

Validation of Data Categorization Using Extensions of Information Systems: Experiments on Melanocytic Skin Lesion Data

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Abstract—The purpose of data categorization is to group similar cases (items, examples, objects, etc.) together under a common label so that information can be acted upon in the aggregate form. Sometimes, this process is made arbitrary by an expert. For each case, an expert determines a class (group) to which the case is classified. In the paper, we propose a method for validation of a categorization process. The method is based on extensions of information systems defined in terms of rough sets. Usefulness of the proposed method is shown for the data used in the synthesis of images of melanocytic skin lesions.

Index Terms—extensions of information systems, data categorization, rough sets, synthesis of images

I. INTRODUCTION

AN INFORMATION system proposed by Z. Pawlak [1] can represent a finite set of cases described by attributes. Each attribute represents one of features of cases. Besides all cases appearing in the original information system, an extension of it can include new cases, which have not been observed yet, but which are consistent to a certain degree with the knowledge included in the original system. The knowledge can be represented in the form of rules (production, association, etc.). One can consider only the so called consistent extensions of information systems [2], when all new cases are totally consistent with the knowledge included in the original information systems. However, in general case, we can consider partially consistent extensions [3], when some new cases are consistent with the knowledge possessed, only to a certain degree. The important problem is to determine consistency factors of new cases taking into consideration different ways of knowledge representation.

In this paper, we use an approach to computing consistency factors based on rough set theory and proposed in [3]. The algorithm using that approach has been presented in [4]. In Section III we recall it. This algorithm makes use of important results of research on extensions of information systems given in [5]. In the algorithm, we assume that the knowledge included in an original information system S is expressed by minimal rules true and realizable in S (see Section III). Computing a consistency factor for a given object is based on determining importance (relevance) of rules extracted from the

system S which are not satisfied by the new case. We assume that if the importance of these rules is greater the consistency factor of a new object with the knowledge is smaller. The importance of a set of rules not satisfied by the new case is determined by means of a strength factor of this set of rules in S .

As an example, we consider phenomena related to melanocytic skin lesions. In the process of validation of categorization of cases for respective types of lesions, we can use extensions of information systems. A consistency factor calculated for a given text vector (case) representing combination of colors and structures can be considered in terms of possibility theory proposed by L. Zadeh [6]. This factor enables us to answer a question about possibility with which a given combination of colors and structures appears in a respective melanocytic skin lesion. If we have an information system consisting of information about combinations of colors and diversities of structures appearing for given lesions, it means that we have collected all cases observed until now. In many situations, we have observed only a part of all possible combinations of colors and diversities of structures. In this case, if we take a new combination which has not been observed yet, then the following question Q arises: "Is it possible that the combination (case) will appear for a given lesion?". In this question, "possible" means "plausible". We wish to answer the question Q on the basis of the possessed information collected until now in the information system S (observed combinations for a given lesion). We can determine a possibility distribution on all combinations of colors and diversities of structures.

We can identify consistency factors of cases in the extension of a given information system with a possibility distribution of cases. Possibility theory [6] is a framework for representing vague and incomplete knowledge. Let X be a set that represents the range of a variable x . A possibility distribution π_x on X is a mapping from X to the unit interval $[0, 1]$ attached to the variable x . The variable x can be treated as some phenomenon \mathcal{P} . The range of x represents the set of cases of \mathcal{P} . The function π_x distinguishes which case of \mathcal{P} is plausible and which one is less plausible. Let $u \in X$, $\pi_x(u) = 0$ means

that $x = u$ is impossible. $\pi_x(u) = 1$ means that $x = u$ is totally possible, i.e., plausible. The quantity $\pi_x(u)$ represents the degree of possibility of the assignment $x = u$.

II. PROBLEM BACKGROUND

In our research, we use a database on melanocytic skin lesions including 548 cases belonging to one of four types of lesions: *Benign nevus*, *Blue nevus*, *Suspicious nevus*, and *Melanoma malignant*. Each case in a database is recorded as a 15-element text vector constituting the input information in the process of synthesis of static images of melanocytic skin lesions (cf. [7], [8], [9]). We have noticed at the current research state that a key role is played by two features of melanocytic skin lesions, namely, *Color* and *Diversity of structure*. These features have a multivalued character and describe the presence or absence of colors and diversities of structures allowed by the ABCD rule [10]. *Color* can have six allowed values: *black*, *blue*, *dark-brown*, *light-brown*, *red*, and *white*, whereas *Diversity of structure* can have five allowed values: *branched streaks*, *pigment dots*, *pigment globules*, *pigment network*, and *structureless area*. Assessment of a tinge of a skin lesion consists in differentiating any number of colors (from the set of six allowed colors). Assessment of structural elements in a skin lesion consists in determining any number of structures (from the set of five allowed diversities of structures). The remaining attributes of the text vector are *Assymetry*, *Border*, and *TDS (Total Dermatoscopy Score)*. In Table I, we present the collation of information about attributes of the text vector used in our experiments. For each allowed *color* and *diversity of structure*, we have one attribute taking one of logical values: 1 (denoting presence) or 0 (denoting absence).

