)$$

$$X_{a,b,d,e} \in C \forall a = 1..A, b = 1..B, d = 1..D, e = 1..E \quad (16)$$

$$X_{b,c,e} \in C \forall a = 1..A, b = 1..B, e = 1..E \quad (17)$$

$$Y_{b,c,d,e} \in C \forall b = 1..B, c = 1..C, d = 1..D, e = 1..E \quad (18)$$

$$Y_{b,c,d} \in C \forall b = 1..B, c = 1..C, e = 1..E \quad (19)$$

$$X_{a,b,e} \in \{0,1\} \forall a = 1..A, b = 1..B, e = 1..E \quad (20)$$

$$Y_{a,b,c,e} \in \{0,1\} \forall b = 1..B, c = 1..C, e = 1..E \quad (21)$$

$$Tc_b \in \{0,1\} \forall b = 1..B \quad (22)$$

$$K = \sum_{b=1}^B (F_b \cdot Tc_b) + \sum_{c=1}^C Od_c \left(\sum_{a=1}^A \sum_{b=1}^B X_{a,b,e} + \sum_{b=1}^B \sum_{c=1}^C Y_{b,c,e} \right) \quad (23)$$

$$+ \sum_{a=1}^A \sum_{b=1}^B \sum_{e=1}^E Koa_{a,b,e} + \sum_{b=1}^B \sum_{c=1}^C \sum_{e=1}^E Kog_{s,j,d} +$$

$$\sum_{a=1}^A \sum_{d=1}^D (Co_{a,d} \cdot \sum_{b=1}^B \sum_{e=1}^E X_{a,b,d,e}) \quad (24)$$

$$K_p = \sum_{a=1}^A \sum_{d=1}^D (Co_{a,d} \cdot \sum_{b=1}^B \sum_{e=1}^E X_{a,b,d,e})$$

$$K_t = \sum_{a=1}^A \sum_{b=1}^B \sum_{e=1}^E Koa_{a,b,e} + \sum_{b=1}^B \sum_{c=1}^C \sum_{e=1}^E Kog_{s,j,d} \quad (25)$$

$$K_s = \sum_{c=1}^C Od_c \left(\sum_{a=1}^A \sum_{b=1}^B X_{a,b,e} + \sum_{b=1}^B \sum_{c=1}^C Y_{b,c,e} \right) \quad (26)$$

TABLE AII.
DECISION VARIABLES AFTER TRANSFORMATION

<i>Decision variables</i>	
delivery quantity of product d for route from manufacturer a to distributor b using mode of transport e_1 and from distributor b to customer c using mode of transport e_2	X_{a,b,c,d,e_1,e_2}^T
the number of courses from manufacturer a to distributor b using mode of transport e	$Xb_{a,b,e}^T$
the number of courses from distributor b to customer c using mode of transport e	$Yb_{b,c,e}^T$
if distributor b participates in deliveries, then $Tc_b=I$, otherwise $Tc_b=0$	Tc_b^T

APPENDIX B

%Products

product (d1, 10, 2). product (d2, 20, 4).
product (d3, 20, 4). product (d4, 20, 5).
product (d5, 10, 2). product (d6, 20, 2).
product (d7, 20, 3). product (d8, 20, 4).
product (d9, 10, 4). product (d10, 20, 5).
product (d11, 20, 6). product (d12, 20, 2).
product (d13, 30, 3). product (d14, 30, 3).
product (d15, 20, 4).

%Factories

factory (a1). factory (a2). factory (a3).
factory (a4). factory (a5).

%Distribution Centers

center (b1, 1000, 100). center (b2, 1000, 200).
center (b3, 1000, 300). center (b4, 1000, 400).

