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Reliability Analysis of Healthcare System

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***Abstract*—Modern system is complex and includes different types of components such as software, hardware, human factor. Reliability is principal property of this system. The importance analysis is one of approaches in reliability engineering. Application of this approach for healthcare system is considered in this paper. The importance reliability analysis allows estimating the influence of every healthcare system component to the system reliability, its functioning and failure.**

I. Introduction

THE PRINCIPAL goal of IT application in medicine is improvement and conditioning of medical care [1]–[3]. Modern healthcare systems have to reduce problems and difficulties in diagnosing and treatment of diseases, and have to perfect patient care. Therefore the healthcare has to be characterised by high reliability first of all and reliability analysis of such system is important problem.

There are investigations in reliability analysis of healthcare system. This area includes some concepts that can be declared as reliability analysis of medical equipment and devices [3]–[5] and human reliability analysis in medicine [4], [6], [7]. Unfortunately, these concepts develop independently, only in paper [4] problems of reliability analysis of technical part and human factor have been considered but different methods for theirs analysis have been proposed. It is caused by reliability engineering state, where there are some independent areas of investigation, for example, as software reliability analysis, hardware reliability analysis, human reliability analysis (HRA). Methods of estimation and quantification of these objectives aren't interchangeable. Therefore reliability analysis system with different types of component needs new methods. These methods have to allow estimation of such system based on unified methodology. Declaration of this problem for the healthcare system has been presented in [8].

According to [8] the healthcare system includes four components of different types (Fig.1): hardware, software, human factor and organization factor. In the paper [8] there have been shown that the hardware and software components unite in one component for the healthcare system. This component is named as technical component, but this component is separated from other two components: special technical component and basic technical component. The first of them includes special equipment, devices and software (for example, magnetic resonance imaging scanners). The second component corresponds with basic equipment and software as personal computer, operating system, database and etc. The human factor and organization factor have been interpreted as two components.

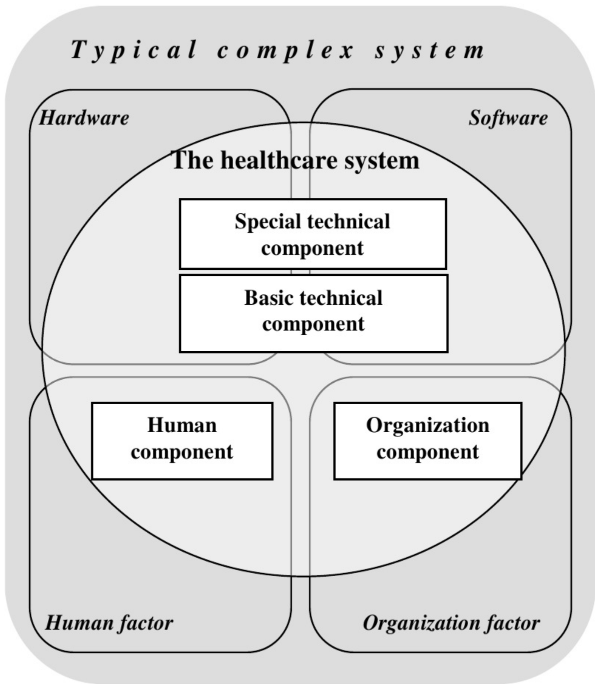

Fig. 1 The healthcare system typical structure for reliability analysis

In this paper reliability analysis of the healthcare system (Fig.1) is developed. The influence of system component states (some levels of functioning and failure) to the system reliability is investigated and quantified based on unified methodology. In other words the probabilities of the healthcare system performance levels are calculated against changes of the system component state changes.

