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Fair flow optimization with advanced aggregation operators in Wireless Mesh Networks

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Abstract—**The problem of fair resource allocation is of considerable importance in many applications. In this paper advanced aggregation operators based on the Ordered Weighted Averaging (OWA) are utilized as consistent and fairness–preserving approach to modeling various preferences with regard to distribution of Internet traffic between network participants. The networking model based on Wireless Mesh Networks is considered. The physical medium properties cause strong interference among simultaneously operating node devices, which makes the optimization problem extremely difficult. We show that in this case OWA–based aggregation operators can be utilized just as easily as traditional lexicographic operators.**

I. Introduction

Wireless Mesh Network (WMN) is an organized cooperating group of network devices communicating with each other by means of wireless media. The nodes are organized in a mesh topology, where each wireless device not only sends and receives its own data but also serves as a relay for other nodes. Some of the nodes can be connected to cable network or mobile network and serve as Internet gateways. This way the whole mesh network constitutes a decentralized way of providing Internet access between attending participants.

This network type poses numerous advantages including setup cost, independence of the hardwired infrastructure and flexibility. However, providing fair and efficient network management, including routing and scheduling, is not a straightforward task. The main source of difficulty lies in physical medium properties that cause strong interference among simultaneously operating devices. Additionally the link quality is a function of the distance and can be affected by obstacles present between the nodes. As a result the efficient network operation requires transmission scheduling, channel assignment and transmission power determination.

Common objective of the optimization is maximization of the total throughput while retaining fairness in its distribution between participants.

In many network optimization problems, fairness is accomplished by simple max-min optimization with regularization through minimization of the second largest outcome (provided that the largest one remains as small as possible), minimization of the third largest (provided that the two largest remain as small as possible), and so on. This approach, called MMF (Max-Min Fairness) prevents some demands with structurally low throughputs from blocking/disabling the max-min function. This is, however, a stiff approach that usually does not allow any other criteria, the overall efficiency (total throughput) in particular. Moreover, it requires sequential repeated optimization of the original problem.

In this paper we show the application of the ordered weighted averaging (OWA) aggregation with extensions as consistent, reasonable and fairness–preserving approach to modeling various preferences (from the extreme pessimistic, through neutral to extreme optimistic) with regard to distribution of Internet throughput between network participants.

In the OWA aggregation ([20], [21]) the weights are assigned to the ordered values (i.e., to the largest value, the second largest and so on) rather than to the specific criteria. The OWA operator provides a parameterized family of aggregation operators, which include many of the well-known operators such as the maximum, the minimum, the k-order statistics (including CVaR), the median and the arithmetic mean. The OWA satisfies the properties of strict monotonicity, impartiality and, in the case of monotonic increasing weights, the property of equitability (satisfies the principle of transfers – equitable transfer of an arbitrary small amount from the larger outcome to a smaller outcome results in a more preferred achievement vector). Thus the OWA-based optimization generates the so-called equitably efficient solutions (cf.[8] for the formal axiomatic definition). According to [8] and [16], equitable efficiency expresses the concept of fairness, in which all system entities have to be treated equally and in the stochastic problems equitability corresponds to the risk aversion [3]. Since its introduction, the OWA aggregation has been successfully applied to many fields of decision making [22], [21], [11]. When applying the OWA aggregation to multicriteria optimization, the weighting of the ordered outcome values causes that the OWA optimization problem is nonlinear even for linear programming formulation of the original constraints and criteria. Yager has shown that the nature of the nonlinearity introduced by the ordering operation allows one to convert the OWA optimization into a mixed integer programming problem. We have shown [13] that the OWA optimization with monotonic weights can be formed as a standard linear program of higher dimension. Its significant extension introduced by Torra [18] incorporates importance weighting into the OWA operator forming the weighted OWA (WOWA) aggregation as a particular case of Choquet integral using a distorted probability as the measure. The WOWA averaging is defined by two weighting vectors: the preferential weights and the importance weights. It covers both the weighted means and the OWA averages as special cases. Some of the example applications of importance weights include definition of the size or importance of processes in a multi-agent environment, setting scenario probability (if uniform objectives represent various possible values of the same uncertain outcome under several scenarios), or job priorities in scheduling problems. In [14] we have shown that in the case of monotonic preferential weights also WOWA aggregation can also be modeled by a mere linear extension of the original problem.