%Products in distribution centers

fact_b_d (b1, d1, 1). fact_b_d (b1, d2, 1).
fact_b_d (b1, d3, 1). fact_b_d (b1, d4, 1).
fact_b_d (b1, d5, 1). fact_b_d (b1, d6, 1).
fact_b_d (b1, d7, 1). fact_b_d (b1, d8, 1).
fact_b_d (b1, d9, 1). fact_b_d (b1, d10, 1).
fact_b_d (b1, d11, 1). fact_b_d (b1, d12, 1).
fact_b_d (b2, d5, 1). fact_b_d (b2, d6, 1).
fact_b_d (b2, d7, 1). fact_b_d (b2, d8, 1).
fact_b_d (b2, d9, 1). fact_b_d (b2, d10, 1).
fact_b_d (b2, d11, 1). fact_b_d (b2, d12, 1).
fact_b_d (b2, d13, 1). fact_b_d (b2, d14, 1).
fact_b_d (b2, d15, 1). fact_b_d (b3, d1, 1).
fact_b_d (b3, d2, 1). fact_b_d (b3, d3, 1).
fact_b_d (b3, d4, 1). fact_b_d (b3, d5, 1).
fact_b_d (b3, d6, 1). fact_b_d (b3, d7, 1).
fact_b_d (b3, d8, 1). fact_b_d (b3, d9, 1).
fact_b_d (b3, d10, 1). fact_b_d (b3, d11, 1).
fact_b_d (b3, d12, 1). fact_b_d (b3, d13, 1).
fact_b_d (b3, d14, 1). fact_b_d (b3, d15, 1).
fact_b_d (b4, d1, 1). fact_b_d (b4, d2, 1).
fact_b_d (b4, d3, 1). fact_b_d (b4, d4, 1).
fact_b_d (b4, d5, 1). fact_b_d (b4, d6, 1).
fact_b_d (b4, d7, 1). fact_b_d (b4, d8, 1).
fact_b_d (b4, d9, 1). fact_b_d (b4, d10, 1).
fact_b_d (b4, d11, 1). fact_b_d (b4, d12, 1).
fact_b_d (b4, d13, 1). fact_b_d (b4, d14, 1).
fact_b_d (b4, d15, 1).

%Products infactories

fact_a_d (a1, d1, 800, 400). fact_a_d (a2, d2, 800, 400).
fact_a_d (a3, d3, 800, 400). fact_a_d (a1, d4, 800, 400).
fact_a_d (a2, d5, 800, 400). fact_a_d (a3, d6, 800, 400).
fact_a_d (a1, d7, 800, 400). fact_a_d (a2, d8, 800, 400).
fact_a_d (a3, d9, 800, 400). fact_a_d (a1, d10, 800, 400).
fact_a_d (a2, d11, 800, 400). fact_a_d (a3, d12, 800, 400).
fact_a_d (a1, d13, 800, 400). fact_a_d (a2, d14, 800, 400).
fact_a_d (a3, d15, 800, 400). fact_a_d (a4, d1, 800, 100).
fact_a_d (a4, d2, 800, 100). fact_a_d (a4, d3, 800, 100).
fact_a_d (a4, d4, 800, 100). fact_a_d (a4, d5, 800, 100).
fact_a_d (a4, d6, 800, 100). fact_a_d (a4, d7, 800, 100).
fact_a_d (a4, d8, 800, 100). fact_a_d (a4, d6, 800, 100).

fact_a_d (a4, d7, 800, 100). fact_a_d (a4, d8, 800, 100).
fact_a_d (a4, d9, 800, 100). fact_a_d (a4, d10, 800, 100).
fact_a_d (a4, d11, 800, 100). fact_a_d (a4, d12, 800, 100).
fact_a_d (a4, d13, 800, 100). fact_a_d (a4, d14, 800, 100).
fact_a_d (a4, d15, 800, 100). fact_a_d (a5, d1, 800, 100).
fact_a_d (a5, d2, 800, 100). fact_a_d (a5, d3, 800, 100).
fact_a_d (a5, d4, 800, 100). fact_a_d (a5, d5, 800, 100).
fact_a_d (a5, d6, 800, 100). fact_a_d (a5, d7, 800, 100).
fact_a_d (a5, d8, 800, 100). fact_a_d (a5, d6, 800, 100).
fact_a_d (a5, d7, 800, 100). fact_a_d (a5, d8, 800, 100).
fact_a_d (a5, d9, 800, 100). fact_a_d (a5, d10, 800, 100).
fact_a_d (a5, d11, 800, 100). fact_a_d (a5, d12, 800, 100).
fact_a_d (a5, d13, 800, 100). fact_a_d (a5, d14, 800, 100).
fact_a_d (a5, d15, 800, 100).

%Transportation modes

trans (e1, 850, 5, 500). trans (e2, 600, 10, 300).
trans (e3, 350, 10, 200). trans (e4, 150, 10, 100).

