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Efficient Coloring of Wireless Ad Hoc Networks With Diminished Transmitter Power.

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***Abstract*—In our work we present a new approach to the problem of channel assignment in Wireless Ad Hoc Network. We introduce a new algorithm which works in distributed model of computations on Unit Disc Graphs modeling the Wireless Ad Hoc Network. The algorithm first modifies the transmitting power of the devices constituting the network (radii of the vertices of the Unit Disc Graph) and then it assigns the frequencies in the network adjusted to our demands. We assume that initially all the devices have the same transmission range and we are able to reduce the transmission range of some of them in order to decrease the number of necessary frequencies. We are able to diminish the number of communication links without loosing connectivity of the network, i.e. reduce the number of possible interference threats without risk of loosing possibility to exchange information. In addition, the diminution of communication range reduce the power consumption of the transmitting devices.**

***Index Terms*—Diminished Transmitter Power, Distributed Algorithm, Wireless Ad Hoc Networks, Coloring**

I. Introduction

THE WIRELESS Ad Hoc Network consists of the devices having computational ability placed on the given area (for example wireless cellphone network formed by the cellphones). We consider the problem of channel assignment in the Wireless Ad Hoc Network. In the proper channel assignment each vertex transmits information using one channel and without interference. An interference may occur due to two possible collisions. The primary one have place when a vertex simultaneously transmits and receives signals over the same channel. The secondary one occurs when a vertex simultaneously receives more than one signals over the same channel. Thus, to prevent the primary collision, two vertices may be assigned the same channel if neither of them is within the transmission range of the other. Similarly, to prevent the secondary collision, two vertices can be assigned the same channel if no other vertex is located in the intersection of their transmission ranges. The crucial problem concerning Wireless Ad Hoc Networks is finding the appropriate channel assignment. This problem is equivalent to determining optimal proper coloring of the square of graph representing the network.

In the standard model of the Ad Hoc Network each device of the network is placed in some point of the given area and all the devices have the same transmitting power. Therefore a widely used model for such a network is the Unit Disc Graph (UDG) model. The Unit Disc Graph is a graph in which the set of vertices is a set of vertices on the plane and two vertices are connected by an edge if and only if they are at the distance at most one in Euclidian norm. The optimal coloring (which bound from below minimum number of channels in channel assignment) of any Unit Disk Graph uses at least $\omega(G)$ colors, where $\omega(G)$ is the size of the largest clique. Moreover the chromatic number of UDG G $(\chi(G))$ is between $\omega(G)$ and $3\omega(G)$ (see [1]). Obviously, for dense graphs, the number $\omega(G)$ may be large, thus the number of channels necessary to ensure appropriate communication is large as well. The algorithms, working in various models, finding proper colorings of the Disk Graphs may be found in [3], [5], [6] and [11].

Surely, the absorbing issue is how to change the characteristics of the Wireless Ad Hoc Network which is represented by the graph with a large chromatic number in order to assure good communication using small number of frequencies. An effective approach to minimize the number of assigned channels is to diminish power of the transmitters installed in the devices (make the radius of transmission smaller). This concept has already appeared to be beneficial in total energy reducing problem (see [4],[9],[8],[10],[14],[17],[18],[19]) and interference reduction (see [2],[7],[12]). Diminishing power of transmitters enables to save energy of the batteries in devices, since during the transmission the full power of the transmitters is not used. Moreover such procedure reduce number of edges in the corresponding Unit Disc Graph and, since the graph is sparser, decrease the chromatic number. However diminishing transmission power may have disadvantageous impact on the connectivity of the network. Due to reduction of the number of active links the network may be partitioned (i.e. the corresponding graph would become disconnected) and some messages may never be delivered. Therefore our goal is to create an efficient algorithm which diminish powers of transmitters keeping strong connectivity of the graph and significantly reducing the chromatic number of the graph. To the best of our knowledge, in our algorithm the concept of diminishing transmitting power for solving the problem of channel assignment is used for the first time. Moreover this is fast distributed topology control algorithm which combine ideas of coloring and reducing total energy.