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II. Reliability Analysis of Healthcare System

A. Background and mathematical model

The basic reliability concept is defined as the probability that the system will perform its intended function during a period of running time without any failure. A fault is an erroneous state of the system. Although the definitions of fault are different for different systems and in different situations, a fault is always an existing part in the system and it can be removed by correcting the erroneous part of the system. New tendencies in reliability engineering have been defined in [9], some of they are:

- detail analysis of changes of the system reliability states from perfect function to failure;
- priority analysis of causes of the system failure, e.g. discover causes and mechanisms of failure and to indentify consequences;
- development of methods for the system reliability analysis in design.

These tendencies have been taken into account in process of any system reliability analysis that includes next steps:

- the quantification of the system model;
- the representation and modelling of the system;
- the quantification of the system reliability (definition of reliability indexes and measures for the system evaluation).

Two steps of the system process analysis considered with definition of the mathematical model. This model has to allow estimation some levels of the system reliability changes. Binary-State System (BSS) and Multi-State System (MSS) are basic mathematical models in reliability analysis. BSS is used for description of initial system as system with two states: reliable and unreliable. But this model doesn't allow quantifying different levels of the system reliability. MSS is mathematical model in reliability analysis that is used for description system with some (more than two) levels of performance (availability, reliability) [9], [10]. MSS allows presenting the analyzable system in more detail than traditional Binary-State System.

The MSS and each of n components can be in one of m possible states: from the complete failure (it is 0) to the perfect functioning (it is m-1). A structure function is one of typical representations of MSS [10], [11]. This function of a MSS of n components is denoted as:

$$\phi(x_1,\ldots,x_n) = \phi(\boldsymbol{x}): \{0,\ldots,m-1\}^n \rightarrow \{0,\ldots,m-1\}, \quad (1)$$

where x_i is the i-th component; $\boldsymbol{x} = (x_1, \ldots, x_n)$ is vector of components states; values of a MSS reliability (structure function $\phi(\boldsymbol{x})$) and its component state (variables x_i, $i = 1, \ldots, n-1$) change from zero to $(m-1)$.

Need to say that for the structure function (1) there are next assumptions that will be used in the system reliability estimation [12]:

- the structure function is monotone and $\phi(\boldsymbol{s}) = \boldsymbol{s}$ ($s \in \{0, \ldots, m-1\}$);
- all components are s-independent and are relevant to the system.

Every system component states x_i is characterized by probability of the performance rate:

$$p_{i,s} = \Pr\{x_i = s\}, \quad s = 0, \ldots, m-1 \quad (2)$$

The principal advantage of the system representation by the structure function (1) is definition of this function for any system with different complexity and structure.

There are different directions for quantification of MSS behaviour. One of them is importance analysis [12]–[14].

Importance analysis is used for MSS reliability estimation depending on the system structure and its components states. Quantification is indicated by importance measure. They have been widely used as tools for identifying system weaknesses, and to prioritise reliability improvement activities. MSS importance measures are probabilities that the system has the reliability level h ($h = 1, ..., m$-1) if the i-th system component states is s ($s = 1, ..., m$-1). Different combinations of the system reliability levels h and components states s allow investigating boundary system state and system states that take priority of failure.

Note one more significant aspect of the importance analysis. Some types of importance measures can be calculated for the system in design. Therefore this system quantification method can be used for the system reliability estimation in design.

The theoretical aspects of MSS importance analysis have been investigated since first paper in MSS analysis [15]. These investigations were developed in papers [12]–[16]. Importance measures for system with two performance level and multi-state components and their definitions by output performance measure have been considered in [12]. Universal generating function method has been used for importance analysis in [12], [16]. Composite importance measures for MSS estimation have been proposed in [14]. New method based on Logical Differential Calculus for importance analysis of MSS has been considered in paper [11], [17] and new type of importance measures has been proposed. These measures have been named as Dynamic Reliability Indices (DRIs). The importance analysis method based on Logical Differential Calculus is demonstrable, intuitive and is characterized by simple calculation.

Therefore MSS importance analysis is actual approach in reliability engineering because allows:

- to investigate the system behaviour in detail that include the quantification of different level of reliability;
- to examine causes of the system failure;
- to estimate the system reliability analysis in design.

