Proceedings of the International Multiconference on
Computer Science and Information Technology, pp. 201 – 205

ISBN 978-83-60810-14-9
ISSN 1896-7094

# A Hybrid Differential Evolution Algorithm to Solve a Real-World Frequency Assignment Problem

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***Abstract*—The Frequency Assignment is a very important task in the planning of the GSM networks, and it still continues to be a critical task for current (and future) mobile communication operators. In this work we present a hybrid Differential Evolution (DE) algorithm to solve a real-world instance of the Frequency Assignment problem. We present a detailed explanation about the hybridization method applied to DE in order to make it more efficient. The results that are shown use accurate interference information. That information was also adopted by other researchers and it represents a real GSM network, granting, therefore, an extremely important applicability. Furthermore, we have analyzed and compared our approach with other algorithms, obtaining good results.**

## I. Introduction

THE Frequency Assignment problem (FAP) represents an important task in the GSM networks *(Global System for Mobile)*. These networks are very used in the telecommunication area (by mid 2006 GSM services were used by more than 1.8 billion subscribers across 210 countries, representing approximately 77% of the world cellular market) [1].

The FAP problem is an NP-hard problem, therefore approaches using metaheuristic algorithms [2] have proven that they are a viable choice in its resolution.

The main goal of this problem is to be able to obtain an efficient use of the scarcely available radio spectrum on a network. The available frequency band is assigned into channels (or frequencies) which have to be allocated to the transceivers (TRXs) installed in each base station of the network. Both these components have an important role in the definition of this problem. This work is focused on the concepts and models used by the current GSM frequency planning [3]. Moreover, we adopted a developed formulation that takes advantage of realistic and accurate interference information from a real-world GSM network [4].

Our approach presented here uses the Differential Evolution (DE) algorithm as the fundamental step to solve this specific problem. Also, this algorithm w as hybridized with a Local Search method, as well as other features. Therefore, several modifications have been proposed to the original DE to improve its performance further. Also, as mentioned previously to this specific problem a Local Search (LS) method was added in order to be possible to obtain better results to solve this specific real-world FAP problem. It was also applied a method to guarantee that all obtained solutions do not have the most severe penalty, which influences the frequencies assignation inside a same sector of the network.

Our study was focused in a real-world instance of a GSM network named Denver which represents the city of Denver in the USA. It uses 711 sectors with 2612 transceivers (TRXs) installed. For each TRX the same 18 channels are available, representing the available frequencies for the assignation. As we have said, both these elements are the key elements of the problem, influencing directly the codification of the problem and the evaluation of the quality of the solutions accomplished. In a GSM network the TRXs give support to communications, making the conversion between the digital traffic data on the network side and radio communication between the mobile terminal and the GSM network [5].

This paper is structured as follows. In the next section we provide some details about the frequency assignment problem in the GSM networks. Section III describes the algorithm proposed and the hybridization methods applied to it. The results of the experiments are analyzed in Section IV. Finally, conclusions are discussed in the last section.

## II. Frequency Assignment in GSM Networks

In the following subsections we will first give a brief description about the elements present in a GSM network that are fundamental to the Frequency Assignment problem (FAP). Finally it will be presented the mathematical formulation followed by this work.

### *A. The FAP Problem within a GSM System*

A GSM network has several components, but the most important ones, to understand the frequency planning, are the antennas, which are more known as base station transceivers (BTSs) and the transceivers (or TRXs). Therefore, transceivers are the main element to be considered. A BTS can be viewed as a set of TRXs, which are organized in sectors. The

TRX is the physical equipment responsible for providing the communications between the mobile terminal and the GSM network.

The frequency assignment problem arises because the number of available frequencies (or channels) to be assigned to each TRX is very scarce. Therefore, the available frequencies need to be reused by many transceivers of the network [3][6]. The reuse of the frequencies can compromise the quality of the service of the network. Hence, it is extremely important to make an adequate reuse of the frequencies to the several TRXs, in such a way that the total sum of the interferences occurring in the network needs to be minimized.