Our work is organized as follows: in Section II we introduce the model of computation in which the algorithm works. In Section III we sketch the main results of our work. In

Section IV we present the main algorithm and show that it works properly. In the last section we show that the number of colors used by our algorithm is the best possible.

II. Model

The presented algorithm will work in distributed synchronous model of computations on Unit Disc Graphs with possible reduction of radii length. The Unit Disc Graphs (UDG) is a wiely used Wireless Ad Hoc Networks. The set of vertices of UDG is a set of vertices on the plane and any vertex v is connected by an edge with the vertex w if and only if the vertex w is contained in the disk of radius 1 with a center in v. In our model we make also additional assumption that every vertex can diminish its radius. We introduce a function $f : V(G) \to (0, 1]$ assigning to each vertex its reduced radius. The value $f(v)$ will be interpreted as the power of the transmitter v. Given the Unit Disc Graph G and the function $f : V(G) \to (0, 1]$ we define the directed Disc Graph $G\langle f\rangle$ as follows: $V(G\langle f\rangle) = V(G)$ and in $G\langle f\rangle$ there is an edge pointing from v to w if and only if w is contained in the circle of radius $f(v)$ and a center in v (i.e. $vw \in E(G\langle f\rangle) \Leftrightarrow \|v, w\| \le f(v)$). The existence of the edge vw in $G\langle f\rangle$ is interpreted as the fact that vertex v can send information to w (w is in the communication range of v).

We make also additional assumption that all the devices constituting the network are equipped with the Global Positioning System (GPS), or know their position on the plane by other sources. We also assume that the network is synchronized and in one round a vertex can send, receive messages from its neighbors and can perform some local computations. Neither the amount of local computations nor the size of messages sent is restricted in any way. Concluding, the algorithm will work in a synchronous, message-passing model of computations introduced in [13], called LOCAL model, in which we additionally know the position on the plane of each element of the network.

III. Previous Work and Our Result

The presented algorithm DiminishPower works in the model introduced in Section II and finds a function f and channel assignment such that the network represented by $G\langle f\rangle$ is connected and in the network the number of necessary frequencies and energy consumption is significantly diminished comparing to the network with initial characteristics.

In order to formalize our arguments we give two additional definitions. We say that the directed graph $G\langle f\rangle$ is *strongly connected* if for all $v, w \in V(G\langle f\rangle)$ there exists a directed path form v to w. Strong connectivity of $G\langle f\rangle$ will imply the possible communication between any two devices constituting the network represented by $G\langle f\rangle$. The second definition concerns the amount of energy consumed by the network. By *total energy cost* we will mean the value $TE(G\langle f\rangle) = \sum_{v\in V(G\langle f\rangle)} f(v)^2$. The concept of saving energy in wireless networks was introduced in [15], [20]. Our definition of total energy cost includes the value f in second power since in the real two dimensional space the area of transmission is of order square of the power of the transmitter.

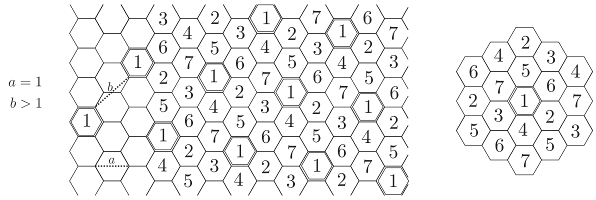

Fig. 1. Division of the plane into 7 classes of hexagons.

The idea of the algorithm DiminishPower is to divide the plane into 7 classes of hexagons with diameter 1 as in Figure 1. To each class we will attribute a different palette of colors. The subgraph of UDG G induced by the vertices contained in one hexagon form a clique. We will use that property of the division and in each hexagon we will run independently the procedure OnClique, which will locally color the vertices of the hexagon with colors from the palette attributed to that hexagon and assign the value of the function f to each vertex v from the given hexagon. The function will be such that the subgraph of $G\langle f\rangle$ induced by the vertices from a hexagon will be strongly connected and the coloring constructed by the procedure will be proper coloring of that subgraph. Finally we will join local colorings and obtain the proper colloring of $G\langle f\rangle$.