The algorithm for the healthcare system reliability estimation by importance analysis based on typical process of the estimation is in Fig.2.

According to the algorithm in Fig.2 number m of performance (reliability or availability) levels for the system and its components for estimation of this system is defined firstly. Then the structure function as mathematical model of this system is determined taking into account the number of performance levels. For example, consider the healthcare system for the Decision Support System for Early

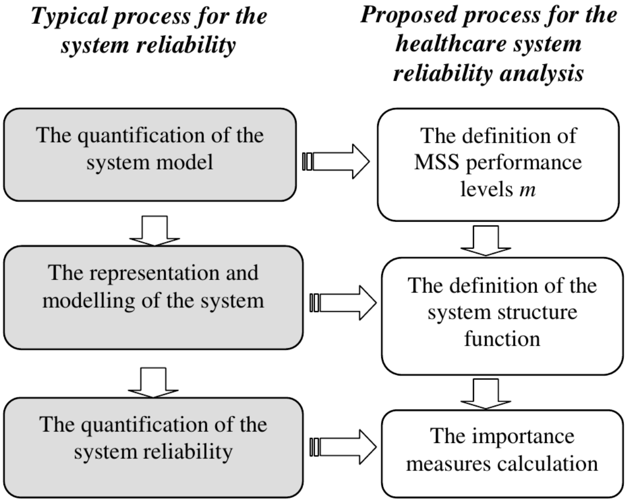

Fig. 2 The healthcare system reliability analysis process

Diagnostics in Oncology (DSSEDO) that have been described in [8]. The system structure can be interpreted as typical for healthcare system in Fig.1. Define for this system number of performance levels as m = 3. The structure function of this system is defined as:

$$\phi(\boldsymbol{x}) = \mathrm{OR}(\mathrm{AND}(x_1, x_2), \mathrm{AND}(x_1, x_3, x_4)), \quad (3)$$

where x_1 is performance level of the special devices; x_2 is performance level of the basic devices; x_3 and x_4 is performance level of the human and organization components of the system; OR(y, z) = max(y, z); AND(y, z) = min(y, z).

Therefore the structure function (3) is mathematical model for the DSSEDO that is used for estimation and quantification of its performance or reliability (the detail description of this function is in Table I).

B. Direct Partial Logic Derivative

The mathematical tool of Multiple-Valued Logic (MVL) as Logical Differential Calculus is used for calculation of importance analysis. The MSS structure function is interpreted as MVL function in this case. The Logical Differential Calculus is mathematical tool that permits to analysis changes in function depending of changes of its variables. Therefore evaluate influence of every system component state change to level of MSS reliability by Direct Partial Logic Derivative (this approach is part of Logical Differential Calculus). Direct Partial Logic Derivative reflects the change in the value of the MVL function when the values of variables change.

A Direct Partial Logic Derivative with respect to i-th variable for a MSS reliability analysis has been defined in [17] as:

$$\partial\varphi(j \to \tilde{j})/\partial x_i(a \to \tilde{a}) = \begin{cases} 1, & if\ \varphi(a_i, x) = j\ and\ \varphi(\tilde{a}_i, x) = \tilde{j} \\ 0, & other \end{cases} ,(4)$$

TABLE I.
TRUTH TABLE OF STRUCTURE FUNCTION (3)

