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New property for rule interestingness measures

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Abstract—**The paper considers interestingness measures for evaluation of rules induced from data with respect to two properties: property of Bayesian confirmation and property Ex_1 concerning the behavior of measures in case of entailment or refutation of the conclusion by the rule's premise. We demonstrate that property Ex_1, even though created for confirmation measures, does not fully reflect the concept of confirmation. We propose a modification of this property, called weak Ex_1, that deploys the concept of confirmation in its larger sense and allows to escape paradoxes that might appear when using measures satisfying the original Ex_1 property.**

I. Introduction

DISCOVERING knowledge from data aims at finding "*valid, novel and potentially useful*" [5] patterns often expressed as "*if…, then…*" rules. Typically, the number of rules generated from massive datasets is quite large, but only some of them are likely to be useful for the domain expert analyzing the data. In order to measure the relevance and utility of the discovered rules, quantitative measures, also known as interestingness or attractiveness measures, have been proposed and studied. Among the most commonly used ones there are: *support*, *confidence*, *lift*, *rule interest function*, *dependency factor*, etc. There is a rich discussion about interestingness measures for rules in data mining (see, for example, [1],[8], [12] for exhaustive reviews of the subject) as each of the measures proposed in the literature reflects different characteristics of rules. The discussion was also extended in [9], by an important issue concerning the possibility of using Bayesian confirmation measures (i.e. measures quantifying the degree to which a piece of evidence provides "support for or against" a hypothesis [7]) as interestingness measures for evaluation of rules. Moreover, the research in [13], [14], [15] shows that discovering patterns in data can be represented in terms of Bayes' theorem. In this context, a variety of non-equivalent confirmation measures should be regarded as a useful tool able to discriminate the most interesting rules discovered by induction from data.

However, due to the plurality of ordinally non-equivalent measures and because there is no agreement which measure is the best, the choice of an interestingness measure for a particular application is non-trivial. To help to analyze measures and overcome the problem of their vast variety, some properties have been proposed. They express the user's expectations towards the behavior of measures in particular situations e.g., one could desire to use only such measures that reward the rules having a greater number of objects supporting the pattern. In general, properties group the measures according to similarities in their characteristics, thus using the measures which satisfy the desirable properties one can avoid considering unimportant rules. Different properties have been proposed and surveyed in [2], [4], [8], [9], [18].

This article concerns Bayesian confirmation measures with respect to their properties. We analyze a property denoted as Ex_1, introduced in [3], assuring that any conclusively confirmatory rule is assigned a higher value of a confirmation measure than any rule which is not conclusively confirmatory, and any conclusively disconfirmatory rule is assigned a lower value than any rule which is not conclusively disconfirmatory. We propose a modification of Ex_1, called weak Ex_1, that deploys the concept of confirmation in its larger sense and allows to escape paradoxes that might occur when using measures with Ex_1 property.

The article is organized as follows. In Section 2 there are preliminaries on rules and their quantitative description. In section 3, we investigate two properties of measures, being the property of Bayesian confirmation and property Ex_1. Section 4 shows specific rankings of rules obtained using measures satisfying the property Ex_1 and explains the paradox appearing in such situations. Section 5 introduces a proposition of modification of property Ex_1 into weak Ex_1 and shows that substituting Ex_1 by weak Ex_1we escape the paradoxes. Finally, Section 6 presents conclusions.

II. Preliminaries

A *rule* induced from a dataset U shall be denoted by $E \rightarrow H$ (read as "*if E, then H*"). It consists of a premise (evidence) E and a conclusion (hypothesis) H.

In general, by $sup(\gamma)$ we denote the number of objects in the dataset for which γ is true, e.g., $sup(E)$ is the number of objects in the dataset satisfying the premise, and $sup(H, E)$ is the number of objects satisfying both the premise and the conclusion of a $E \rightarrow H$ rule.

Moreover, the following notation shall be used throughout the paper: $a=sup(H, E)$, $b=sup(H, \neg E)$, $c=sup(\neg H, E)$, $d=sup(\neg H, \neg E)$. It corresponds to a 2x2 contingency table of the premise and the conclusion.