We will prove that for a given connected Unit Disc Graph G the algorithm DiminishPower finds in constant number of synchronous rounds in LOCAL model (see Theorem 8) a function $f : V(G) \to (0, 1]$ such that:

1) $G\langle f\rangle$ is strongly connected (see Theorem 5);
2) $G\langle f\rangle$ has proper coloring with $O(\log(\chi(G)))$ colors (see Theorem 6);
3) total energy cost of the wireless network represented by $G\langle f\rangle$ is of range
$$TE(G\langle f\rangle) = O\left(|MIS(G)| \log \frac{|V(G)|}{|MIS(G)|}\right)$$
(see Theorem IV) , where $|MIS(G)|$ is the size of the maximum independent set in G.

Moreover there exists a channel assignment with the above properties (see Remark 9). Another important property of the construction is possibility of defining an effective routing protocol in $G\langle f\rangle$ (see Remark 10).

In comparison M. Fussen, R. Wattenhofer and A. Zollinger in [7] present the Algorithm NCC which finds strongly connected $G_{ncc}\langle f\rangle$, i.e. the graph which has property 1. However the maximum degree in $G_{ncc}\langle f\rangle$ is $O(\log(|V(G)|))$ (in $G\langle f\rangle$ it is $O(\log(\omega(G)))$). In their approach they do not solve problems of coloring, channel assignment and do not compute total energy. The Algorithm NCC strictly base on one predefined global sink and its distributed implementation takes $O(diameter(G))$ synchronous rounds (our algorithm takes $O(1)$ time). Also the graph constructed by the Algorithm NCC do not allow to define fast routing protocol (see Remark 10).

Moreover vertices in one hop distance in G can be at distance of $O(diameter(G)\log(|V(G)|))$ hops in $G_{ncc}\langle f\rangle$ (in $G\langle f\rangle$ they are at most $O(\log(\omega(G)))$ hops away).

IV. Main algorithm

First we present the procedure OnClique, which will be implemented independently on each subgraph induced by the vertices contained in a hexagon. We assume that the input graph K is a clique, therefore information about all the vertices may be send to one vertex of the clique and all the computations may be performed by that vertex. In the algorithm and later on we will call a pair of vertices vw a *double directed edge* if in the directed graph there are edges pointing from v to w and pointing from w to v.

OnClique
Input: Unit Disc Graph K which is a clique and palette $P(K)$ of colors.
Output: Disc Graph $K\langle f\rangle$ and its coloring.

(1) Let $i := 1$, $M_i = V(K)$, $f \equiv 1$ and $K\langle f\rangle[M_i]$ be the graph induced by the set of vertices M_i.
(2) For all $v \in M_i$ set $f(v) := \min_{w\in M_i, w\neq v}\|v,w\|$.
(3) Color graph $K\langle f\rangle[M_i]$ using new 5 colors from palette $P(K)$.
(4) Let N be a subgraph of $K\langle f\rangle[M_i]$ induced by all double directed edges.
(5) $i := i+1$.
(6) Set M_i to be the set of leaders of the connected components of N.
(7) If $|M_i| \geq 2$ then go to step (2).
(8) If $M_i = \{v\}$ then $f(v) := 1$ and color v with a new color from $P(K)$.
(9) Return $K\langle f\rangle[V(K)]$ with coloring.

It should be mentioned that it is possible to implement the step (3) of the procedure, since the graph $K\langle f\rangle[M_i]$ is planar (see the proof of Lemma 3).

In addition the graph N is always well defined (i.e. has at least one vertex) and, as a consequence, for all i the set M_i contains at least one element. It is necessary for correctness of the algorithm. It is enough to notice that for all M_i such that $|M_i| \geq 2$ and f defined as in (2) the graph $K\langle f\rangle[M_i]$ has at least one double directed edge. From definition of f in $K\langle f\rangle[M_i]$ each vertex has out–degree at least one, therefore in $K\langle f\rangle[M_i]$ there is at least one cycle (the last vertex from the longest path has an out–neighbour on that path). Moreover, from definition of f each cycle in $K\langle f\rangle[M_i]$ consists of double directed edge. More precisely if $v_1,\ldots,v_t,v_1$ is a cycle, then: $f(v_1) = \min_{w\in M_i,w\neq v_1}\|v_1,w\| = \|v_1,v_2\| \geq f(v_2) = \|v_2,v_3\| \geq \ldots \geq f(v_t) = \|v_t,v_1\| \geq f(v_1)$ thus all the inequalities above are equalities and all the edges of the cycle are double directed.