$x_1 x_2\ x_3\ x_4$	$\phi(\boldsymbol{x})$	$x_1 x_2\ x_3\ x_4$	$\phi(\boldsymbol{x})$	$x_1 x_2\ x_3\ x_4$	$\phi(\boldsymbol{x})$
0 0 0 0	0	1 0 0 0	0	2 0 0 0	0
0 0 0 1	0	1 0 0 1	0	2 0 0 1	0
0 0 0 2	0	1 0 0 2	0	2 0 0 2	0
0 0 1 0	0	1 0 1 0	0	2 0 1 0	0
0 0 1 1	0	1 0 1 1	1	2 0 1 1	1
0 0 1 2	0	1 0 1 2	1	2 0 1 2	1
0 0 2 0	0	1 0 2 0	0	2 0 2 0	0
0 0 2 1	0	1 0 2 1	1	2 0 2 1	1
0 0 2 2	0	1 0 2 2	1	2 0 2 2	2
0 1 0 0	0	1 1 0 0	1	2 1 0 0	1
0 1 0 1	0	1 1 0 1	1	2 1 0 1	1
0 1 0 2	0	1 1 0 2	1	2 1 0 2	1
0 1 1 0	0	1 1 1 0	1	2 1 1 0	1
0 1 1 1	0	1 1 1 1	1	2 1 1 1	1
0 1 1 2	0	1 1 1 2	1	2 1 1 2	1
0 1 2 0	0	1 1 2 0	1	2 1 2 0	1
0 1 2 1	0	1 1 2 1	1	2 1 2 1	1
0 1 2 2	0	1 1 2 2	1	2 1 2 2	2
0 2 0 0	0	1 2 0 0	1	2 2 0 0	2
0 2 0 1	0	1 2 0 1	1	2 2 0 1	2
0 2 0 2	0	1 2 0 2	1	2 2 0 2	2
0 2 1 0	0	1 2 1 0	1	2 2 1 0	2
0 2 1 1	0	1 2 1 1	1	2 2 1 1	2
0 2 1 2	0	1 2 1 2	1	2 2 1 2	2
0 2 2 0	0	1 2 2 0	1	2 2 2 0	2
0 2 2 1	0	1 2 2 1	1	2 2 2 1	2
0 2 2 2	0	1 2 2 2	1	2 2 2 2	2

where $\phi(\bullet_i, \boldsymbol{x})=\phi(x_1,\ldots,x_{i-1}, \bullet_i, x_{i+1},\ldots,x_n)$ is value of structure function; $\tilde{a} \neq a$, $\tilde{j} \neq j$ and a, j, $\tilde{a}$, $\tilde{j}$ = {0, …, m-1}.

For monotone structure function the changes from a to $\tilde{a}$ and from j to $\tilde{j}$ can be defined as changes from a to $\tilde{a}$ = (a-1) or $\tilde{a}$ = (a+1) and from j to $\tilde{j}$ = (j-1) or $\tilde{j}$ = (j+1) accordingly. These changes are caused by gradual type of reliability changes without jumps too.

The Direct Partial Logic Derivative allows to calculate the system boundary sates for which change the i-th component state from a to $\tilde{a}$ cause changes of the system performance level from j to $\tilde{j}$. These states correspond to the nonzero values of the derivative (4). For example, for the healthcare system with structure function (3) boundary states of the system performance level reduction for the first component are in Table II. These states are computed by Direct Partial Logic Derivative that is indicated in Table II too.

Therefore according to the Table II the first component state change from 2 to 1 doesn't cause the system failure (change from 1 to 0) and the failure of this component doesn't influence to the system performance level chance from 2 to 1, because the Direct Partial Logic Derivatives $\partial\phi(1\to0)/x_1(2\to1)$ and $\partial\phi(2\to1)/x_1(1\to0)$ have zero value only. But break down of the first component and its deterioration cause failure and degradation of the system

TABLE II.
BOUNDARY STATES FOR THE FIRST COMPONENT OF HEALTHCARE SYSTEM WITH STRUCTURE FUNCTION (3)