Consequently, it becomes extremely important to quantify the interferences provoked by an assignation of a frequency to a TRX and its influence on the remaining TRXs of the other sectors of the network. To quantify this value an *interference matrix* is used, denoted $M$ [7]. Each element $M(i,j)$ of $M$ represents the degradation of the network quality if sector $i$ and $j$ operate with the same frequency value. This represents the *co-channel* interferences. It also need to be considered the *adjacent-channel* interferences, which occurs when two TRXs, in two different sectors, operate on adjacent channels (i.e., when one TRX operates on channel $f$ and the other on channel $f+1$ or $f–1$). Therefore, the interference matrix assumes an import role in the formulation of the FAP problem, which intents to minimize the sum of interferences occurring in the network. Thus, it is only possible by using the interference matrix to compute the cost function (see Eq. 1).

### B. Mathematical Formulation

Let $T = \{t_1, t_2, ..., t_n\}$ be a set of $n$ transceivers (TRXs), and let $F_i = \{f_{i1}, \ldots, f_{ik}\} \subset N$ be the set of valid frequencies that can be assigned to a transceiver $t_i \in T, i=1,\ldots,n$ (the cardinality of $F_i$ could be different to each TRX). Furthermore, let $S = \{S_1, S_2, ..., S_m\}$ be a set of given *sectors* (or cells) of cardinality $m$. Each transceiver $t_i \in T$ is installed in exactly one of the $m$ sectors and is denoted as $s(t_i) \in S$. It is also necessary the interference matrix, $M$, defined as: $M = \{(\mu_{ij}, \sigma_{ij})\}_{mxm}$. The two elements $\mu_{ij}$ and $\sigma_{ij}$ of a matrix entry $M(i,j) = (\mu_{ij}, \sigma_{ij})$ are numerical values and they represent the mean and standard deviation respectively, of a Gaussian probability distribution used to quantify the interferences on the GSM network when sector $i$ and $j$ operate on a same frequency. Therefore, the higher the mean value is, the lower interferences are, and thus it will have a superior communication quality.

A solution to the problem lies in assigning to all the TRXs $(t_i)$ a valid frequency from its domain $(F_i)$, in order to minimize the following cost function:

$$C(p) = \sum_{t \in T} \sum_{u \in T, u \neq t} C_{sig}(p, t, u) \qquad (1)$$

where $C_{sig}$ will compute the *co-channel* interferences $(C_{co})$ and the *adjacent-channel* interferences $(C_{adj})$ for all sector $t$ and $u$, in which the transceivers $t$ and $u$ are installed, that is, $s(t)$ and $s(u)$, respectively. $p \in F_1 x F2 x \ldots x F_n$ denotes a solution (or frequency plan), where $p(t_i) \in F_i$ is the frequency assigned to the transceiver $t_i$. Moreover, $\mu_{s_t s_u}$ and $\sigma_{s_t s_u}$ are the interference matrix values at the entry $M(s_t,s_u)$ for the sectors $s_t$ and $s_u$ . In order to obtain the $C_{sig}$ cost from equation 1, the following conditions are considered:

$$\begin{cases} k & \text{if } s_t = s_u, |p(t)-p(u)| < 2 \\ c_{co}(\mu_{s_t s_u}, \sigma_{s_t s_u}) & \text{if } s_t \neq s_u, \mu_{s_t s_u} > 0, |p(t)-p(u)| = 0 \\ c_{adj}(\mu_{s_t s_u}, \sigma_{s_t s_u}) & \text{if } s_t \neq s_u, \mu_{s_t s_u} > 0, |p(t)-p(u)| = 1 \\ 0 & \text{otherwise} \end{cases} \qquad (2)$$

where $K$ is a very large value, defined in the configuration files of the network. The $K$ value makes it undesirable to allocate the same or adjacent frequencies to TRXs that are installed in the same sector. In our approach to solve this problem, this restriction was incorporated in the creation of the new solution (frequency plan) produced by the DE algorithm. Therefore, we assure that the solution does not have this severe penalty, which causes the most undesirable interferences (see section III).

## III. Differential Evolution Algorithm

This section is devoted to present the DE algorithm and also its hybridization made by applying a Local Search (LS) method. We also explain the optimization made in the assignment of frequencies to TRXs that are inside the same sector.