Lemma 1. *Let $V(K) = M_1 \supseteq M_2 \supseteq M_3 \supseteq \ldots \supseteq M_k$ be the sequence of all the subsets of $V(K)$ constructed by the algorithm* OnClique. *Then $|M_i| \geq 2\left|M_{(i+1)}\right|$, $k \leq \log_2(|V(K)|)$ and $|M_k| = 1$.*

Proof: Since N is a subgraph of $K\langle f\rangle[M_i]$ induced by some edges, each connected component of N contains at least two vertices. M_{i+1} contains exactly one vertex from each component of N. Therefore $|M_i| \geq 2\left|M_{(i+1)}\right|$ for all $1 \leq i \leq k$. Consequently we have $k \leq \log_2(|V(K)|)$. $|M_k| = 1$ since for all i the set M_i contains at least one element. ∎

Lemma 2. *The graph $K\langle f\rangle$ constructed by the algorithm* OnClique *is strongly connected.*

Proof: Let $V(K) = M_1 \supseteq M_2 \supseteq M_3 \supseteq \ldots \supseteq M_k = \{m\}$ be the sequence of all the subsets of $V(K)$ constructed by the algorithm OnClique. Let $w = w_1$ be any vertex from $V(K)$. We only need to prove that in $K\langle f\rangle[V(K)]$ after the last iteration there is a directed path from w to m. This combined with the fact that in $K\langle f\rangle$ there are directed edges pointing from m to all other vertices (since $f(m) = 1$) implies strong connectivity.

In the first iteration in (2) we set $f(v) := \min_{w\in K,w\neq v}\|v,w\|$ for all $v \in V(K) = M_1$. By definition of $f(v)$ every vertex v in $K\langle f\rangle[M_1]$ have out-degree at least 1. Notice that this implies that there exists a directed path from $w = w_1$ to a vertex w'_1 which is incident to a double directed edge. We just have to take w'_1 – the last vertex of the longest directed path with origin in w_1 and notice that w'_1 have to have a neighbour on that path. Thus w'_1 is contained in directed cycle and, as mentioned before, cycles in $K\langle f\rangle[M_1]$ consist of double directed edges or are a double directed edges. If w_2 is the leader of the connected component of N (defined as in (4) during the first iteration) containing w'_1, then there exists a directed path from w_1 to $w_2 \in M_2$.

Using the same reasoning but replacing M_1 by M_i ($i > 1$) and the first iteration by the i-th one, we can prove that there exists a directed path from $w_i \in M_i$ to $w_{i+1} \in M_{i+1}$ in $K\langle f\rangle[M_i]$ defined in the i-th iteration. Since $f(v)$ in the next iterations may only grow, therefore the above–considered directed paths from $w = w_1$ to w_2, from w_2 to w_3, …, from w_{k-1} to $w_k = m$ are also directed paths in the final graph $K\langle f\rangle[V(K)]$. ∎

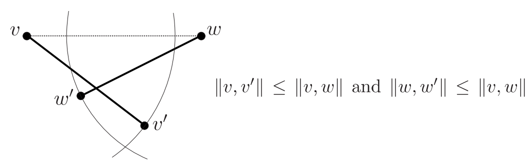

Fig. 2. $\|v,w'\| < \|v,v'\|$

Lemma 3. *The algorithm* OnClique *finds a coloring of the graph $K\langle f\rangle$ using at most $5\log_2(|V(K)|)$ colors from the palette.*

Proof: First observe that in the i-th iteration of the algorithm the graph $K\langle f\rangle[M_i]$ is plane. Otherwise there would exist two directed edges vv' and ww' which would cross each other. So by definition of f we know that $\|v, v'\| \le \|v, w\| \,\&\, \|v, v'\| \le \|v, w'\|$ and $\|w, w'\| \le \|w, v\| \,\&\, \|w, w'\| \le \|w, v'\|$. However if we suppose that $\|v, v'\| \le \|v, w\|$ and $\|w, w'\| \le \|v, w\|$ then w' would lie inside the circle with a center in v and the radius of length $\|v, v'\|$ (see Figure 2). So $\|v, w'\| < \|v, v'\|$, a contradiction.