$x_2\ x_3\ x_4$	$\partial\phi(1\to0)/x_1(1\to0)$	$\partial\phi(1\to0)/x_1(2\to1)$	$\partial\phi(2\to1)/x_1(1\to0)$	$\partial\phi(2\to1)/x_1(2\to1)$
0 0 0	0	0	0	0
0 0 1	0	0	0	0
0 0 2	0	0	0	0
0 1 0	0	0	0	0
0 1 1	1	0	0	0
0 1 2	1	0	0	0
0 2 0	0	0	0	0
0 2 1	1	0	0	0
0 2 2	1	0	0	1
1 0 0	1	0	0	0
1 0 1	1	0	0	0
1 0 2	1	0	0	0
1 1 0	1	0	0	0
1 1 1	1	0	0	0
1 1 2	1	0	0	0
1 2 0	1	0	0	0
1 2 1	1	0	0	0
1 2 2	1	0	0	1
2 0 0	1	0	0	1
2 0 1	1	0	0	1
2 0 2	1	0	0	1
2 1 0	1	0	0	1
2 1 1	1	0	0	1
2 1 2	1	0	0	1
2 2 0	1	0	0	1
2 2 1	1	0	0	1
2 2 2	1	0	0	1

accordantly (derivatives $\partial\phi(1\to0)/x_1(1\to0)$ and $\partial\phi(2\to1)/x_1(2\to1)$ have nonzero values that correspond to the boundary system states).

Investigation of the boundary states of the system is important problem but set of boundary state has high dimensionality and isn't acceptable well for practical application. Therefore in engineering problem probability measures for the system reliability or performance are used as a rule.

III. HEALTHCARE SYSTEM RELIABILITY ANALYSIS

A. Healthcare system probability state

MSS probability state, $R(j)$, is one of the best known MSS reliability measures [12]. It is the probability that system performance level is equal to the level j:

$$R(j) = \Pr\{\phi(\boldsymbol{x}) = j\},\ j \in \{0, 1, \ldots, m\text{-}1\}. \tag{5}$$

For example, for the healthcare system with structure function (3) can be computed system state probabilities based on its structure function:

$$R(0) = p_{1,0} + (p_{1,1}+ p_{1,2})\cdot p_{2,0}\cdot(p_{3,0}+ (p_{3,1}+p_{3,2})\cdot p_{4,0}),$$

$$R(1) = (p_{1,1}+p_{1,2})\cdot p_{2,0}\cdot(p_{3,1}+p_{3,2})\cdot(p_{4,1}+p_{4,2}) + p_{1,1}\cdot(p_{2,1}+p_{2,2}) + +p_{1,2}\cdot p_{2,1}\cdot(p_{3,0}+p_{3,1})\cdot(p_{4,0}+p_{4,1}),$$

$$R(2) = p_{1,2}\cdot \mathrm{p}_{2,1}\cdot p_{3,2}\cdot p_{4,2}+p_{1,2}\cdot p_{2,2}.$$

B MSS Importance Measures

Probability states (5) don't enable the analysis of the change in system reliability that is caused by a change in component states. Importance analysis of the healthcare system allows estimating the influence of every system component state changes to system performance. Consider some of importance measures and their calculation by Direct Partial Logic Derivative.

Structural Importance (SI) is one of the simplest measures of component importance and this measure is concentrated on the topological aspects of the system. According to definition in papers [13], [18] this measure determines the proportion of working states of system in which the working of the i-th component makes the difference between system failure and its working. SI of MSS for the i-th component state s is probability of this system performance level j decrement if the i-th component state changes from s to s-1 depending on topological properties of system:

$$I_S(s_i|j) = \frac{\rho_i^{s,j}}{m^{n-1}} \quad , \tag{6}$$

where $\rho_i^{s,j}$ is number of system states when the change component state from s to s-1 results the system performance level decrement and this number is calculated as numbers of nonzero values of Direct Partial Logic Derivatives (4).

There is one more definition of SI [11]. It is modified SI that represent of the i-th system component state change influence to MSS performance level decrement for boundary system state. In terms of Direct Partial Logic Derivatives (4) modified SI is determined as:

$$I_{MS}(s_i|j) = \frac{\rho_i^{s,j}}{\rho_i^{(s,j)}} \tag{7}$$

where $\rho_i^{s,j}$ is defined in (6); $\rho_i^{(s_i,j)}$ is number of boundary system states when $\phi(s_i, \boldsymbol{x}) = j$ (it is computed by structure function (1)).

