

# The influence of a matrix condition number on iterative methods' convergence

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**Abstract**—We investigate condition numbers of matrices that appear during solving systems of linear equations. We consider iterative methods to solve the equations, namely Jacobi and Gauss-Seidel methods. We examine the influence of the condition number on convergence of these iterative methods. We study numerical aspects of relations between the condition number and the size of the matrix and the number of iterations experimentally. We analyze random matrices, the Hilbert matrix and a strictly diagonally dominant matrix.

## I. INTRODUCTION

THE CONDITION number plays an important role in the numerical linear algebra. The condition number measures the sensitivity of the solution of a problem to perturbations in the data. It provides an approximate upper bound on the error in a computed solution. The condition number depends on the norm used. The condition number can also be used to predict the convergence of iterative methods.

A lot of articles [1], [2], [3], [7] deal with problems associated with the condition number in terms of mathematical theory. In those papers authors did not examine experimental and numerical aspects of the condition number. This paper investigates numerical aspects of relations between the condition number and the size of the matrix and the number of iterations experimentally.

The convergence of iterative methods depends on the condition number of the coefficient matrix describing the system. That is the subject of this work. In order to determine its impact on the convergence of an iterative process, the accuracy of the Jacobi and Gauss-Seidel method will be tested. We examine some kinds of special matrices, namely random matrices, the Hilbert matrix, a strictly diagonally dominant matrix and relations between the size of the matrix, the condition number and the convergence of iterative methods.

The paper is set up as follows. In Section II the condition number of a matrix have been presented [4], [5], [6]. Section III is a report from an experimental research about the condition number. It contains the results of tests on the condition number of random matrices, the Hilbert matrix [2], a strictly diagonally dominant matrix. It shows the results of

testing the accuracy of iterative methods on the basis of which the convergence of these methods is determined. Section IV concludes the paper.

## II. THE CONDITION NUMBER

The convergence of an iteration process and the existence of an equation linear system solution depends on the form of the matrix  $A$ . If the square matrix of the system  $Ax = b$  is singular, then this system does not have a solution. On the contrary, when  $\det A \neq 0$ , it is the condition number of a matrix that decides about the convergence of the approximate solution, obtained in the iteration process, to the correct solution of the equation system. The condition number is defined by a formula:

$$\kappa(A) = \|A\| \cdot \|A^{-1}\|.$$

The value of condition number is dependent on the choice of a matrix norm, and indirectly on the choice of a vector norm. When it is necessary to show this choice we use the following symbols:  $\kappa_{\infty}(A)$  (the chosen norm is infinity norm — of  $l_{\infty}$ ),  $\kappa_1(A)$  (for the matrix norm of  $l_1$ ); and so on.

The condition number is strongly connected to the accuracy of the approximate solution of the equation system. In order to calculate the accuracy, we must appoint a residual vector, with the aid of a formula:

$$r = b - Ax^{(k)}$$

where  $A$  is a square matrix of the system  $Ax = b$ ,  $b$  is a free elements' vector, and  $x^{(k)}$  is the  $k$ -th approximation solution of the system, which is calculated with the use of numerical methods. Taking the norm of the residual vector, we get a number which defines the accuracy with which the obtained solution approximates the correct value of the solution. This accuracy can be calculated with the use of:

$$\varepsilon = \|r\| = \|b - Ax^{(k)}\|.$$

A condition number of a matrix is always bigger than or equal to 1. Equality is reached when the obtained solution is the correct solution; for example, a condition number of the unit matrix is equal to 1. If a condition number is not “too big”, then the matrix  $A$  is said to be well-conditioned. It means that the result was obtained with a good accuracy. A matrix

This work was partially supported within the project N N516 479640 of the Ministry of Science and Higher Education of the Polish Republic (MNiSW) *Modele dynamiki transmisji, sterowania zatkanieniem i jakością usług w Internecie*.

with a big condition number  $\kappa(A)$  (called an ill-conditioned matrix) can generate approximations with a huge error.

It is not precisely defined what means “too big” in the above context. This paper hopes to help understand it.