### A. Original DE Algorithm

Differential Evolution is an Evolutionary Algorithm proposed by Storn and Price, and has been used successfully to solve optimization problems [8]. The DE has three major steps which are executed in each generation. They are the generation, evaluation and selection, each one involving different operations. These three steps are performed until a stop criterion is not reached. In DE each individual of the current population (named $x_{target}$ or $x_i$) will compete with a new individual $(x_{trial})$ generated by a mutation factor. An individual represents a solution, and it is encoded as a vector of integer values (of frequencies in our case). The DE starts with the creation of an initial population, normally at random.

New individuals are created applying the mutation and the recombination operators. At each generation, for each individual of the population is created a new solution $(x_{trial})$, using a weighted vector difference between two other individuals, selected randomly from the current population. This new individual is obtained e.g. using the equation 3, whose DE scheme name is DE/rand/1/$\beta$, where $\beta$ represents the crossover scheme that can be binomial *(bin)* or exponential *(exp)*. Different classifications of DE schemes are also available [9]. The available schemes use the notation DE/α/γ/β. The α represents the way in which individuals (necessary to obtain the new weighted differences vector, that is, the *trial* individual) are selected from the current population. They can be selected randomly *(rand)* or as the best individual from the current population *(best)*. The $\gamma$ is the value of difference vector pairs used, which normally are 1 or 2.

$$x_{trial}=x^t_{r3}+F\left(x^t_{r1}-x^t_{r2}\right) \tag{3}$$

Equation 3 is used to create a new solution, where $r1 \neq r2 \neq r3 \neq i$ are used as indices to index each parent vector. F value is the scaling factor, which controls the differential variation (mutation).

Besides the F parameter, it is also necessary the crossover parameter (CR). This last parameter represents a probability that influences the generation of the *trial* individual, by controlling the amount of genes which will be changed from the *target* individual to the new one (after apply the equation 3). It is also necessary to guaranty that all the genes changed will have a value inside the permitted limits. This corresponds to a very important step, in order to guaranty the creation of a valid solution. This characteristic was implemented by using a list of frequency values, containing all the ones that are not been used. Therefore they are available to be assigned to other TRX inside the same sector (each sector of this network problem has one list). Therefore, every time a TRX value is changed, its new value, which is generated by the DE (line 10, Fig. 2), is selected from that list containing the frequencies. This way we guaranty that the values are always inside the valid boundary values.

TABLE I.
FORMULATION FOR THE DIFFERENT SCHEMES OF THE DE ALGORITHM

| **Sheme Name** | **DE mutation definition** |
|---|---|
| DE/Rand/1/$\beta$ | $x^t_i=x^t_{r3}+F\left(x^t_{r1}-x^t_{r2}\right)$ |
| DE/Best/1/$\beta$ | $x^t_i=x^t_{best}+F\left(x^t_{r1}-x^t_{r2}\right)$ |
| DE/Rand/2/$\beta$ | $x^t_i=x^t_{r5}+F\left(x^t_{r1}+x^t_{r2}-x^t_{r3}-x^t_{r4}\right)$ |
| DE/Best/2/$\beta$ | $x^t_i=x^t_{best}+F\left(x^t_{r1}+x^t_{r2}-x^t_{r3}-x^t_{r4}\right)$ |
| DE/RandToBest/1 | $x^t_i=x^t_{r3}+F\left(x^t_{best}-x^t_{r3}\right)+F\left(x^t_{r1}-x^t_{r2}\right)$ |

The creation of the trial individual is the main characteristic of DE. As we have mentioned previously, several schemas are available [8][9]. The main difference between them is the fact that: they can use different vectors pairs; and also that they could use only randomly-selected individuals or choose to use the best individual obtained so far. Table 1 summarizes the several available schemes, where *t* represents the $t^{th}$ generation and *r1, r2,* etc. are randomly selected individuals from the population. Steps from line 6 to line 14 (see Fig. 2) may vary depending on the scheme which is being used.

## *B. Hybrid Characteristics*

In order to optimize the results (the frequency plan) two methods have been implemented on the original DE. The first optimization consisted in guarantying that for all the TRXs inside the same sector will not be assigned the same or an adjacent frequency value, because it will provoke severe penalties – these co-channel and adjacent-channel interferences are the highest-cost interferences, as shown in Eq. 2). The second method consisted in applying a Local Search (LS) method, which was adapted to this FAP problem [10]. As described above, this method permits to optimize the assignment of frequencies to every TRX in each sector of a given solution. Also, a more detailed explanation about the LS applied to the FAP problem can be consulted in the reference [10].