Modified SI I_{MS} is probability of MSS performance decrement depending on the i-th component state change and boundary system states. A system component with maximal value of the SI measure (I_S and I_{MS}) has most influence to MSS and this component failure causes high possibility of MSS failure [11], [13].

SI and modified SI measures don't depend on components state probability (2) and characterize only topological aspects of MSS performance. These measures are used for prevention system analysis or reliability analysis in step of a system design previously.

Birnbaum Importance (BI) of a given component is defined as the probability that such component is critical to MSS functioning [13], [14], [19]. This measure has been defined for traditional system with two states firstly as:

$$I_B(x_i) = |\Pr\{\varphi(x)=1,\ x_i=1\} - \Pr\{\varphi(x)=1,\ x_i=0\}| \quad .$$

But mathematical and logical generalization of this measure for MSS has some interpretations. So in paper [12] proposed definition of BI for system with two performance level that consists of multi-state components. Authors of the paper [14] considered definition of BI of MSS failure analysis. Than in paper [18], [20] new modifications of BI and algorithms for calculation based on different methodological approach have been proposed. One more interpretation of BI for MSS in terms of Logical Differential Calculus has been presented in paper [11]. According to this definition, BI is probabilistic measure that can be interpreted as rate at which the MSS fails as the i-th system component state decreases:

$$I_B(s_i \mid j) = \left|\Pr\{\phi(\boldsymbol{x})\,|_{x_i=s} = j\} - \Pr\{\phi(\boldsymbol{x})\,|_{x_i=s-1} = j\}\right|, \tag{8}$$

where

$$\Pr\{\varphi(x)|_{x_i=s}=j\}=\sum p_{1,a_1}\cdots p_{i-1,a_{i-1}}p_{i+1,a_{i+1}}\cdots p_{n,a_n}$$

if $\phi(\boldsymbol{x}) = j$ and $x_i = s$ for $a_w = \{0, \ldots, m\text{-}1\}$, $w = 1,\ldots, n$ and $w\neq i$; $s = \{1, \ldots, m_i\text{-}1\}$.

BI measures (8) depend on the structure of the system and states of the other components, but is independent of the actual state of the i-th component.

Consider the definition of *Criticality Importance* (CI) that is the probability that the i-th system component is relevant to MSS performance decrement if it has failed or has diminished state. For the system with two performance level this measure is considered in [19] in detail. For MSS this measure can be defined as probability of the MSS performance reduction if the state of the i-th system component has changed from s to s-1:

$$I_C(s_i)=I_B(s_i|j)\cdot\frac{p_{i,s-1}}{R(j)}, \tag{9}$$

where $I_B(s_i|j)$ is the i-th system component BI measure (8); $p_{i,s\text{-}1}$ is probability of the i-th system component state s-1 (2) and $R(j)$ is probability of system state j that is defined in accordance with (5); $s = \{1, \ldots, m_i\text{-}1\}$

The CI measure (9) correct BI for unreliability or lower state of the i-th component relative. This measure is useful, if the component has high BI and low probability of investigated state with respect of MSS performance decrement. In this case the i-th component CI is low.