An example of an ill-conditioned matrix is the matrix of the following system:

$$\begin{cases} 10^{-18}x_1 + x_2 = 1, \\ x_1 + 2x_2 = 4. \end{cases} \quad (1)$$

Its correct solution is

$$\begin{aligned} x_1 &= 2.000000000000000004, \\ x_2 &= 0.9999999999999997999999999999999996. \end{aligned}$$

On the other hand, with the use of the iterative Jacobi method, after 10 iteration steps, we get

$$\begin{aligned} x_1 &= -6.21875000000000 \cdot 10^{88}, \\ x_2 &= -3.09375000000000 \cdot 10^{88}. \end{aligned}$$

We can easily notice that these approximations differ considerably from the real solutions.

### III. NUMERICAL EXPERIMENTS

The main goal of the tests was to check the behavior of the condition number for some matrices and its influence on the accuracy of the iterative Jacobi and Gauss-Seidel methods. The tests were carried out for some matrices generated by the authors. In the following subsection we show the test matrices.

#### A. The studied matrices

##### Matrix 1

$$\begin{bmatrix} 2 & -1 & 0 \\ 1 & 6 & -2 \\ 4 & -3 & 8 \end{bmatrix}$$

In order to examine the condition number of this matrix, we examine the system:

$$\begin{bmatrix} 2 & -1 & 0 \\ 1 & 6 & -2 \\ 4 & -3 & 8 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \\ 5 \end{bmatrix}.$$

##### Matrix 2

The diagonally dominant matrix is generated by a computer program which random numbers from the range  $\langle 0; 1 \rangle$  assigns to the system’s coefficients. Additionally, to make the matrix diagonally dominant, the elements on diagonal are equal to the sum of the elements in the given row, increased by 30. The size of the matrix is  $50 \times 50$ .

##### Matrix 3

$$\begin{bmatrix} 4 & -1 & 3 \\ -7 & 8 & 1.5 \\ -8 & 2 & -6 \end{bmatrix}$$

We try to solve the system:

$$\begin{bmatrix} 4 & -1 & 3 \\ -7 & 8 & 1.5 \\ -8 & 2 & -6 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 7.5 \\ -4 \end{bmatrix}.$$

TABLE I

THE MATRICES’ CONDITION NUMBER VALUES CALCULATED WITH THE USE OF TWO DIFFERENT NORMS

#	singular?	$\kappa_1$	$\kappa_\infty$
1	No	8.50E+00	7.80E+00
2	No	2.30E+00	2.26E+00
3	Yes	—	—
4	No	2.84E+04	2.84E+04
5	No	6.11E+19	7.06E+19
6	No	9.00E+00	9.00E+00

This matrix is a singular matrix, so this system does not have a normal solution.

##### Matrix 4

$$\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} \\ \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} \end{bmatrix}$$

Matrix 4 is a  $4 \times 4$  Hilbert matrix. Its elements are calculated like this:

$$h_{ij} = \frac{1}{i + j - 1}.$$

##### Matrix 5

Matrix 5 is a  $50 \times 50$  Hilbert matrix.

##### Matrix 6

$$\begin{bmatrix} 10^{-18} & 1 \\ 1 & 2 \end{bmatrix}$$

Here we solve the system (1):

$$\begin{bmatrix} 10^{-18} & 1 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}.$$

#### B. The condition number study

Table I presents the researched matrices’ condition number values calculated with the use of two norms.

We can see that there is some correlation between the kind of the matrix and its condition number’s value. Among the researched ones, the only matrix with the determinant equal to zero is Matrix 3, and that is why this linear system does not have any solution, and we cannot compute its condition number.

We should notice considerably large condition numbers of the Hilbert matrices (Matrices 4 and 5). They are much bigger than the other ones what suggests a hypothesis that for Hilbert matrices iterative methods are not convergent. On the other hand, system 2 is certainly convergent, whose condition number in any norm does not exceed 2.5, while the condition number of the Hilbert matrix with the size  $4 \times 4$  is as large as ten thousand.

Quite interesting are solutions given for Matrices 1 and 6. These matrices have very small sizes and their condition numbers in the same norms are very similar. However, Matrix 1 is convergent, and Matrix 6 is divergent in terms of iterative methods. In order to explain this situation more precisely, we carried out next tests, results of which are given in the next part of this article.