TABLE I
ATTRIBUTES CORRESPONDING TO FEATURES: *Color* AND *Diversity of structure*

Feature	Attribute	Value set
<i>Color</i>	<i>black</i>	{0, 1}
	<i>blue</i>	{0, 1}
	<i>dark-brown</i>	{0, 1}
	<i>light-brown</i>	{0, 1}
	<i>red</i>	{0, 1}
	<i>white</i>	{0, 1}
<i>Diversity of structure</i>	<i>branched streaks</i>	{0, 1}
	<i>pigment dots</i>	{0, 1}
	<i>pigment globules</i>	{0, 1}
	<i>pigment network</i>	{0, 1}
	<i>structureless area</i>	{0, 1}

Synthesis of colors and structures of a lesion should consider multi-valued character of *Color* and *Diversity of structure* features, capable to create a considerable number of combinations of these parameters, which can simultaneously appear in a given lesion.

According to the ABCD rule, a number of all possible mappings of colors and structures is $2^{11} - 95$. 95 is a number of inadmissible combinations. Inadmissible combinations are: combinations without any color (64 cases) and combinations without any structure (32 cases). One of combinations does not have both any color and any structure, hence 95. A real number

of mappings can be greater in view of different mappings of given colors in respective structures. The ABCD rule does not define such mappings. Therefore, a number of all possible combinations is $2^{30} + 1$ minus 95 inadmissible combinations. Generating over one billion textures is virtually almost impossible. An additional disadvantage of such mapping would be a very frequent repetition of occurrence of selected structures in particular colors, because of its shape, size and place in the generated image. Taking all these circumstances into consideration the synthesis requires a special approach, we should initially (before generating textures) find which colors and structures occur simultaneously in real lesions.

In the process of validation of choices of combinations for respective types of lesions, we use extensions of information systems described in the next section. A consistency factor calculated for a given text vector representing the combination of colors and structures is considered in terms of possibility theory. This factor enables us to answer a question about possibility with which a given combination of colors and structures appears in a respective melanocytic skin lesion.

III. EXTENSIONS OF INFORMATION SYSTEMS

In this section, we recall crucial notions concerning rough sets, information systems, rules, as well as, extensions of information systems. For more exact description and explanation we refer readers to [5], [11].

An information system is an ordered pair $S = (U, A)$, where U is a non-empty, finite set of objects which is also called universum, A is a non-empty, finite set of attributes. Each attribute $a \in A$ is a total function $a : U \rightarrow V_a$, where V_a is a set of values of the attribute a . Each information system $S = (U, A)$ can be presented in the form of a data table. Columns are labeled with attributes from A whereas rows are labeled with objects from U . Cells of the table include values of appropriate attributes. A decision system is an information system $S = (U, A)$, where $A = C \cup D$ and $C \cap D = \emptyset$. C is a set of condition attributes (in short, conditions) whereas D is a set of decision attributes (in short, decisions).

Let $S = (U, A)$ be an information system. Each subset $B \subseteq A$ of attributes determines an equivalence relation on U , called an *indiscernibility relation* $Ind(B)$, defined as

$$Ind(B) = \{(u, v) \in U \times U : \forall a \in B a(u) = a(v)\}.$$

The equivalence class containing $u \in U$ will be denoted by $[u]_B$.

Let $X \subseteq U$ and $B \subseteq A$. The *B-lower approximation* $\underline{B}X$ of X and the *B-upper approximation* $\overline{B}X$ of X are defined as

$$\underline{B}X = \{u \in U : [u]_B \subseteq X\}$$

and

$$\overline{B}X = \{u \in U : [u]_B \cap X \neq \emptyset\},$$

respectively.