It is possible to color any planar graph using 5 colors in polynomial time. In each iteration we use at most 5 new colors and if in iteration we enlarge the value of $f(v)$ then in the same iteration the color of v is changed and a new color have not been used in previous iterations. Therefore the output coloring is a proper coloring of $K\langle f \rangle$. By Lemma 1 we know that there are at most $\log_2(|V(K)|)$ iterations, thus the coloring use at most $5\log_2(|V(K)|)$ colors. ■

Lemma 4. *The graph $K\langle f \rangle$ constructed by the algorithm* OnClique *has total energy cost $TE(K\langle f \rangle) \le 16\log_2(|V(K)|)$*

Proof: Let $V(K) = M_1 \supseteq M_2 \supseteq M_3 \supseteq \ldots \supseteq M_k$ be the sequence of the subsets of $V(K)$ constructed by the algorithm OnClique. Let $1 \le i \le k$ and $\{v_1, v_2, \ldots, v_{|M_i|}\} = M_i$. During the i-th iteration we set $f(v) := \min_{w \in M_i, w \ne v} \|v, w\|$ for all $v \in M_i$, therefore $f(v_t) \le \|v_t, v_l\|$ and $f(v_l) \le \|v_t, v_l\|$ for all $1 \le k \ne l \le |M_i|$. This implies $f(v_t)/2 + f(v_l)/2 \le \|v_t, v_l\|$ for all $1 \le k \ne l \le |M_i|$. So, if we denote by $c(v, r)$ the disk with the center in v and the radius of length r, then all the disks $c(v_1, f(v_1)/2)$, $c(v_2, f(v_2)/2)$, …, $c(v_{|M_i|}, f\left(v_{|M_i|}\right)/2)$ are pairwise disjoint. Moreover K is a clique, thus v_1 is connected by an edge with all other vertices of K. Therefore, since K is UDG, all of the vertices from K are contained in $c(v_1, 1)$. This implies that all the disks $c(v_1, f(v_1)/2)$, $c(v_2, f(v_2)/2)$, …, $c(v_{|M_i|}, f\left(v_{|M_i|}\right)/2)$ lie inside $c(v_1, 2)$. Therefore

$$\pi 2^2 \ge \sum_{j=1,2,\ldots|M_i|} \pi \left(\frac{f(v_j)}{2}\right)^2 .$$

So $TE(M_i) \le 16$.

We use the same reasoning for all iterations and for all $v \in V(K)$ the value $f(v)$ from the outcome graph is set in one of those iterations, therefore we have $TE(K\langle f \rangle) \le \sum_{j=0,1,\ldots,k} TE(M_i) \le \sum_{j=0,1,\ldots,k} 16 = 16\log_2(|V(K)|)$. By Theorem 1 the number of iterations $k \le \log_2(|V(K)|)$. ■

Now we are ready to introduce the main algorithm. In the algorithm, for all $i = 1, 2, \ldots 7$, we will define C_i to be the set of all connected components of the subgraph of the input UDG G induced by the vertices contained in the haxagons of the i-th class (see Figure 1). Obviously each of those connected components is a clique induced by the vertices from one hexagon of the i-th class.

DiminishPower
Input: Connected Unit Disc Graph G
Output: Disc Graph $G\langle f \rangle$ with proper coloring.