The paper is organized as follows.

II. Flow optimization in WMN

The WMN networking technology has been drawing an increased attention over the last years (see literature overview in [15] and references therein). Due to complexity of the problem usually some sort of simplifications are assumed. The problem considered in this paper can be stated as follows. There is given a WMN network with a number of nodes – routers and gateways. The nodes are interconnected wirelessly in compliance with all the physical constraints and requirements including signal loss with increasing distance and interference occurring during simultaneous operation. Each node can be either sending or receiving data, but not both at the same time. There is a number of modulation and coding schemes (MCS) used for communication between the nodes with different properties with regard to speed, maximum allowable interference and the distance. Each MCS has its signal to interference plus noise ratio (SINR) requirement that must be fulfilled in order to successfully transmit data. Only one fixed transmitting power and single channel are assumed, but MCS can be dynamically allocated. The network model consists only of links for which at least one MCS can be applied, and this requirement reduces to the maximum allowable distance between the nodes.

Only downstream communication direction from gateways to routers is considered. For each router there is a single predefined path leading to a chosen gateway. The routers have elastic traffic demand, which means they can consume all the possible network capacity. The demands compete for network resources to get as much throughput as possible.

The objective is to maximize total throughput preserving fairness among competing demands.

The solution approach is based on the concept of compatible sets introduced in [4]. Compatible set consists of links that can operate at the same time within given interference model. The basic solution concept consists in linear approximation of the model and consecutive generation of the compatible sets improving current solution within the column generation schema. The approximation is needed if the time horizon is divided into fixed-length time slots, if not the solution is optimal.

Although we consider only specific problem, the solution concepts involving application of WOWA operators can be utilized for many other variants of WMN problems including different capacity reservation models (see [15]).

A. Notation

Wireless mesh network topology is represented by a directed graph $\mathcal{N} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \mathcal{G} \cup \mathcal{R}$ is the set of nodes from which we distinguish the set of gateways and set of mesh routers denoted respectively by $\mathcal{G}$ and $\mathcal{R}$, and $\mathcal{E}$ is the set of (radio) links.

The (potential) link between two nodes $v, w \in \mathcal{V}$ is modeled by a directed arc $e = (v, w) \in \mathcal{E}$, where $a(e) = v$ is the originating node that can transmit a signal of a given power P_{vw} to its terminating node $b(e) = w$. Additionally, we assume that if arc $e = (v, w) \in \mathcal{E}$ exists then an opposite arc $e' = (w, v) \in \mathcal{E}$ also exists. Furthermore, the sets of outgoing and incoming arcs from/to node $v \in \mathcal{V}$ are denoted, respectively, by $\delta^+(v)$ and $\delta^-(v)$, while $\delta(v) = \delta^+(v) \cup \delta^-(v)$ is the set of all arcs incident to node v.

Nodes are transmitting using one of the available *modulation and coding schemes* (MCSs) denoted by $m \in \mathcal{M}$, where $\mathcal{M}$ is the set of all MCSs (to simplify the considerations, we assume that the set of available MCSs $\mathcal{M}(e) = \mathcal{M},\ e \in \mathcal{E}$). The (raw) data rate of transmission using MCS m is denoted by B^m.

The (radio) link $e = (v, w)$ can successfully transmit if the signal to noise ratio (SNR) for the arc e is greater than a certain threshold value denoted by γ^m for at least one MCS $m \in \mathcal{M}$:

$$\Gamma'_e = \frac{P_{vw}}{N} \geq \gamma^m, \tag{1}$$

where $N = 10^{-10.1}$ mW is the ambient noise power.

At any arbitrary time instance the transmission of other nodes can interfere with transmission on e. The corresponding signal to interference plus noise ratio (SINR) condition for successful transmission on e using MCS m reads as follows:

$$\Gamma_e = \frac{P_{vw}}{N + \sum_{a \in \mathcal{A} \setminus \{v\}} P_{aw}} \geq \gamma^m, \tag{2}$$

where $\mathcal{A} \subseteq \mathcal{V}$ is the set of active nodes which are transmitting at the same time.