Dependencies among values of attributes in an information system may be expressed by means of the so-called rules. Each rule ρ considered by us in the information system $S = (U, A)$

has the form $(a_{i_1}, v_{i_1}) \wedge (a_{i_2}, v_{i_2}) \wedge \dots \wedge (a_{i_r}, v_{i_r}) \Rightarrow (a_d, v_d)$, where $a_d \in A$ and $v_d \in V_{a_d}$, while $a_{i_j} \in B \subseteq A - \{a_d\}$ and $v_{i_j} \in V_{a_{i_j}}$ for $j = 1, 2, \dots, r$. The rule ρ is a satisfiable (true) rule if for each object $u \in U$: if $a_{i_1}(u) = v_{i_1} \wedge a_{i_2}(u) = v_{i_2} \wedge \dots \wedge a_{i_r}(u) = v_{i_r}$, then $a_d(u) = v_d$. The rule ρ is a minimal rule if removing any atomic formula (a_{i_j}, v_{i_j}) , where $j = 1, 2, \dots, r$, from the predecessor of a rule makes this rule not true in S . The rule ρ is a realizable rule if there exists any object $u \in U$ such that $a_{i_1}(u) = v_{i_1} \wedge a_{i_2}(u) = v_{i_2} \wedge \dots \wedge a_{i_r}(u) = v_{i_r}$. A set of all minimal rules satisfiable and realizable in the information system S will be denoted by $Rul(S)$. By $Rul_a(S)$ we will denote the set of all rules from $Rul(S)$ having an atomic formula containing the attribute a in their successors.

Let $S = (U, A)$ be an information system. An information system $S^* = (U^*, A^*)$ is an extension of S if and only if:

- $U \subseteq U^*$,
- $card(A) = card(A^*)$,
- for each $a \in A$, there exists $a^* \in A^*$ such that a function $a^* : U^* \rightarrow V_{a^*}$ is an extension of a function $a : U \rightarrow V_a$ to U^* .

We may admit also situation when $a^* : U^* \rightarrow V_{a^*}$, where $V_{a^*} \subset V_a$, for any $a^* \in A^*$. It means that we can add new objects to a given information system S that have new values of attributes not existing yet in S . If $V_{a^*} = V_a$ for each $a^* \in A^*$, then S^* will be called a proper extension of S , otherwise S^* will be called a non-proper extension of S .

A set A^* of attributes in the extension $S^* = (U^*, A^*)$ of an information system $S = (U, A)$ can be also denoted by A like in the original system S . So, we write $S^* = (U^*, A)$ instead of $S^* = (U^*, A^*)$. The same applies to attributes of A^* , i.e., $a_1^*, a_2^*, \dots, a_m^* \in A^*$. So, we write a_1, a_2, \dots, a_m instead of $a_1^*, a_2^*, \dots, a_m^*$, where $a_1, a_2, \dots, a_m \in A$.

For any object u from the extension S^* of a given information system S , we define a coefficient called a consistency factor. This coefficient expresses a degree of consistency of u with the knowledge (expressed by $Rul(S)$) included in the original system S . The procedure for computing a consistency factor is described here.

For each attribute $a \in A$ of a given information system S and a new object u^* added to S we can translate an information system S into the information system $S_{a,u^*} = (U_a, C_a \cup \{a\})$ with irrelevant values of attributes. Such a system will be called the a - u^* match of S .

Let $S = (U, A)$ be an information system, $S^* = (U^*, A)$ its extension, and $u^* \in U^*$ a new object from the extension S^* . The a - u^* match of S is an information system $S_{a,u^*} = (U_a, C_a \cup \{a\})$ with irrelevant values of attributes created in the following way. Each attribute $c' \in C_a$ corresponds exactly to one attribute $c \in A - \{a\}$. Each object $u' \in U_a$ corresponds exactly to one object $u \in U$ and moreover:

$$c'(u') = \begin{cases} c(u) & \text{if } c(u) = c(u^*) \\ * & \text{otherwise} \end{cases}$$

for each $c' \in C_a$, and $a(u') = a(u)$.

If we create the a - u^* match of S , then we create a new information system for which appropriate sets of attribute values are extended by the value $*$. The symbol $*$ means that a given value of the attribute is not relevant.

For simplicity, the attribute $c \in A - \{a\}$ in S and the attribute c' in S_{a,u^*} corresponding to c will be marked with the same symbol, i.e., c' will be marked in S_{a,u^*} with c .

The system S_{a,u^*} can be treated as a decision system with condition attributes constituting the set C_a and the decision attribute a .