%Deliveries from factories to distribution centers

fa_a_b_e (a1, b1, e1, 270, 1). fa_a_b_e (a2, b1, e1, 300, 1).
fa_a_b_e (a3, b1, e1, 400, 1). fa_a_b_e (a4, b1, e1, 500, 8).
fa_a_b_e (a5, b1, e1, 400, 8). fa_a_b_e (a1, b1, e2, 170, 2).
fa_a_b_e (a2, b1, e2, 200, 2). fa_a_b_e (a3, b1, e2, 300, 2).
fa_a_b_e (a4, b1, e2, 400, 8). fa_a_b_e (a5, b1, e2, 300, 8).
fa_a_b_e (a1, b2, e1, 270, 1). fa_a_b_e (a2, b2, e1, 300, 1).
fa_a_b_e (a3, b2, e1, 400, 1). fa_a_b_e (a4, b2, e1, 500, 8).
fa_a_b_e (a5, b2, e1, 400, 8). fa_a_b_e (a1, b2, e2, 170, 1).
fa_a_b_e (a2, b2, e2, 200, 1). fa_a_b_e (a3, b2, e2, 300, 1).
fa_a_b_e (a4, b2, e2, 300, 8). fa_a_b_e (a5, b2, e2, 300, 8).
fa_a_b_e (a1, b3, e1, 270, 1). fa_a_b_e (a2, b3, e1, 300, 1).
fa_a_b_e (a3, b3, e1, 400, 1). fa_a_b_e (a4, b3, e1, 500, 8).
fa_a_b_e (a5, b3, e1, 400, 8). fa_a_b_e (a1, b3, e2, 170, 1).
fa_a_b_e (a2, b3, e2, 200, 1). fa_a_b_e (a3, b3, e2, 300, 1).
fa_a_b_e (a4, b3, e2, 400, 8). fa_a_b_e (a5, b3, e2, 300, 8).
fa_a_b_e (a1, b4, e1, 270, 1). fa_a_b_e (a2, b4, e1, 300, 1).
fa_a_b_e (a3, b4, e1, 400, 1). fa_a_b_e (a4, b4, e1, 500, 8).
fa_a_b_e (a5, b4, e1, 400, 8). fa_a_b_e (a1, b4, e2, 170, 1).
fa_a_b_e (a2, b4, e2, 200, 1). fa_a_b_e (a3, b4, e2, 300, 1).
fa_a_b_e (a4, b4, e2, 400, 8). fa_a_b_e (a5, b4, e2, 300, 8).

%Customers

customer (c2). customer (c3). customer (c4).
customer (c5). customer (c6). customer (c7).
customer (c8). customer (c9). customer (m10).
customer (m11). customer (m12). customer (m13).
customer (m14). customer (m15).

%Deliveries from distribution centers to customers

fa_b_c_e (b1, c1, e4, 30, 1). fa_b_c_e (b1, c2, e4, 30, 1).
fa_b_c_e (b1, c3, e4, 30, 1). fa_b_c_e (b1, c4, e4, 30, 1).
fa_b_c_e (b1, c5, e4, 30, 1). fa_b_c_e (b1, c6, e4, 30, 1).
fa_b_c_e (b1, c7, e4, 30, 1). fa_b_c_e (b1, c8, e4, 30, 1).
fa_b_c_e (b1, c9, e4, 30, 1). fa_b_c_e (b1, m10, e4, 30, 1).
fa_b_c_e (b1, m11, e4, 30, 1). fa_b_c_e (b1, m12, e4, 30, 1).
fa_b_c_e (b1, m13, e4, 30, 1). fa_b_c_e (b1, m14, e4, 30, 1).
fa_b_c_e (b1, m15, e4, 30, 1). fa_b_c_e (b2, c1, e4, 30, 1).
fa_b_c_e (b2, c2, e4, 30, 1). fa_b_c_e (b2, c3, e4, 30, 1).
fa_b_c_e (b2, c4, e4, 30, 1). fa_b_c_e (b2, c5, e4, 30, 1).
fa_b_c_e (b2, c6, e4, 30, 1). fa_b_c_e (b2, c7, e4, 30, 1).
fa_b_c_e (b2, c8, e4, 30, 1). fa_b_c_e (b2, c9, e4, 30, 1).
fa_b_c_e (b2, m10, e4, 30, 1). fa_b_c_e (b2, m11, e4, 30, 1).
fa_b_c_e (b2, m12, e4, 30, 1). fa_b_c_e (b2, m13, e4, 30, 1).
fa_b_c_e (b2, m14, e4, 30, 1). fa_b_c_e (b2, m15, e4, 30, 1).
fa_b_c_e (b3, c1, e4, 30, 1). fa_b_c_e (b3, c2, e4, 30, 1).
fa_b_c_e (b3, c3, e4, 30, 1). fa_b_c_e (b3, c4, e4, 30, 1).
fa_b_c_e (b3, c5, e4, 30, 1). fa_b_c_e (b3, c6, e4, 30, 1).
fa_b_c_e (b3, c7, e4, 30, 1). fa_b_c_e (b3, c8, e4, 30, 1).
fa_b_c_e (b3, c9, e4, 30, 1). fa_b_c_e (b3, m10, e4, 30, 1).
fa_b_c_e (b3, m11, e4, 30, 1). fa_b_c_e (b3, m12, e4, 30, 1).
fa_b_c_e (b3, m13, e4, 30, 1). fa_b_c_e (b3, m14, e4, 30, 1).
fa_b_c_e (b3, m15, e4, 30, 1). fa_b_c_e (b4, c1, e4, 30, 1).
fa_b_c_e (b4, c2, e4, 30, 1). fa_b_c_e (b4, c3, e4, 30, 1).
fa_b_c_e (b4, c4, e4, 30, 1). fa_b_c_e (b4, c5, e4, 30, 1).
fa_b_c_e (b4, c6, e4, 30, 1). fa_b_c_e (b4, c7, e4, 30, 1).
fa_b_c_e (b4, c8, e4, 30, 1). fa_b_c_e (b4, c9, e4, 30, 1).
fa_b_c_e (b4, m10, e4, 30, 1). fa_b_c_e (b4, m11, e4, 30, 1).
fa_b_c_e (b4, m12, e4, 30, 1). fa_b_c_e (b4, m13, e4, 30, 1).
fa_b_c_e (b4, m14, e4, 30, 1). fa_b_c_e (b4, m15, e4, 30, 1).