(1) Divide the set of vertices of G into 7 classes as in Figure 1.
(2) For each $K \in C_i$ define palette of colors $P(K)$ as follows $P(K) = \{x \in \mathbb{N} : x = i \pmod 7\}$.
(3) Let $\mathcal{C} := C_1 \cup C_2 \cup \ldots \cup C_7$ For each $K \in \mathcal{C}$ parallel do:
    (a) Run algorithm OnClique on graph K and color it using palette $P(K)$.
    (b) Define $\mathcal{K}' := \{K' \in \mathcal{C} : K' \ne K$ and $\exists_{k \in V(K), k' \in V(K')} kk' \in E(G)\}$. Denote the graphs from $\mathcal{K}'$ by $K'_1, K'_2, \ldots, K'_{|\mathcal{K}'|}$.
    (c) For $j = 0, 1, \ldots |\mathcal{K}'|$ do:
        (c1) Select one vertex $w_j \in V(K)$ such that $\exists_{w'_j \in V(K'_j)} w_j w'_j \in E(G)$.
        (c2) Color w_j taking first free color from the palette $P(K)$
        (c3) Set $f(w_j) = 1$.

The following theorems show that the algorithm has the properties claimed in Section III.

Theorem 5. *Given the connected Unit Disk Graph G as an input, the algorithm* DiminishPower *finds the graph $G\langle f \rangle$ which is strongly connected.*

Proof: Let $K, K' \in \mathcal{C}$. By Lemma 2 and the fact that during the algorithm DiminishPower in step 3(c) we only may enlarge the radius of some vertices from $V(K)$ and $V(K')$, thus in the output graph $G\langle f \rangle$ the subgraphs induced on vertices from $V(K)$ and $V(K')$ are strongly connected. Moreover, if in G there were edges between $V(K)$ and $V(K')$, then after step 3(c3) in $G\langle f \rangle$ there is at least one edge pointing from $V(K)$ to $V(K')$ and at least one edge pointing from $V(K')$ to $V(K)$. Therefore, if G was connected, then $G\langle f \rangle$ is strongly connected. ■

Theorem 6. *Given Unit Disc Graph G algorithm* DiminishPower *finds a proper coloring for graph $G\langle f \rangle$ which use $O(\log(\chi(G)))$ colors.*

Proof: By Lemma 3 we know that in step (a) of the algorithm DiminishPower for each $K \in \mathcal{C}$ we use at most $5\log_2(|V(K)|)$ colors from palette $P(K)$. Moreover $|\mathcal{K}'| \le 18$ (see Figure 1), therefore for any graph $K \in \mathcal{C}$ in step 3(c2) we use at most 18 colors from palette. Since there are 7 types of palettes and $|V(K)| \le \omega(G)$ (K is a clique), during the whole algorithm at most $\max_{K \in \mathcal{C}} 7\left(5\log_2(|V(K)|) + 18\right) = O\left(\log(\omega(G))\right)$ colors are used. The result follows from the fact that $\omega(G) \le \chi(G) \le 3\omega(G)$ (see [1]).

By Lemma 3 the coloring constructed by the procedure OnClique is a proper coloring of the output graph of the procedure. Moreover in step 3(c), if we change the radius of the vertex, then we color that vertex with a new color, which have not been used before in K. Therefore, for any $K \in \mathcal{C}$, the coloring constructed by the algorithm DiminishPower is a proper coloring of the subgraph of $G\langle f \rangle$ induced by the vertices from $V(K)$. To conclude that this coloring is also a proper coloring of $G\langle f \rangle$ we only have to notice that in the communication range of the vertices from K exept other vertices from K there are only vertices from neighboring hexagons (see Figure 1). Those vertices are from different type then K, therefore their colors are from different palettes. ■

Theorem 7. *Given as an input a connected Unit Disc Graph G the algorithm* DIMINISHPOWER *finds graph $G\langle f\rangle$ such that the total energy cost of $G\langle f\rangle$ equals*

$$TE(G\langle f\rangle) = O\left(|MIS(G)|\log\left(\frac{|V(G)|}{|MIS(G)|}\right)\right),$$

where $|MIS(G)|$ is the size of the maximum independent set in G.