Fussell-Vesely Importance (FVI) measure quantifying the maximum decrement in system reliability caused by the i-th system component state deterioration [12], [14]. By other words this measure represents the contribution of each component to the system and for the system with two performance levels is calculated by next equation [21]:

$$I_{FV}(x_i)=\frac{\Pr\{\varphi(x)=0\}-\Pr\{\varphi(x)=0|x_i=1\}}{\Pr\{\varphi(x)=0\}}.$$

FVI for MSS represents probabilistic measure of the i-th component state deterioration influence to the system performance level decrement:

$$I_{FV}(s_i|j)=1-\frac{\Pr\{\varphi(s_i,\boldsymbol{x})=j\}}{R(j)} \tag{10}$$

where

$\Pr\{s_i, \varphi(x)=j\}=\sum p_{1,a_1}\cdots p_{i,s_i}\cdots p_{n,a_n}$ if $\phi(\boldsymbol{x})=j$ and $x_i=s$ for $a_w = \{0, \ldots, m\text{-}1\}$, $w = 1,\ldots, n$ and $w\neq i$; $s = \{1, \ldots, m\text{-}1\}$.

The calculation of FVI measure is similar to algorithm for computation of BI measure.

Reliability Achievement Worth (RAW) and *Reliability Reduction Worth* (RRW) are two importance measures and both represent adjustments of the improvement potential to MSS unreliability. RAW for Binary-State System (system that has only two performance level as function and failure) indicates the increase in the system unreliability when the i-th component is failed and this measure is defined as [21]:

$$I_{RAW}(x_i)=\frac{\Pr\{\varphi(0_i,x)=0\}}{F}$$

According to the papers [12], [14] RAW for MSS is defined as the ratio of MSS unreliability if the i-th component state has decrease:

$$I_{RAW}(s_i|j)=\frac{\Pr\{\varphi(s_i-1,x)=j\}}{R(j)}, \tag{11}$$

where $s= \{1, \ldots, m\text{-}1\}$.

RRW can be interpreted as opposite importance measure to RAW and for Binary-State System is defined as [21]:

$$I_{RRW}(x_i)=\frac{F}{\Pr\{\varphi(1_i,x)=0\}}.$$

Generalization of this equation and representation of RRW in [12], [14] allows defining RRW for MSS as importance measure quantifies potential damage caused to the MSS by the i-th system component:

$$I_{RRW}(s_i|j)=\frac{R(j)}{\Pr\{\varphi(s_i,x)=j\}}, \tag{12}$$

where $s = \{1, \ldots, m\text{-}1\}$.

There is one more type of importance measures for MSS that are *Dynamic Reliability Indices* (DRIs). These measures have been defined in paper [11], [17]. DRIs allow to estimate component relevant to MSS and to quantify the influence of this component state change to the MSS performance. There are two groups of DRIs: *Component Dynamic Reliability Indices* (CDRIs) and *Dynamic Integrated Reliability Indices* (DIRIs).

CDRI indicates the influence of the i-th component state change to MSS performance level change [17]. This definition of CDRI is similar to definition of modified SI, but CDRIs for MSS failure take into consideration two probabilities: (*a*) the probability of MSS performance level decrease caused by the *i*-th component state reduction and (*b*) the probability of this component state:

$$I_{CDRI}(s_i|j)=I_{MS}(s_i|j)\cdot p_{i,s_i-1} \qquad (13)$$

where $I_S(x_i|j)$ is the modified SI (7); p_{i,s_i} is probability of component (2).

DIRI is the probability of MSS performance level decrement that caused by the one of system components state deterioration. DIRIs allow estimate probability of MSS failure caused by some system component (one of *n*):

$$\overset{q\neq i}{\text{¿}}$$

$$I_{DIRI}(s|j)=\sum_{\text{¿}}^{n}\text{¿¿}_{n} \qquad (14)$$

IV. Example of Healthcare system Importance Analysis

Consider the healthcare system in Fig. 1. The structure function of such healthcare system is declared based on an expertise for every real system. This function is defined based on the expert knowledge and influence form the area of the system application. For example, the Decision Support System for Early Diagnostics in Oncology (DSSEDO) has structure function (3) that is described in Table I. The MSS mathematical model of this system has three levels of performance (m = 3) and four components (n = 4). In Table I the system component state 0 considers to the component failure; the component state 1 is component functioning with some unimportant restriction; the component state 2 is perfect functioning. The component probabilities in Table III have been determined for this system by the expertise.