The first optimization is used in the creation of the initial population (line 2, Fig. 2) and every time a new solution is created. That occurs every time some of the genes of the solution are changed (line 10 and line 11, Fig. 2). We have optimized this implementation through the incorporation, for each solution, of a dynamic list containing all the available frequencies in each sector. Also, in the LS method we guaranty that the same frequency value or an adjacent frequency is not assigned to two TRXs within the same sector. Therefore, any time a change is made in a solution it is assured that the co-channel and the adjacent-channel interferences are not present. This requires that every time there is made a change to the frequency assigned to a TRX, the list of available frequencies to which belongs the TRX need to be updated (each sector of the network has is one list of available frequencies). Assigning a frequency value *f* to a TRX, originates that also the *f-1* and *f+1* values will be eliminated from the list of available frequencies in the sector of the TRX. To the reverse process (release a frequency value from a TRX) it is necessary to verify that no others TRXs in the same sector are using the *f+1* and *f-1* frequencies values.

Using this approach it is no longer necessary to verify, for each sector, which are the frequencies that are not been used. All that information is always updated in the list of the available frequencies of that sector. Using this approach it was possible to make the process much more efficient.

The second optimization consists in apply the LS method to a solution (frequency plan). Therefore, in a previous work we have conducted several experiments in order to identify what is the most efficient way to apply the LS in the DE [11]. Two hypotheses were considered. The first one consisted in applying the LS to all individuals in the initial population (line 2, Fig. 2) and also after creating every $x_{trial}$ individual (apply the DE scheme, line 10, Fig. 2). The second approach consisted in only applying the LS method after creating every $x_{trial}$ individual.

Due to the experiments carried out the selected strategy was the second approach, because it was the one that permitted obtain the most successfully results, that is solutions with a lower cost [11]. To accomplish these results, and as mentioned previous, several runs using the same DE parameters, were performed in order to identify the location to apply the LS method into the DE algorithm. This experiments were made using a common set of configuration parameters of DE (NP=10; CR=0.4 and F=0.5 and the scheme *DE/Rand/2/Bin*).

```
generation t = 0
population P_t = createInitialPopulation(NP)
evaluate(P_t)
while (not reached stop criterion)
   for i = 0 to NP do
      select randomly r1≠r2≠r3≠i
      x_target = x_i
      for j = 1 to D do //dimension of the problem
         if (randj[0,1] < CR or j = j_rand) then
            x_trial,j = x_r3,j + F (x_r1,j – x_r2,j)        //DE scheme
            x_trial,j = applyLocalSearch( x_trial,j )
         else
            x_trial,j = x_i,j
         end if
      end for
      if ( f(x_trial) < f(x_i)) then
         x_i = x_trial
      else
         x_i = x_i
      end if
   end for
   t = t + 1
end while
```

Fig 1.DE algorithm with the LS method, using the DE/Rand/1/Bin schema. NP, CR and F are user-defined parameters. NP is the population size.

### C. Solution Encoding

The solution encoding carried out to this problem incorporates the characteristics of this instance of the FAP problem. Therefore, a solution has a dimension equal to the total number of TRXs of the GSM network instance used (in this case 2612, representing the number of TRXs from the network). For every TRX, a frequency (also named channel) has to be assigned. Each TRX has a set of available frequencies. Hence, a solution plan is encoded as a list of integers values $p$, where $p(t)$ is the frequency value assigned to TRX $t$.

Each time the DE algorithm changes the value assigned to a TRX, it ensures that the new value is inside the set of the available frequencies for that specific TRX. It also assures that the new value is not assigned to other TRXs installed inside the same sector. According to the made experiments, with this approach it was possible to achieve better results because it is always available an updated list of the unused frequencies.

## IV. Experiments

Our algorithm was implemented in C# using also the Microsoft .NET Framework 3.5. Results were obtained on a PC with a Pentium-4 CPU at 3.2GHz and with 2 GB of RAM, running Windows XP.

The experiments were developed over a real-world instance, named Denver, which has 711 sectors with 2612 TRXs to be assigned a frequency. Each TRX has 18 available channels (from 134 to 151). We only use this dataset in order to be able to compare our results with the results already accomplished for this problem. At this moment is only available this single instance. In future work, when available more real-world instances of the problem we intent to use them.