Table I. Contingency table of E and H

	H	$\neg H$	
E	a	c	$a+c$
$\neg E$	b	d	$b+d$
	$a+b$	$c+d$	$\lvert U \rvert$

Observe that b can be interpreted as the number of objects that do not satisfy the premise but satisfy the conclusion of the $E \rightarrow H$ rule. Analogously, $c=sup(\neg H, E)$ can be construed as the number of objects in the dataset that satisfy the premise but do not satisfy the conclusion of the $E \rightarrow H$ rule, and $d=sup(\neg H, \neg E)$ can be interpreted as the number of objects in the dataset that satisfy neither the premise nor the conclusion of the $E \rightarrow H$ rule. Moreover, the following relations occur: $a+c=sup(E)$, $a+b=sup(H)$, $b+d=sup(\neg E)$, $c+d=sup(\neg H)$, and the cardinality of the dataset U, denoted by $|U|$, is the sum of a, b, c and d.

Reasoning in terms of a, b, c and d is natural and intuitive for data mining techniques since all observations are gathered in some kind of an information table describing each object by a set of attributes. However, a, b, c and d can also be regarded as frequencies that can be used to estimate probabilities: e.g., $\Pr(E)=(a+c)/|U|$ or $\Pr(H)=(a+b)/|U|$.

III. Properties of interestingness measures

The problem of choosing an appropriate interestingness measure for a certain application is non-trivial because the number and variety of measures proposed in the literature is overwhelming. To help to analyze measures, some properties have been proposed, expressing the user's expectations towards the behavior of measures in particular situations. Properties of measures group them according to similarities in their characteristics. Using the measures which satisfy the desirable properties, one can avoid considering unimportant rules.

Our analysis of properties is conducted from the view point of Bayesian confirmation theory. We propose a modification of property Ex_1, called *weak* Ex_1, that deploys the concept of confirmation in its larger sense. We demonstrate that using property Ex_1 can lead to paradoxical situations and thus, we propose to substitute Ex_1 by weak Ex_1.

A. Property of Bayesian confirmation

Bayesian confirmation theory assumes the existence of probability Pr. Given a proposition X, $\Pr(X)$ represents the probability of X, and given X and Y, $\Pr(X|Y)$ is the probability of X given Y, i.e. $\Pr(X|Y) = \Pr(X \wedge Y)/\Pr(Y)$.

Generally speaking, a measures possessing the property of Bayesian confirmation is expected to obtain values greater than 0 when the premise of a rule confirms the conclusion of a rule, values equal to 0 when the rule's premise and conclusion are neutral to each other, and finally, values smaller than 0 when the premise disconfirms the conclusion.

Formally, an interestingness measure $c(H, E)$ has the property of Bayesian confirmation if and only if it satisfies the following conditions (BC):

$$c(H,E)\begin{cases} >0 \ \ if \ \Pr(H \mid E) > \Pr(H), \\ =0 \ \ if \ \Pr(H \mid E) = \Pr(H), \\ <0 \ \ if \ \Pr(H \mid E) < \Pr(H). \end{cases} \quad (1)$$

The (BC) definition identifies confirmation with an increase in the probability of the conclusion H provided by the premise E, neutrality with the lack of influence of the premise E on the probability of conclusion H, and finally disconfirmation with a decrease of probability of the conclusion H imposed by the premise E [2].

It is important to note that there are many different, but logically equivalent, ways of expressing that E confirms H:

- $\Pr(H|E) > \Pr(H)$
- $\Pr(H|E) > \Pr(H|\neg E)$
- $\Pr(H|E) > \Pr(E|\neg H)$.

Since they are equivalent (see also [7], [11]), one can also express the (BC) conditions as:

$$c(H,E)\begin{cases} >0 \ \ if \ \Pr(H \mid E) > \Pr(H \mid \neg E), \\ =0 \ \ if \ \Pr(H \mid E) = \Pr(H \mid \neg E), \\ <0 \ \ if \ \Pr(H \mid E) < \Pr(H \mid \neg E). \end{cases} \quad (2)$$

To avoid ambiguity, we shall denote the above formulation as (BC'). According to it E confirms H when E raises the probability of H, and E raises the probability of H if the probability of H given E is higher than the probability of H given non E.