TABLE II

THE RELATIONSHIP OF THE NUMERICAL METHODS' ACCURACY WITH THE CONDITION NUMBER OF MATRICES

#	Jacobi method		Gauss-Seidel method		$\approx_1$
	10 iter.	100 iter.	10 iter.	100 iter.	
1	5.264E-03	0.00E+00	1.47E-05	0.00E+00	8.50E+00
2	1.93E-02	1.00E-17	4.15E-09	8.24E-18	2.30E+00
3	—	—	—	—	—
4	4.29E+05	5.12E+42	7.67E+00	1.23E+00	2.84E+04
5	1.67E+19	8.71E+165	4.34E+02	4.34E+02	6.11E+19
6	1.28E+84	3.64E+835	9.67E+166	7.81E+1669	9.00E+00

C. The iterative methods' convergence study

We can easily notice the relationship between the condition number of matrices and the iterative methods' convergence when we analyze the accuracy with which the obtained solution approaches a correct solution of the system. Table II shows this well.

We know that the smaller the accuracy approximation value is, the better the obtained result approaches the correct solution's value. "The accuracy equal to zero" means that a given approximation is the correct solution of the equation system. Comparing obtained information, we can notice that for Matrices 1 and 2, the accuracy's values are very small and they decrease in the successive iterations, going to zero, meaning the convergence of the methods.

It is different in the case of Matrices 4, 5 and 6 where the error increases very fast with every iteration. It means divergence.

For the Hilbert matrices (4 and 5) and Matrix 2, the relationship between a condition number of the matrix and the accuracy, or the iterative methods' convergence is very visible. This relationship is not clear in the case of Matrices 1 and 6. The condition numbers of these matrices are very similar, but it is clearly visible that for Matrix 1 numerical methods are convergent and from some moment they give a correct solution of the linear equation system while Matrix 6 acquires immense errors after some iterations.

Therefore, in order to show the relationship between the condition number of matrices and iterative methods' convergence, we carry out next tests, the solutions of which can be found in the next part of this article. For Matrix 3, we can not set any values, because its determinant is equal to 0 and the calculations are impossible, therefore in the next part of this article singular matrices are not be considered.

D. The condition number of Hilbert matrices and of diagonally dominant matrices study

In order to better present the relationship of the condition number and the kind of the matrix, we made a more detailed examination, the results of which can be found in Tables III and IV. We made tests for Hilbert matrices and for matrices generated in an analogous way to Matrix 2. The calculations were made in norms of  $l_1$  and  $l_\infty$  for matrices of different sizes.

When we analyze the data in Tables III and IV, we can easily notice that the condition number of Hilbert matrices

TABLE III

THE CONDITION NUMBER VALUES OF HILBERT MATRICES AND OF DIAGONALLY DOMINANT MATRICES IN THE NORM OF  $l_1$

size of matrix	$\approx_1$	
	Hilbert matrix	matrix of type 2
1	1.00E+00	1.00E+00
2	2.70E+01	1.04E+00
3	7.48E+02	1.10E+00
4	2.84E+04	1.15E+00
5	9.44E+05	1.19E+00
6	2.91E+07	1.22E+00
7	9.85E+08	1.31E+00
8	3.39E+10	1.31E+00
9	1.10E+12	1.35E+00
10	3.54E+13	1.40E+00
15	2.81E+21	1.62E+00
20	1.30E+22	1.69E+00
25	5.88E+21	1.86E+00
30	5.71E+22	1.92E+00
35	1.73E+23	2.00E+00
40	1.06E+23	2.09E+00
45	1.95E+23	2.27E+00
50	1.35E+23	2.36E+00

TABLE IV

THE CONDITION NUMBER VALUES OF HILBERT MATRICES AND OF DIAGONALLY DOMINANT MATRICES IN THE NORM OF  $l_\infty$

size of matrix	$\approx_\infty$	
	Hilbert matrix	matrix of type 2
1	1.00E+00	1.00E+00
2	2.70E+01	1.04E+00
3	7.48E+02	1.09E+00
4	2.84E+04	1.13E+00
5	9.44E+05	1.19E+00
6	2.91E+07	1.24E+00
7	9.85E+08	1.28E+00
8	3.39E+10	1.29E+00
9	1.10E+12	1.30E+00
10	3.54E+13	1.35E+00
15	2.81E+21	1.66E+00
20	1.27E+22	1.66E+00
25	5.84E+21	1.79E+00
30	5.92E+22	1.89E+00
35	1.21E+23	2.00E+00
40	1.22E+23	2.06E+00
45	1.27E+23	2.09E+00
50	1.03E+23	2.26E+00

is much bigger than the condition number of diagonally dominant matrices (Fig. 1).