Fig. 2 displays the network topology, and every triangle represents a sectorized antenna in which operate several TRXs. The interference matrix is the one also used by [4] [10].

All the available information about this instance problem is available in separated configuration files.

### A. Parameterization

Several user-defined parameters are necessary and in the following we present the results achieved with experiments that were performed in order to identify the best configuration parameters to be used by DE. In a previous work [11], we have identify what were the best parameters to be used bt the DE algorithm. The parameters used are the population size (NP), crossover probability (CR), mutation factor/weighting factor (F) and the the DE strategy/scheme. Also, we have determined the best way to apply the local search method in the DE.

The best parameter settings used are a population size (NP) of 10 individuals, a crossover probability (CR) of 0.2, a mutation factor (F) of 0.1, the DE/Rand/2/bin scheme and applying the LS method after the creation of every $x_{trial}$ individual.

We specified as stopping criterion a 30 minutes of execution. In order to provide the results with statistical confidence, we have considered 30 independent runs for each experiment. For each run, and also at every 2 minutes, we have obtained the average, best and standard deviation values.

### B. Results

In order to provide results to be compared with other authors, we have focused on three different time limits (120, 600 and 1800 seconds). Therefore, it is possible to compare different algorithms within short and long time ranges.

According to previous results [10] the more efficient way to create the initial population is randomly but removing the most costly interferences (co-channel and adjacent-channel) within the same sector. After proving the efficiency of this approach to the DE, we used it to initiate the DE algorithm. Initially, without using this approach the accomplished results were not so satisfactory.

After tuning the DE parameters [11] and incorporating also some optimized methods, the conducted experiments showed that using the Local Search method within the algorithm has a very important role in its performance.

According to the formulation presented in section II, the results are given by using the cost function, meaning that a frequency plan with a small value cost is a better frequency plan.

Our results were obtained with the hybrid version of the DE algorithm, which has been described in section III. The best result obtained was 87845.9 cost units (after 30 independent runs, each one with 30 minutes).

Table 2 shows a resume of the results accomplished for this problem by other algorithms with similar characteristics (implemented by other authors), and also our DE algorithm. Here, we show the Ant Colony Optimization algorithm (ACO) and the Scatter Search (SS) [10]. Both have already

TABLE II.
EMPIRICAL RESULTS OF THE METAHEURISTICS FOR 3 DIFFERENT TIME LIMITS. IT SHOWS THE BEST, AVERAGE AND STANDARD DEVIATION OF 30 EXECUTIONS

| Time Limit | 120 seconds | | | 600 seconds | | | 1800 seconds | | |
|---|---|---|---|---|---|---|---|---|---|
| | Best | Average | Std. | Best | Average | Std. | Best | Average | Std. |
| **ACO** | **90736.3** | **93439.5** | 1318.9 | 89946.3 | 92325.4 | 1092.8 | 89305.9 | 90649.9 | 727.5 |
| **SS** | 91216.7 | 94199.6 | 1172.3 | 91069.8 | 93953.9 | 1178.6 | 91069.8 | 93820.4 | 1192.3 |
| **DE** | 92145.8 | 95414.2 | 1080.4 | **89386.4** | **90587.2** | 682.3 | **87845.9** | **89116.8** | 563.8 |

been tested to solve the FAP problem, and they have shown a good performance for this problem.

Analyzing these results, we can conclude that DE obtains the best results for 600 and 1800 seconds, although it is not the algorithm with the best start. Furthermore, with our approach it was also possible to improve the results obtained in [4][12] (where other metaheuristics were used: (1,10) Evolutionary Algorithm, PBIL –Population Based Incremental Learning-, …). In conclusion, the results are quite better that others already achieved for this instance of the problem.

## V. CONCLUSION

In this paper we present a hybrid Differential Evolution (DE) algorithm for solving the frequency assignment problem (FAP) for a real GSM network. It was used a mathematical formulation adopted by other researchers [4]. In this way, it is possible the make comparisons between the results accomplished by this algorithm and others.

In this work, DE was modified to incorporate a Local Search method (i.e., optimizing the assignment of frequencies to every TRX in each sector of the solution).