Measures that possess the property of confirmation are referred to as *confirmation measures* or *measures of confirmation*. For a given rule $E \rightarrow H$, interestingness measures with the property of confirmation express the credibility of the following proposition: *H is satisfied more frequently when E is satisfied, rather than when E is not satisfied*. By using interestingness measures that possess this property one can filter out rules which are misleading and disconfirm the user, and this way, limit the set of induced rules only to those that are meaningful [17]. Let us also stress that the catalogue of confirmation measures available in the literature is quite large and the condition (BC) (or (BC') equivalently) does not favor one single measure as the most adequate [3], [6].

The discussion brought up in [9] about using the confirmation measures as interestingness measures for decision rules within rough set approach and, more generally, within data mining, machine learning and knowledge discovery, leads to the conclusion that the group of measures with property of Bayesian confirmation should be considered a valuable and meaningful tool for assessing the quality of rules induced from data. Using the quantitative confirmation theory for data analysis allows to benefit from the ideas of such prominent researchers as Carnap [2], Hempel [10] and Popper [16].

B. *Property* Ex_1

To handle the plurality of alternative confirmation measures, Crupi, Tentori and Gonzalez [3] have proposed a property (principle) Ex_1 resorting to considering inductive logic as an extrapolation from classical deductive logic. On the basis of classical deductive logic they construct a function v:

$$v(H,E)=\begin{cases}\text{the same positive value, denoted as } V, \\ \text{if } E \models H; \\ \text{the same negative value, denoted as } -V, \\ \text{if } E \models \neg H; \\ \\ 0, \text{ otherwise.}\end{cases} \tag{3}$$

For any argument (H, E) function v assigns it the same positive value V (e.g., +1) if and only if the premise E of the rule entails the conclusion H (i.e. $E \models H$). The same value but of opposite sign $-V$ (e.g., -1) is assigned if and only if the premise E refutes the conclusion H (i.e. $E \models \neg H$). In all other cases (i.e. when the premise is not conclusively confirmatory nor conclusively disconfirmatory for the conclusion) function v obtains value 0.

Let us observe, that any confirmation measure obtains positive (negative) values whenever function $v(H, E)$ is positive (negative). However, according to Crupi et al., the relationship between the logical implication or refutation of H by E, and the conditional probability of H subject to E should go further and demand fulfillment of the following principle (Ex_1):

$$\text{if } v(H_1,E_1)>v(H_2,E_2) \text{ then } c(H_1,E_1)>c(H_2,E_2) \tag{4}$$

Property Ex_1 guarantees that the measure will assign a greater value to any conclusively confirmatory rule (i.e. such that $E \models H$) than to any rule which is not conclusively confirmatory. Moreover, rules that are conclusively disconfirmatory (i.e. such that $E \models \neg H$) will obtain smaller values of interestingness measures than any rule which is not conclusively disconfirmatory.

Let us consider an example of drawing cards from a standard deck to review the consequences of property Ex_1 in three possible situations: conclusively confirmatory, non-conclusively confirmatory (or disconfirmatory), and conclusively disconfirmatory.

A rule r_1*: if x is a jack then x is a face-card* is conclusively confirmatory as the premise (drawing a jack) entails (i.e. confirms in 100%) the conclusion that the drawn card is a face-card. The entailment of the conclusion H by the premise E ($E \models H$), implies that there cannot be any counterexamples to the rule (i.e. c=0). Such conclusively confirmatory rules should be assigned a maximal value V of a function $v(H, E)$.

An inverse rule r_2: *if x is a face-card then x is a jack* should be regarded as non conclusively confirmatory. Drawing a face-card one can be lucky to get a jack, but it is not a 100% sure situation, therefore the premise does not entail the conclusion and the rule is not conclusively confirmatory. Moreover, the rule is also non conclusively disconfirmatory as the premise does not refute (i.e. disconfirm in 100%) the conclusion. For rules like r_2 function $v(H, E)$ obtains value 0, which implies that confirmation measures with property Ex_1 assign to such rules smaller values than to conclusively confirmatory rules.

A conclusively disconfirmatory rule could be r_3*: if x is seven of spades then x is a face-card*. Here, the premise of drawing the seven of spades disconfirms in 100% the conclusion that the drawn card is a face-card. The refutation of the conclusion H by the premise E ($E \models \neg H$), implies that there cannot be any positive examples to the rule (i.e. a=0). Such conclusively disconfirmatory rules should be assigned a minimal value $-V$ of a function $v(H, E)$.