Proof: For each $K \in \mathcal{C}$, by Lemma 4 and the fact that in step 3(c) we enlarge the radius of at most 18 vertices we have that for the function f constructed by the algorithm DIMINISHPOWER

$$TE\left(G\langle f\rangle[V(K)]\right) \leq 16(\log_2(|V(K)|) + 18.$$

Therefore

$$\begin{aligned} TE(G\langle f\rangle) &= \sum_{K\in\mathcal{C}} [16(\log_2(|V(K)|) + 18] \\ &= 16\log_2\left(\prod_{K\in\mathcal{C}} |V(K)|\right) + 18|\mathcal{C}|. \end{aligned}$$

Now notice that $|MIS(G)| \leq |\mathcal{C}| \leq 7|MIS(G)|$, since in each $K \in \mathcal{C}$ there is at most one vertex from $MIS(G)$ and we can construct an independent set by choosing one vertex from each hexagon from one class. Moreover for any positive numbers a_i and a natural number n we have: $((a_1+a_2+\ldots+a_n)/n)^n \geq a_1a_2\ldots a_n$, therefore:

$$\begin{aligned} TE(G\langle f\rangle) &\leq 16\log_2\left(\frac{|V(G)|}{|\mathcal{C}|}\right)^{|\mathcal{C}|} + 18|\mathcal{C}| \\ &\leq 16\cdot 7|MIS(G)|\log_2\left(\frac{|V(G)|}{|MIS(G)|}\right) + 18|\mathcal{C}| \end{aligned}$$

■

Theorem 8. *The algorithm* DIMINISHPOWER *can be implemented in $LOCAL$ model in constant number of synchronous rounds.*

Proof: Since we assume that each vertex knows its position on the plane, step (1) (dividing into 7 classes) and step (2) (defining palettes of colors) of the algorithm DIMINISHPOWER can be easily implemented in one synchronous round. In addition each $K \in \mathcal{C}$ is a clique, thus information about the whole graph K may be send to a leader of K, which run the algorithm ONCLIQUE. From the properties of the $LOCAL$ model the leader can execute the algorithm ONCLIQUE in one synchronous round. Therefore ONCLIQUE can be implemented in four synchronous rounds: first we select a leader, next we send to him all information about the graph, then the leader run the algorithm ONCLIQUE and finally the leader send to the vertices of K the output of the algorithm). Finally $|\mathcal{K}'| \leq 18$, therefore the step (c) consists of at most 18 iterations. ■

As we claimed in Section I channel assignment problem is equivalent to coloring of the square of the graph. To obtain a proper channel assignment we will modify two steps in algorithm DIMINISHPOWER.

- In step (1) of the algorithm DIMINISHPOWER divide the set of vertices of G into 12 (instead of 7) hexagons (see Figure 1 in [16]).
- In step (3) of the algorithm ONCLIQUE we color square of the graph $K\langle f\rangle[M_i]$ with 37 colors from palette $P(K)$ (instead of 5) using greedy algorithm.

Remark 9. *The algorithm* DIMINISHPOWER *with above modifications constructs in constant number of synchonous rounds a graph $G\langle f\rangle$ with a proper channel assignment. Moreover it uses $O(\log(\chi(G)))$ channels and have properties claimed in Theorems IV and 5.*

Proof:

Firstly, in the partition into 12 classes any two vertices in the different hexagons of the same class are at least two hops away. So there is no channel collision between channels of vertices from different hexagons.

Secondly, in the i-th iteration of the algorithm ONCLIQUE the graph $K\langle f\rangle[M_i]$ has maximum degree bounded by 6. It follows from this observation that the maximal angle between edges in the graph $K\langle f\rangle[M_i]$ is $\frac{\pi}{3}$. Suppose that there would be two directed edges $wv, w'v \in E(K\langle f\rangle[M_i])$ such that angle $\angle(wvw') < \frac{\pi}{3}$ and $\|w,v\| \geq \|w',v\|$ then $\|w,w'\| < \|w,v\|$ and we arrive at the contradiction with a fact that $wv \in K\langle f\rangle[M_i]$. Therefore square of the graph $K\langle f\rangle[M_i]$ have degree bounded by 36. Thus using standard greedy algorithm it is simple to color square of $K\langle f\rangle[M_i]$ graph using 37 colors in polynomial time.