The results have shown that this hybrid version of the DE algorithm, with the additional features incorporated, clearly makes possible to obtain a viable solution, when compared with other algorithms proposed by other authors. It also shows that the results of the modified DE are better compared to the original one. The proposed modifications have improved the computational efficiency of the DE algorithm to solve the approached problem.

Therefore, it is possible to conclude that the results evolution and the final results for the DE algorithm are very positives.

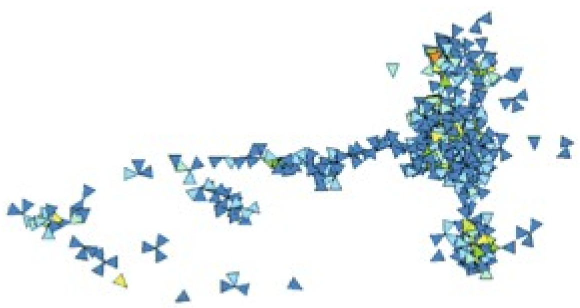

Fig 2.Topology of the GSM instance used

Future work includes the study of other evolutionary algorithms (like VNS -Variable Neighborhood Search-) to make deeper analysis with more different metaheuristics. Furthermore, we will work with more real-world instances, in order to evaluate the algorithms using different instances. Finally, the formulation of the FAP problem as a multiobjective optimization problem will be investigated as well.

## ACKNOWLEDGMENT

This work was partially funded by the Spanish Ministry of Education and Science and FEDER under contract TIN2005-08818-C04-03 (the OPLINK project). Thanks also to the Polytechnic Institute of Leiria, for the economic support offered to Marisa Maximiano to make this research.

## REFERENCES

[1] Wireless Intelligence (2006), http://www.wirelessintelligence.com
[2] C. Blum, A. Roli, "Metaheuristics in Combinatorial Optimization: Overview and Conceptual Comparison", ACM Computing Surveys 35, 2003, pp: 268–308.
[3] A. Eisenblätter, "Frequency Assignment in GSM Networks: Models, Heuristics, and Lower Bounds", PhD thesis, Technische Universität Berlin, 2001.
[4] F. Luna, C. Blum, E. Alba, A.J. Nebro, "ACO vs EAs for Solving a Real-World Frequency Assignment Problem in GSM Networks.", GECCO'07, 2007, pp: 94 – 101, London, UK.
[5] F. Luna, E. Alba, A. J. Nebro & S. Pedraza, Heidelberg, S. B. /. (ed.) Evolutionary Algorithms for Real-World Instances of the Automatic Frequency Planning Problem in GSM Networks Springer Berlin / Heidelberg, 2007, 4446/2007, 108-120.
[6] A. R. Mishra, "Fundamentals of Cellular Network Planning and Optimisation: 2G/2.5G/3G... Evolution to 4G", chapter: Radio Network Planning and Opt., pp: 21-54. Wiley, 2004.
[7] A. M. J. Kuurne, "On GSM mobile measurement based interference matrix generation", IEEE 55th Vehicular Technology Conference, VTC Spring 2002, 2002, pp: 1965-1969.
[8] K. Price, R. Storn., DE website. http://www.ICSI.Berkeley.edu/~storn/code.html, 2008.
[9] K. Price, R. Storn, "Differential Evolution: A Simple Evolution Strategy for Fast Optimization.", Dr. Dobb's Journal, 1997, 22(4): 18-24.
[10] F. Luna, C .Estébanez, et al., "Metaheuristics for Solving a Real-World Frequency Assignment Problem in GSM Networks". GECCO'08, July 2008, pp: 1579 – 1586, Atlanta, GE, USA.
[11] M. Maximiano, M.A. Vega-Rodríguez, et al., "Analysis of Parameter Settings for Differential Evolution Algorithm to Solve a Real-World Frequency Assignment Problem in GSM Networks", The Second International Conference on Advanced Engineering Computing and Applications in Sciences (ADVCOMP'08),Valencia, Spain, 2008.
[12] J. M. Chaves-González, M.A. Vega-Rodríguez, et al.. "SS vs PBIL to Solve a Real-World Frequency Assignment Problem in GSM Networks", Applications of Evolutionary Computing. Springer. LNCS, volume 4974/2008, pp: 21-30, Berlin / Heidelberg, 2008