Table III.
Component probabilities

m / *i*	0	1	2
1	0.1	0.2	0.7
2	0.1	0.4	0.5
3	0.2	0.4	0.3
4	0.3	0.5	0.3

So the system state probabilities (5) for this healthcare system are calculated based on component probabilities in Table III:

$$R(0) = p_{1,0}+(p_{1,1}+p_{1,2})\cdot p_{2,0}\cdot(p_{3,0}+(p_{3,1}+p_{3,2})\cdot p_{4,0})=0.137,$$

$$R(1) = (p_{1,1}+p_{1,2})\cdot p_{2,0}\cdot(p_{3,1}+p_{3,2})\cdot(p_{4,1}+p_{4,2}) + p_{1,1}\cdot(p_{2,1}+p_{2,2}) + +p_{1,2}\cdot p_{2,1}\cdot(p_{3,0}+p_{3,1})\cdot(p_{4,0}+p_{4,1})=0.488,$$

$$R(2) = p_{1,2}\cdot p_{2,1}\cdot p_{3,2}\cdot p_{4,2}+p_{1,2}\cdot p_{2,2}=0.375$$

Therefore the performance level 1 of the healthcare system is more probably than system perfect functioning (the performance level 2) and system failure that have probabilities 0.137 and 0.375 accordingly.

Importance measures of this system are in Table IV. According to the data in Table IV the first system component change has maximal influence to the system reliability. Therefore correct functioning of special devices is important condition for reliability of the healthcare system with structure function (3). But need to say that the modification of the structure function of this system causes change of the importance analysis result. The positive result of importance analysis will be obtained based on investigation of some structure function of this system. The impediment for this analysis is caused by generation of structure function based on expert knowledge only that is subjective.

Table II.
Importance Measures for the system with structure function (3)

i / Importance measures	1	2	3	4
$I_S(x_i\|1)$	0.222	0.123	0.049	0.049
$I_S(x_i\|2)$	0.123	0.099	0.025	0.025
$I_S(x_i)$	0.173	0.111	0.027	0.027
$I_{MS}(x_i\|1)$	1	0.588	0.308	0.308
$I_{MS}(x_i\|2)$	1	0.889	0.400	0.400
$I_{MS}(x_i)$	1	0.739	0.354	0.354
$I_B(x_i\|1)$	0.456	0.248	0.063	0.010
$I_B(x_i\|2)$	0.800	0.452	0.050	0.105
$I_B(x_i)$	0.628	0.350	0.032	0.058
$I_C(x_i\|1)$	0.095	0.052	0.026	0.006
$I_C(x_i\|2)$	0.419	0.473	0.053	0.137
$I_C(x_i)$	0.257	0.263	0.040	0.072
$I_{CDRI}(x_i\|1)$	0.100	0.059	0.062	0.062
$I_{CDRI}(x_i\|2)$	0.200	0.356	0.160	0.200
$I_{CDRI}(x_i)$	0.150	0.208	0.111	0.131

V. Conclusion

In this paper new algorithms of IM calculation for MSS analysis are considered. These algorithms are implemented based on methods of MVL as Logical Differential Calculus and MDD. But investigated MSS has one principal assumption: system and all its components have m-1 different performance levels and state unreliability. In next investigation we are going to develop this mathematical approach for estimation of MSS without this assumption. Structure function of such MSS is defined as:

$$\phi(\boldsymbol{x})\text{: } \{0,\ldots, m_1-1\}\times\ldots\times\{0, \ldots, m_n-1\}\rightarrow\{0,\ldots,M-1\}.$$

In this case MSS consists of n components and has M levels of the performance rate from complete failure (this level corresponds with 0) to the perfect functioning (this level is interpreted as M-1). Each of n MSS components is characterized by different performance level and the i-th component has m_i possible states: from the complete failure (it is 0) to the perfect functioning (it is m_i-1).

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