However, a condition number's value does not depend only on the kind of the matrix but also on the size of the system that is examined. This relationship is direct, but for a randomly generated diagonally dominant matrices the condition number increases very slowly, while for Hilbert matrices it grows very rapidly. As early as for the size  $8 \times 8$ , the condition number of the Hilbert matrix reaches value  $3.39E+10$ , while for the diagonally dominant matrix of the same size, the condition number is only 1.3121 in the norm of  $l_1$  and 1.3166 in the norm of  $l_\infty$ . We can see that for Hilbert matrices, beginning from the size  $25 \times 15$ , the condition number, though huge, does not grow so fast but rather oscillates around some very high value.

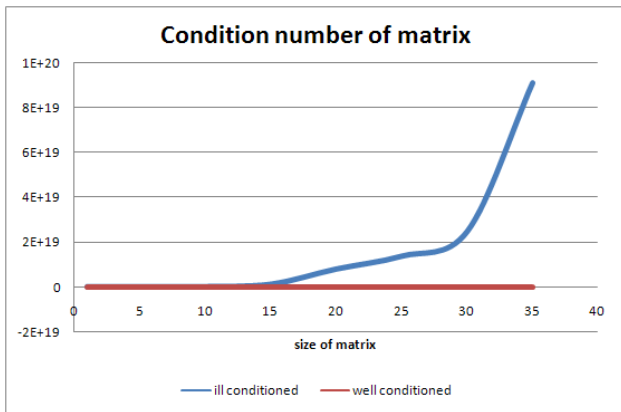


Fig. 1. The graph of the condition number of Hilbert matrices and diagonally dominant matrices

TABLE V  
THE RELATIONSHIP OF THE ITERATIVE METHODS' ACCURACY AND HILBERT MATRICES' CONDITION NUMBER

size of matrix	number of iterations	Jacobi method	Gauss-Seidel method	$\approx_1$
5	10	7.61E+06	8.16E+00	9.442E+05
	25	8.66E+14	3.02E+00	
	50	2.32E+28	1.98E+00	
	75	6.19E+41	2.02E+00	
	100	1.65E+55	1.93E+00	
10	10	6.94E+10	2.52E+01	3.54E+13
	25	1.61E+24	1.99E+01	
	50	3.02E+46	1.40E+01	
	75	5.68E+68	9.54E+00	
20	10	3.28E+14	7.69E+01	1.30E+22
	25	5.95E+32	4.66E+01	
	50	1.61E+63	2.63E+01	
	75	4.35E+93	2.65E+01	
40	10	1.22E+18	2.97E+02	1.06E+2
	25	1.12E+41	1.44E+02	
	50	2.08E+79	1.18E+02	
	75	3.88E+117	7.70E+01	
	100	7.22E+155	6.25E+01	

The obtained results also show that the condition number's values calculated in different norms for the same matrices, which are of the same size, are very close to each other. We can say that the kind of the norm has little influence on the value of the condition number of a matrix.

*E. The examination of the iterative methods' convergence for Hilbert matrices and diagonally dominant matrices*

The results of examination presented in the previous subsection show how strongly a condition number depends on the kind of the matrix. The tests of the iterative methods' accuracy given in Tables V and VI show, that there exists very strong relationship between the condition number's value and the iterative methods' convergence.

Iterative methods for this system of equations are divergent what results clearly from calculation's accuracy analysis for the Hilbert matrices (Table V). The Jacobi method just after some iterations gives solutions with a serious error. The Gauss-

TABLE VI  
THE RELATIONSHIP OF THE ITERATIVE METHODS' ACCURACY AND DIAGONALLY DOMINANT MATRICES' CONDITION NUMBER

size of matrix	number of iterations	Jacobi method	Gauss-Seidel method	$\approx_1$
5	5	3.13E-05	3.37E-08	1.13E+00
	10	4.66E-11	8.67E-19	
	15	6.91E-17	0.00E+00	
	25	0.00E+00	0.00E+00	
	50	0.00E+00	0.00E+00	
10	5	1.79E-03	1.32E-06	1.32E+00
	10	9.95E-08	8.46E-15	
	15	5.53E-12	2.49E-18	
	25	1.62E-18	2.49E-18	
	50	1.62E-18	2.49E-18	
20	5	2.95E-02	1.70E-05	1.76E+00
	10	2.04E-05	2.68E-12	
	15	1.41E-08	4.13E-18	
	25	6.69E-15	4.13E-18	
	50	5.25E-18	4.13E-18	
40	5	4.98E-01	4.02E-04	2.20E+00
	10	4.84E-03	1.89E-09	
	15	4.70E-05	8.37E-15	
	25	4.44E-09	6.02E-18	
	50	5.30E-18	6.02E-18	