The ordering based on function v: $v(r_1) > v(r_2) > v(r_3)$, implies the following relations for any confirmation measure possessing the Ex_1 property: $c(r_1) > c(r_2) > c(r_3)$.

Concluding, measures satisfying property Ex_1 have the ability to rank the rules in such a way that those in which the premise entails the conclusion (e.g., the rule: *if x is a jack then x is a face-card*) are on top of the ranking, those in which the premise refutes the conclusion (e.g., *if x is seven of spades then x is a face-card*) are on the very bottom, and rules which are neither 100% sure nor 100% false are in between. Let us also remark, that entailment is equivalent to $\Pr(H|E)$=1, i.e. to situations when there are no counterexamples to the rule (c=0), and that refutation is equivalent to $\Pr(H|E)$=0, i.e. to situations when there are no positive examples to the rule (a=0).

IV. PARADOXES OF Ex_1 PROPERTY

Property Ex_1 was introduced to assure that rules for which the premise entails the conclusion (i.e. conclusively confirmatory rules) are assigned a higher value of a confirmation measure than any rule which is not conclusively confirmatory. Furthermore, rules for which the premise refutes the conclusion (i.e. conclusively disconfirmatory rules) are assigned a lower value than any rule which is not conclusively disconfirmatory. Ranking of rules depending on entailment and refutation seems naturally desirable, however boiling the consideration down to only two situations: when there are no counterexamples to the rule (i.e. $E \models H$, c=0), and when there are no positive examples to the rule (i.e. $E \models \neg H$, a=0) can result in paradoxes.

Let us explain our point of view by taking into account the formulation of (BC') conditions stating that:

E confirms H if the probability of H given E is higher than the probability of H not given E. We believe that it is reasonable to conclude that, *in case of confirmation*, a confirmation measure $c(H, E)$ should express *how much it is more probable to have H when E is present rather than when E is absent*. In this context, the following example shows a paradox caused by the property of Ex_1.

Let us consider two cases in which the number of objects in U is distributed over a, b, c and d in the following manner:

Case α: a_α =10, b_α=9, c_α=0, d_α=1;

Case β: a_β =9, b_β=0, c_β=1, d_β=10.

In case α a rule r_α: $E_\alpha \rightarrow H_\alpha$ is supported by a_α=10 objects from U, there are 9 objects supporting the rule's conclusion but not its premise (b_α=9), there no counterexamples to the rule (c_α=0), and there is 1 object not supporting the rule's premise nor its conclusion (d_α=0). Analogously, in case β a rule r_β: $E_\beta \rightarrow H_\beta$ is supported by a_β=9 objects from U, there are no objects supporting the rule's conclusion but not its premise (b_β=0), there is only 1 counterexamples to the rule (c_β=1), and there are 10 object not supporting the rule's premise nor its conclusion (d_β=10). The rule r_α is conclusively confirmatory, as it has no counterexamples in U (i.e. the premise entails the conclusion). On the other hand, the rule r_β is non conclusively confirmatory because there exists in U one counterexample to that rule. Thus, in case α the value of a confirmation measure should be greater than in case β if Ex_1 holds.

Let us now also incorporate the idea that a confirmation measure $c(H, E)$ should express how much it is more probable to have H when E is present rather than when E is absent. One can see that $\Pr(H_\alpha|E_\alpha) = a_\alpha/(a_\alpha+c_\alpha) = 1$ and $\Pr(H_\alpha|\neg E_\alpha) = b_\alpha/(b_\alpha+d_\alpha) = 0.9$ in case α, while in case β $\Pr(H_\beta|E_\beta) = a_\beta/(a_\beta+c_\beta)$=0.9 and $\Pr(H_\beta|\neg E_\beta) = b_\beta/(b_\beta+d_\beta)$=0.

For case α and β, if Ex_1 holds, passing from the situation when the premise is absent to the situation when the premise is present, we assign a greater value of a confirmation measure when we have a 10% increment of the probability of the conclusion (case α) rather than when the same increment is of 90% (case β). A confirmation measure possessing property Ex_1 favors rule r_α over r_β, which is a paradox when we analyze how much more probable it is to have the rule's conclusion when the premise is present rather than when it is absent.