fa_b_c_e(b1,c1,e3,50,1) . fa_b_c_e(b1,c6,e3,50,1) .
 fa_b_c_e(b1,c7,e3,50,1) . fa_b_c_e(b1,c8,e3,50,1) .
 fa_b_c_e(b1,c9,e3,50,1) . fa_b_c_e(b2,c3,e3,50,1) .
 fa_b_c_e(b2,c4,e3,50,1) . fa_b_c_e(b2,m13,e3,50,1) .
 fa_b_c_e(b2,m14,e3,50,1) . fa_b_c_e(b3,c7,e3,50,1) .
 fa_b_c_e(b3,c8,e3,50,1) . fa_b_c_e(b3,m13,e3,50,1) .
 fa_b_c_e(b3,m14,e3,50,1) . fa_b_c_e(b3,m15,e3,50,1) .
 fa_b_c_e(b4,c1,e3,50,1) . fa_b_c_e(b4,c2,e3,50,1) .
 fa_b_c_e(b4,c3,e3,50,1) . fa_b_c_e(b4,c4,e3,50,1) .
 fa_b_c_e(b4,c5,e3,50,1) . fa_b_c_e(b4,c9,e3,50,1) .
 fa_b_c_e(b4,m10,e3,50,1) . fa_b_c_e(b4,c3,e1,50,1) .
 fa_b_c_e(b4,c4,e1,50,1) . fa_b_c_e(b4,c5,e1,50,1) .
 fa_b_c_e(b4,c9,e1,50,1) . fa_b_c_e(b4,m10,e1,50,1) .

Orders

orders(1,d1,c1,30,2) . orders(2,d1,c2,30,2) .
 orders(3,d2,c3,30,2) . orders(4,d1,c4,30,2) .
 orders(5,d3,c5,30,2) . orders(6,d1,c6,30,2) .
 orders(7,d4,c7,30,2) . orders(8,d1,c8,30,2) .
 orders(9,d5,c9,30,2) . orders(10,d1,c10,30,2) .
 orders(11,d6,c1,30,2) . orders(12,d2,c2,30,2) .
 orders(13,d8,c3,30,2) . orders(14,d2,c4,30,2) .
 orders(15,d9,c5,30,2) . orders(16,d2,c6,30,2) .
 orders(17,d10,c7,30,2) . orders(18,d2,c8,30,2) .
 orders(19,d2,c9,30,2) . orders(20,d2,c10,30,2) .
 orders(21,d3,c1,30,2) . orders(22,d3,c2,30,2) .
 orders(23,d3,c3,30,2) . orders(24,d3,c4,30,2) .
 orders(25,d3,c5,30,2) . orders(26,d3,c6,30,2) .
 orders(27,d3,c7,30,2) . orders(28,d3,c8,30,2) .
 orders(29,d3,c9,30,2) . orders(30,d3,c10,30,2) .
 orders(41,d4,c1,30,2) . orders(42,d4,c2,30,2) .
 orders(43,d4,c3,30,2) . orders(44,d4,c4,30,2) .
 orders(45,d4,c5,30,2) . orders(46,d4,c6,30,2) .
 orders(47,d4,c7,30,2) . orders(48,d4,c8,30,2) .
 orders(49,d4,c9,30,2) . orders(50,d10,m10,30,2) .
 orders(51,d5,c1,30,2) . orders(52,d11,c2,30,2) .
 orders(53,d5,c3,30,2) . orders(54,d12,c4,30,2) .
 orders(55,d5,c5,30,2) . orders(56,d13,c6,30,2) .
 orders(57,d5,c7,30,2) . orders(58,d14,c8,30,2) .
 orders(59,d5,c9,30,2) . orders(60,d15,c10,30,2) .

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