Seidel method can create a impression of the convergence because he accuracy's value does not grow as visibly as in the Jacobi method. It can be even supposed that the solutions are better and better approximated. However, after more detailed analysis, we can see that after a lot of iterations, the accuracy's value oscillates around the number range of tenth or units. This number is definitely too big to recognize the obtained information as the correct solution's approximation.

We can see that a big condition number of a matrix can show the iterative methods' divergence. Additionally, because the size of the system has an influence on the condition number's value, the bigger the size of a matrix is, the faster the iterative methods' divergence is.

The research of the iterative methods' accuracy for randomly generated diagonally dominant matrices shows that these methods are convergent (Table VI). These methods, after a few iterations, give the correct solution or a solution of the equation system with a minute error, which is after some iterations very close to zero.

The analysis of the obtained information clearly shows that the Gauss-Seidel method approaches the correct solution much faster than the Jacobi method. The convergence speed of the iterative methods depends also on the size of a matrix (the bigger the matrix, the slower the method converges), and also on the condition number of a matrix. This relationship is inverse because the smaller the condition number is, the faster the iterative methods approach the correct linear equation system's solution.

The data in Table V and VI shows that Hilbert matrices, the condition number of which is very high, are ill-conditioned, so the iterative methods are not convergent for them. On the contrary, diagonally dominant matrices have small condition numbers, which means that they are well-conditioned matrices for which the iterative methods are convergent.

TABLE VII

THE CONDITION NUMBERS IN NORM OF  $l_1$  OF SMALL HILBERT MATRICES AND SMALL DIAGONALLY DOMINANT MATRICES

size of matrix	Hilbert matrix	Matrix 2a	Matrix 2b
1	1.00E+00	1.00E+00	1.00E+00
2	2.70E+01	1.53E+00	1.04E+00
3	7.48E+02	1.92E+00	1.10E+00
4	2.84E+04	2.74E+00	1.15E+00
5	9.44E+05	3.08E+00	1.19E+00
6	2.91E+07	4.24E+00	1.22E+00
7	9.85E+08	4.45E+00	1.31E+00
8	3.39E+10	4.79E+00	1.31E+00
9	1.10E+12	4.92E+00	1.35E+00

F. The examination of condition numbers of small matrices

The previous tests do not explain why the condition numbers of Matrices 1 and 6 are so similar although the first matrix is well-conditioned and the second one is ill-conditioned. In order to explain this problem, we have to analyze the results in Table VII.

The first matrix is a Hilbert matrix. Matrices 2a and 2b are generated in the same way as Matrix 2. Matrix 2a differs from matrix 2b in such a way that its elements are random numbers from the range  $(0; 10)$  — not from the range  $(0; 1)$ . Such a generation of matrices causes that elements of the Hilbert matrices and of Matrices 2b are much smaller than the elements of Matrices 2a. We can see from Table VII, that the condition numbers of Matrices 2a are much bigger than the condition numbers of Matrices 2b, and nevertheless very small in relation to the Hilbert matrices. The accuracy tests' results of the Matrices 2a presented in the Table VIII show that iterative methods for this matrix are convergent, and therefore the matrix is well-conditioned.

Comparing the information from Tables VI and VIII, we can see that the iterative methods for a matrix of bigger elements (and thereby of a bigger condition number) converge slower than for a matrix of a small condition number, and also approach the correct solution of the linear equation system faster.

The condition number of a matrix do not always define whether the matrix is well-conditioned or ill-conditioned. That happens when the matrix is of a small size. For every system whose size is equal to 1, a condition number of the matrix is equal to 1, and the iterative methods give the correct solution just after the first iteration. Analyzing Tables VII and IX, we can notice that for a matrix of a small size (2 or 3) but of big elements' values the condition number is similar and we cannot decide on its basis if the iterative method for this matrix is convergent or divergent. This fact is evidently confirmed by the results shown in Table X. It is clear that Matrix 1 is well-conditioned and Matrix 6 is not convergent in the iteration process although the condition number of these matrices calculated in the norm of  $l_1$  differs very little from each other.