Analogously, let us interpret (BC') conditions as: E disconfirms H if the probability of H given E is smaller than the probability of H not given E. Thus, *in case of disconfirmation* a confirmation measure $c(H, E)$ should express *how much it is less probable to have H when E is present rather than when E is absent.* In this context, the following example shows a paradox caused by the property of Ex_1.

Let us consider two cases in which the number of objects in U is distributed over a, b, c and d in the following manner:

Case γ: a_γ=0, b_γ=1, c_γ=10, d_γ=9;

Case δ: a_δ=1, b_δ=10, c_δ=9, d_δ=0.

In case γ a rule r_γ: $E_\gamma \rightarrow H_\gamma$ is not supported by any object from U (a_γ=0), there is 1 object supporting the rule's conclusion but not its premise (b_γ=1), there are 10 counterexamples to the rule (c_γ=10), and there are 9 objects not supporting the rule's premise nor its conclusion (d_γ=9). Analogously, in case δ a rule r_δ: $E_\delta \rightarrow H_\delta$ is supported by one object from U (a_δ=1), there are 10 objects supporting the rule's conclusion but not its premise (b_δ=10), there are 9 counterexamples to the rule (c_δ=9), and there are no object not supporting the rule's premise nor its conclusion (d_δ=0). The rule r_γ is conclusively disconfirmatory, as it has no positive examples in U (i.e. the premise refutes the conclusion). On the other hand, the rule r_δ is non conclusively disconfirmatory because there exists in U one positive example to that rule. Thus, in case γ the disconfirmation should be greater than in case δ if Ex_1 holds, i.e. the value of a confirmation measure should be smaller in case γ than in case δ.

From the view point of (BC') condition concerning disconfirmation, measure $c(H, E)$ should express how much it is less probable to have H when E is present rather than when E is absent. The conditional probabilities for the two exemplary cases γ and δ are: $\Pr(H_\gamma|E_\gamma) = a_\gamma/(a_\gamma+c_\gamma) = 0$ and $\Pr(H_\gamma|\neg E_\gamma) = b_\gamma/(b_\gamma+d_\gamma)$=0.1, and $\Pr(H_\delta|E_\delta) = a_\delta/(a_\delta+c_\delta)$=0.1 and $\Pr(H_\delta|\neg E_\delta) = b_\delta/(b_\delta+d_\delta) = 1$.

For case γ and δ, if Ex_1 holds, passing from the situation when the premise is absent to the situation when the premise is present, we should have a smaller value of confirmation measure (greater disconfirmation) when we have a 10% decrement of probability of the conclusion (case γ) rather than when the same decrement is of 90% (case δ). A confirmation measure possessing property Ex_1 treats rule r_δ as less disconfirmatory than r_γ, which is a paradox when we analyze how much less probable it is to have the rule's conclusion when the premise is present rather than when it is absent.

The considerations for cases α-δ show that the requirements forming Ex_1 are not sufficient as using measures with this property can lead to paradoxes. Remark that in case of confirmation, Ex_1 concerns situations of entailment, which is equivalent to $\Pr(H|E)$=1. However, confirmation should express how much it is more probable to have H when E is present rather than when E is absent. Thus, the requirement $\Pr(H|E)$=1 is not sufficient, and property Ex_1 should be modified to take into account also the value of $\Pr(H|\neg E)$. Analogical requirements concern the case of disconfirmation. These considerations lead to important modifications of property Ex_1, called weak Ex_1.

V. Modification of Ex_1 into Weak Ex_1 Property

Property Ex_1 can be regarded as one-sided because it focuses on situations when $E \models H$ (i.e. there are no counterexamples to a rule and c=0), and situations when $E \models \neg H$ (i.e. there are no positive examples to a rule and a=0). In our opinion, the concept of confirmation is too complex and rich to be boiled down simply to verification whether there are no counterexamples or no positive examples.

To formulate the proposition of modification of the Ex_1 property, let us recall the interpretation of (BC') conditions:

- *in case of confirmation*, a confirmation measure $c(H, E)$ should express *how much it is more probable to have H when E is present rather than when E is absent*,
- *in case of disconfirmation* a confirmation measure $c(H, E)$ should express *how much it is less probable to have H when E is present rather than when E is absent*.