For the information system $S_{a,u^*} = (U_a, C_a \cup \{a\})$, we define a characteristic relation $R(C_a)$ similarly to the definition of a characteristic relation in information systems with missing attribute values (cf. [12]). $R(C_a)$ is a binary relation on U_a defined as follows $R(C_a) = \{(u, v) \in U_a \times U_a : \exists c \in C_a c(u) \neq * \text{ and } \forall c \in C_a (c(u) \neq *) \Rightarrow (c(u) = c(v))\}$. For each $u \in U_a$, a characteristic set $K_{C_a}(u)$ has the form $K_{C_a}(u) = \{v \in U_a : (u, v) \in R(C_a)\}$. Let $X \subseteq U_a$. The C_a -lower approximation of X is determined as $\underline{C_a}X = \{u \in U_a : K_{C_a}(u) \neq \emptyset \text{ and } K_{C_a}(u) \subseteq X\}$. Let $S = (\overline{U}, A)$ be an information system, $a \in A$, and $v_a \in V_a$. By $X_a^{v_a}$ we denote the subset of U such that $X_a^{v_a} = \{u \in U : a(u) = v_a\}$.

Let $S = (U, A)$ be an information system, $S^* = (U^*, A)$ its extension, $u^* \in U^*$ a new object from the extension S^* , and $a \in A$. The object u^* satisfies a rule $\rho \in Rul_a(S)$ if and only if for each $v_a \in V_a$ if $\underline{C_a}X_a^{v_a} \neq \emptyset$, then $a(u^*) = v_a$. A proof can be found in [4].

An approach recalled here does not involve computing any rules from an original information system. The algorithm presented here allows us to determine a set of objects from an original information system S supporting minimal rules from $Rul(S)$, but not satisfied by the object u^* . A consistency factor is computed as a complement to 1 of the strength of the set of rules not satisfied. Let $S = (U, A)$ be an information system, $\{V_a\}_{a \in A}$ is a family of sets of attribute of values in S , $Rul(S)$ a set of all minimal rules true and realizable in S , $S^* = (U^*, A)$ an extension of S , and $u^* \in U^*$. The consistency factor $\xi_S(u^*)$ of u^* with the knowledge (expressed by $Rul(S)$) is defined as follows:

$$\xi_S(u^*) = \xi'_S(u^*) \omega_S(u^*),$$

where:

- $\xi'_S(u^*) = 1 - \frac{card(\tilde{U})}{card(U)}$ is a proper consistency,
- $\omega_S(u^*) = \frac{card(\{a \in A : a(u^*) \in V_a\})}{card(A)}$ is a resemblance factor determining some affinity between the object u^* and objects from S with respect to values of attributes,

and $\tilde{U} = \bigcup_{a \in A} \bigcup_{v_a \in V_a} \{\underline{C_a}X_a^{v_a} : \underline{C_a}X_a^{v_a} \neq \emptyset \wedge a(u^*) \neq v_a\}$.

The estimated time complexity of the algorithm has the form:

$$\Theta(|A|^2|U| + |A||V_a||U|^2),$$

where $|A|$ is the cardinality of a set of attributes, $|U|$ is the cardinality of a set of cases, $|V_a|$ is the maximal cardinality of a set of attribute values.

Algorithm 1: Algorithm for computing a consistency factor $\xi_S(u^*)$ of u^* with the knowledge expressed by $Rul(S)$

Input : An information system $S = (U, A)$, a new object u^* added to S .

Output: A consistency factor $\xi_S(u^*)$ of u^* with the knowledge expressed by $Rul(S)$.

$\tilde{U} \leftarrow \emptyset;$

$i \leftarrow 0;$

for each $a \in A$ **do**

if $a(u^*) \notin V_a$ **then**

$i \leftarrow i + 1;$

end

 Create the match $S_{a,u^*} = (U_a, C_a \cup \{a\})$ of S ;

for each $v_a \in V_a$ **do**

$X_a^{v_a} \leftarrow \{u \in U : a(u) = v_a\};$

if $C_a X_a^{v_a} \neq \emptyset$ **then**

if $a(u^*) \neq v_a$ **then**

$\tilde{U} \leftarrow \tilde{U} \cup C_a X_a^{v_a};$

end

end

end

$\xi'_S(u^*) \leftarrow 1 - \frac{\text{card}(\tilde{U})}{\text{card}(U)};$

$\omega_S(u^*) \leftarrow \frac{i}{\text{card}(A)};$

$\xi_S(u^*) \leftarrow \xi'_S(u^*) \omega_S(u^*);$

end

return $\xi_S(u^*);$

IV. EXPERIMENTS

In our experiments, we have used a database on melanocytic skin lesions including 548 cases, each belonging to one of four types of lesions:

- *Benign nevus* - 248 cases,
- *Blue nevus* - 78 cases,
- *Suspicious nevus* - 108 cases,
- *Melanoma malignant* - 114 cases.