A problem with recognizing, with the use of a condition number, if an iteration process at a given matrix is convergent or not, refers only to a matrix of a very small size because for

TABLE VIII

THE RELATIONSHIP OF THE ITERATIVE METHODS' ACCURACY AND THE CONDITION NUMBER OF DIAGONALLY DOMINANT MATRICES WITH BIG ELEMENTS (2A)

size of matrix	number of iterations	Jacobi method	Gauss-Seidel method	$\approx_1$
3	5	4.64E-02	6.65E-06	1.92E+00
	10	1.45E-04	1.63E-12	
	15	4.58E-07	1.79E-18	
	25	4.57E-12	1.73E-18	
5	50	1.73E-18	1.73E-18	3.08E+00
	5	1.73E+00	1.06E-02	
	10	1.41E-01	2.73E-06	
	15	1.15E-02	1.11E-09	
7	25	7.62E-05	9.44E-17	4.45E+00
	50	2.73E-10	1.23E-18	
	5	1.46E+01	1.80E-02	
	10	6.35E+00	8.32E-06	
9	15	2.77E+00	3.66E-09	4.92E+00
	25	5.27E-01	5.71E-16	
	50	8.30E-03	1.11E-18	
	5	1.76E+01	9.60E-03	
9	10	1.29E+01	3.94E-06	4.92E+00
	15	9.50E+00	1.86E-09	
	25	5.12E+00	2.75E-16	
	50	1.09E+00	1.06E-18	

TABLE IX

THE VALUES OF THE CONDITION NUMBER OF THE MATRICES WHICH HAVE SMALL SIZES, CALCULATED WITH THE USE OF TWO DIFFERENT NORMS

#	size	$\approx_1$	$\approx_\infty$
1	3	8.50E+00	7.80E+00
6	2	9.00E+00	9.00E+00

TABLE X

THE RELATIONSHIP OF THE ITERATIVE METHODS ACCURACY AND THE CONDITION NUMBER OF THE MATRICES WHICH HAVE SMALL SIZES

#	Jacobi method		Gauss-Seidel method		$\approx_1$
	10 iter.	50 iter.	10 iter.	50 iter.	
1	5.26E-03	0.00E+00	1.47E-05	0.00E+00	8.50E+00
6	1.28E+84	1.22E+443	9.67E+166	8.80E+884	9.00E+00

ill-conditioned matrices of the size  $4 \times 4$  the condition number is incomparably bigger than a condition of a well-conditioned matrix of 4th degree, which has big elements' values. This happens, because even for big elements a condition number of a well-conditioned matrix grows incomparably slower than a condition of an ill-conditioned matrix, which is confirmed by the information in Table VI.

IV. CONCLUSIONS

These experiments show that there exists a strong relationship between the size of a system and a condition number of a matrix as well as between the condition number's value and the iterative methods' convergence.

Iterative methods are convergent for a matrix of a small condition number (well-conditioned matrices) and divergent for matrices which have a big condition number (ill-conditioned matrices). The condition number of a matrix is directly connected to its size, but for well-conditioned matrices it grows incomparably slower than for ill-conditioned matrices. Fig. 1 visibly shows the difference between the condition number of

a well-condition matrix and the condition number of an ill-conditioned matrix.

The size of a system also indirectly influences the iterative process's convergence. In the case of well-conditioned matrices, the bigger size of a matrix is, the slower the methods approach the correct solution's value. On the contrary, for ill-conditioned matrices, the bigger size of a matrix is (a bigger condition number thereby), the faster the iterative process diverges.

For well-conditioned matrices the condition number can be influenced by the size of elements of a matrix, although the condition number is incomparably smaller than the condition number of ill-conditioned matrices.

It is also easily seen (Fig. 1), that a value of a condition number is growing with the growth of the size of the matrix.

So we can see, that the condition number of a well-conditioned matrix is always small and that the big condition number proves the matrix to be ill-conditioned. However, we

can find some ill-conditioned matrices with small condition numbers.

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