Taking into account such interpretations we can formulate a property called *weak* Ex_1, which generalizes the original Ex_1 property:

$$\begin{aligned}&\textit{if } v(H_1,E_1) > v(H_2,E_2) \textit{ and } v(H_1,\neg E_1) < v(H_2,\neg E_2)\\&\textit{then } c(H_1,E_1) > c(H_2,E_2)\end{aligned} \tag{5}$$

Property weak Ex_1 guarantees that a confirmation measure $c(H, E)$ cannot attain its maximal value unless the two following conditions are satisfied:

- $E \models H$
- $\neg E \models \neg H$

Let us also remark that the condition $E \models H$ is equivalent to $\Pr(H|E)$=1 and to $c=sup(\neg H, E)$=0, because

$$\Pr(H|E)= \frac{sup(H,E)}{sup(H,E)+sup(\neg H,E)} = \frac{a}{a+c} =1 \Leftrightarrow c=0.$$

Furthermore, the condition $\neg E \models \neg H$ can be equivalently expressed as $\Pr(H|\neg E)$=0 or $b=sup(H, \neg E)$=0, since

$$\Pr(H|\neg E)= \frac{sup(H,\neg E)}{sup(H,\neg E)+sup(\neg H,\neg E)} = \frac{b}{b+d} =0 \Leftrightarrow b=0.$$

Analogously, property weak Ex_1 guarantees that the confirmation measure $c(H, E)$ cannot attain its minimal value unless the two following conditions are satisfied:

- $E \not\models \neg H$
- $\neg E \models H$

Let us note that the condition $E \not\models \neg H$ is equivalent $\Pr(H|E)$=0 and to $a=sup(H, E)$=0, as

$$\Pr(H|E)= \frac{sup(H,E)}{sup(H,E)+sup(\neg H,E)} = \frac{a}{a+c} =0 \Leftrightarrow a=0.$$

Moreover, the condition $\neg E \models H$ can be equivalently expressed as $\Pr(H|\neg E)$=1 or as $d=sup(\neg H, \neg E)$=0, because

$$\Pr(H|\neg E)= \frac{sup(H,\neg E)}{sup(H,\neg E)+sup(\neg H,\neg E)} = \frac{b}{b+d} =1 \Leftrightarrow d=0.$$

Using the property Ex_1 can lead to unwanted situations where we favor one rule over another contrary to the increase in the confirmation or decrease of disconfirmation. Our modification of Ex_1 into weak Ex_1 escapes that problem, because the condition c=0 (in case of confirmation) and a=0 (in case of disconfirmation) that were present in original formulation of Ex_1 are now extended to $c=b$=0 (in case of confirmation) and $a=d$=0 (in case of disconfirmation). If we consider a confirmation measure that satisfies weak Ex_1, we do not demand that it should have a greater value in case α rather than in case β, nor vice versa. Thus, the paradox disappears under conditions of weak Ex_1 property. Moreover, if we consider a confirmation measure that satisfies weak Ex_1, we do not demand that it should have a smaller value in case γ rather than in case δ, nor vice versa. Thus, the paradox disappears under conditions of weak Ex_1.

The modifications introduced to the Ex_1 property are indispensable and allow to deploy the concept of confirmation in its larger sense. Therefore we postulate to substitute Ex_1 by weak Ex_1 property.

VI. Conclusion

Analysis of rule interestingness measures with respect to their properties is an important research area. It helps to identify groups of measures that are truly meaningful. A group of measures satisfying the Bayesian confirmation property has been identified as important and useful for evaluation of patters in form of rules [9], [13]. To handle the plurality of alternative, ordinally non-equivalent confirmation measures, property Ex_1 has been proposed [3]. It relates to entailment or refutation of the rule's conclusion by its premise. The formulation of Ex_1 reacts only to the absence of counterexamples or positive examples to a rule. Such approach does not reflect the deep meaning of confirmation stating that a confirmation measure should give an account of the credibility that it is more probable to have the rule's conclusion when the premise is present, rather than when the premise is absent. On the basis of such understanding of the concept of confirmation, we propose modification of Ex_1 property, called weak Ex_1. It takes into account not only the value of $\Pr(H|E)$ but also $\Pr(H|\neg E)$, and this way fully relates to confirmation concept.

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