Categories have been assigned to cases by clinicians.

The experiments have been carried out according to Procedure 2. We have distinguished two tests:

- 1) One information system containing cases belonging to the same category (melanocytic skin lesion) was an original information systems S , another one containing cases belonging to another category was treated as an extension of S (possibly non-proper).
- 2) An information system containing cases belonging to the same category (melanocytic skin lesion) was split randomly into two disjoint subsystems. The first part (greater) constituted an original system, the second one - its extension (possibly non-proper).

For each case from the extension of the original information system S , we have calculated a consistency factor with the knowledge included in S according to Algorithm 1. In the

Procedure for experiments

Input : An original (training) information system $S_{train} = (U_{train}, A)$, a testing information system $S_{test} = (U_{test}, A)$.

Output: A testing information system S_{test} with consistency factors assigned to objects in S_{test} with the knowledge expressed by $Rul(S_{train})$ using Algorithm 1.

for each $u \in U_{train}$ **do**

 Calculate a consistency factor $\xi_S(u)$ of u with the knowledge expressed by $Rul(S_{train})$ according to Algorithm 1;

 Assign $\xi_S(u)$ to u ;

end

return $S_{test};$

approach presented in this paper, we expect the following situations:

- 1) If a given case belongs to a different category from the category of cases in the original system, then the consistency factor should be smaller.
- 2) If a given case belongs to the same category as cases in the original system, then the consistency factor should be greater.

In Table II and III, we present aggregated results (average consistency factors) of our experiments. Results have partly confirmed our expectations. Some exceptions can indicate two directions for further research:

- 1) Some combinations (cases) of colors and diversities of structures used in the synthesis of images of melanocytic skin lesions are incorrectly categorized. It is indication that an informational data base should be verified.
- 2) The proposed method needs some improvement (tuning) for better differentiating between cases belonging to different categories.

V. CONCLUSION

In the paper, we have proposed a method for validation of a categorization process. The method is based on extensions of information systems defined in terms of rough sets. Usefulness of the proposed method has been shown for the data used in the synthesis of images of melanocytic skin lesions. Obtained results have indicated directions for further research. The first direction is a verification of an informational data base of combinations of colors and diversities of structures used in the synthesis of images of melanocytic skin lesions. The second one concerns further developing of the proposed methodology.

ACKNOWLEDGMENTS

This paper has been partially supported by the grant No. N 516 482640 from the National Science Centre in Poland.

TABLE II
RESULTS OF EXPERIMENTS CARRIED OUT FOR CASES BELONGING TO DIFFERENT CATEGORIES

Original information system	Tested information system	Average coefficient factor
<i>Benign nevus</i>	<i>Blue nevus</i>	0.28
<i>Benign nevus</i>	<i>Suspicious nevus</i>	0.87
<i>Benign nevus</i>	<i>Melanoma malignant</i>	0.74
<i>Blue nevus</i>	<i>Benign nevus</i>	0.30
<i>Blue nevus</i>	<i>Suspicious nevus</i>	0.39
<i>Blue nevus</i>	<i>Melanoma malignant</i>	0.41
<i>Suspicious nevus</i>	<i>Benign nevus</i>	0.64
<i>Suspicious nevus</i>	<i>Blue nevus</i>	0.27
<i>Suspicious nevus</i>	<i>Melanoma malignant</i>	0.72
<i>Melanoma malignant</i>	<i>Benign nevus</i>	0.61
<i>Melanoma malignant</i>	<i>Blue nevus</i>	0.36
<i>Melanoma malignant</i>	<i>Suspicious nevus</i>	0.81

TABLE III
RESULTS OF EXPERIMENTS CARRIED OUT FOR CASES BELONGING TO THE SAME CATEGORIES

Original information system	Tested information system	Average coefficient factor
<i>Benign nevus</i>	<i>Benign nevus</i>	0.89
<i>Blue nevus</i>	<i>Blue nevus</i>	0.74
<i>Suspicious nevus</i>	<i>Suspicious nevus</i>	0.82
<i>Melanoma malignant</i>	<i>Melanoma malignant</i>	0.84

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