Concluding in channel assignment constructed by the modified algorithm DIMINISHPOWER we use at most $12\,(37\log_2(|\omega(G)|) + 18)$ channels. ■

Remark 10. *If we have a routing protocol in Unit Disc Graph G then we can easily define the routing protocol in $G\langle f\rangle$. Namely, if the routing protocol use the edge vw then we can use the directed path from v to w constructed as in the proof of Theorem 5. More precisely as w_i' we may take the first vertex from any path with origin in w_i, which is incident to a double directed edge in $G\langle f\rangle[M_i]$ defined as in the i-th iteration.*

V. LOWER BOUND

The chromatic number (and analogously channel assignment) of the graph constructed by the algorithm DIMINISHPOWER is best up to a constant factor. Namely there exists a connected Unit Disc Graph graph G such that for any function $f : V(G) \to (0,1]$ such that $G\langle f\rangle$ is strongly connected we have $\chi(G\langle f\rangle) = \Omega(\log(\chi(G)))$. Our lower bound on chromatic number (and also channel assignment) is based on the idea of Theorem (4.1) [7] where authors shown a lower bound for maximum interference of $G_{ncc}\langle f\rangle$.

The construction of the example will be inspired by the construction of the Cantor Set. More precisely the set A_i will be the set of the end points of the segments of the i-th step of the construction of the Cantor Set. Formally $A_1 = \{0,1\}$, $A_2 = \left\{0, \frac{1}{3}, \frac{2}{3}, 1\right\}$, $A_3 = \left\{0, \frac{1}{9}, \frac{2}{9}, \frac{1}{3}, \frac{2}{3}, \frac{7}{9}, \frac{8}{9}, 1\right\}$ etc. as on Figure 3.

Let G be Unit Disc Graph with the vertex set A_k. Obviously G is a clique and have exactly 2^k vertices.

Theorem 11. *Let G be the Unit Disc Graph with the vertex set A_k and $f : V(G) \to (0,1]$ be any function. If $G\langle f\rangle$ is strongly connected then $\chi(G\langle f\rangle) = \Omega(\log(\chi(G)))$.*

Proof: First define $G\{x_1, x_2\}$ to be the set of vertices of the graph G with x-coordinate between x_1 and x_2. Since $G\langle f\rangle$ is connected then there exists at least one directed edge e_1 from $v_1 \in G\langle f\rangle\{\frac{2}{3}, 1\}$ to $w_1 \in G\langle f\rangle\{0, \frac{1}{3}\}$. Without loosing generality suppose that $v_1 \in G\langle f\rangle\{\frac{2}{3}, \frac{7}{9}\}$ have color 1 (see Figure 3). Observe now that since $f(v_1) \geq \frac{1}{3}$ then all other vertices from $G\langle f\rangle\{\frac{2}{3}, 1\}$ cannot have color 1. Analogously there exists at least one directed edge e_2 from $v_2 \in G\langle f\rangle\{\frac{8}{9}, 1\}$ to $w_2 \in G\langle f\rangle\{\frac{2}{3}, \frac{7}{9}\}$. Without loosing generality suppose that $v_2 \in G\langle f\rangle\{\frac{26}{27}, 1\}$ and have color 2 (see Figure 3). Observe now that since $f(v_2) \geq \frac{1}{9}$ then all others vertices from $G\langle f\rangle\{\frac{8}{9}, 1\}$ cannot have color 2. If we continue the reasoning, then in the k-th step we notice that we must use at least k colors to color $G\langle f\rangle$. Since G is a clique, $\chi(G) = V(G) = 2^k$, so $\chi(G\langle f\rangle) \geq \log_2(\chi(G))$. ■

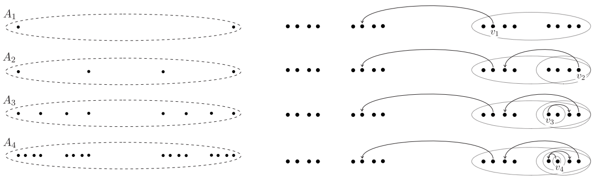

Fig. 3.

VI. Conclusion

In this paper we have studied the problem of channel assignment in Wireless Ad Hoc Network represented by unit disk graphs. We have diminished the number of communication links without loosing connectivity of the network. In addition we have shown, how the diminution of communication range reduces power consumption of transmitting devices. We also show that the number of colors used by our algorithm is the best possible.

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