Moreover, we assume that a node can either transmit or receive, or be inactive, that is,

$$\mathcal{A} \cap \{b(\delta^+(v))\} \neq \emptyset \Rightarrow \mathcal{A} \cap \{a(\delta^-(v))\} = \emptyset \quad v \in \mathcal{V} \tag{3}$$

$$\mathcal{A} \cap \{a(\delta^-(v))\} \neq \emptyset \Rightarrow \mathcal{A} \cap \{b(\delta^+(v))\} = \emptyset \quad v \in \mathcal{V} \tag{4}$$

Each router $r \in \mathcal{R}$ is connected with a selected gateway $g \in \mathcal{G}$ by a directed path p_d (i.e. a subset of links, $p_d \subseteq \mathcal{E}$) that is supposed to carry the entire downstream flow f_d from gateway g to router r (to simplify the formulations we do not consider the upstream direction). The set of routers is considered as demands and denoted by $d \in \mathcal{D}$, where $\mathcal{D} = \mathcal{V} \setminus \mathcal{G}$. Let

$\mathcal{P} = \{p_1, \ldots, p_D\}$ be the given set of paths between routers and gateways, where $D = |\mathcal{D}|$. For each link $e \in \mathcal{E}$, the set of all indices of paths in $\mathcal{P}$ that contain this link will be denoted by $\mathcal{Q}_e = \{d : e \in p_d, 1 \leq d \leq D\}$.

B. Compatible sets

A compatible set (CS) is defined as a subset $\mathcal{E}_i$ of links $\mathcal{E}_i \subseteq \mathcal{E}$ together with a particular MCS $m_e, e \in \mathcal{E}_i$ that each link is using so that each link can be active simultaneously (i.e. transmit without generating too much interfering with other links). In other words, a compatible set is defined by $\mathcal{E}_i = \{(e, m) \in \mathcal{E} \times \mathcal{M} : y_e^m = 1\}$, where variables y_e^m form a feasible solution that satisfy (2) and (3)–(4).

1) Master problem: Using the family of compatible sets denoted by $\mathcal{I}$, the formulation of max-min fair (MMF) flow optimization problem reads as follows:

$$\max f \tag{5}$$

$$f \leq f_d \quad d \in \mathcal{D} \tag{6}$$

$$\sum_{d \in \mathcal{Q}_e} f_d \leq c_e \quad e \in \mathcal{E} \tag{7}$$

$$c_e = \sum_{i \in \mathcal{I}} B_{ei} z_i \quad e \in \mathcal{E} \tag{8}$$

$$\sum_{i \in \mathcal{I}} z_i = T \tag{9}$$

$$z \geq 0 \tag{10}$$

In the presented formulation, T is the time of network operation, B_{ei} is the (raw) data rate of a transmission using MCS $m \in \mathcal{M}$ allocated to link $e \in \mathcal{E}$ in compatible set $i \in \mathcal{I}$, i.e. either B^m or 0, depending on whether link e is active or not in the compatible set i, and c_e is total amount of data that can be transmitted over link $e \in \mathcal{E}$ in a time interval T. This formulation is a non-compact ($|\mathcal{I}|$ grows exponentially in the network size), continuous approximations of the MIP problem involving time slots (see [15]) - continuous variables z_i define the number of time slots assigned to a compatible set within the time T.

2) Pricing problem: The pricing problem we consider corresponds to a WMN system in which there are multiple MCSs available and each node can use different MCS in different compatible set. The following formulation is referred to as dynamic allocation of MCSs to nodes:

$$\max \sum_{e \in \mathcal{E}} \pi_e^* B_e \tag{11}$$

$$X_v = \sum_{m \in \mathcal{M}} x_v^m \quad v \in \mathcal{V} \tag{12}$$

$$Y_e = \sum_{m \in \mathcal{M}} y_e^m \quad e \in \mathcal{E} \tag{13}$$

$$\sum_{e \in \delta(v)} Y_e \leq 1 \quad v \in \mathcal{V} \tag{14}$$

$$\sum_{e \in \delta^+(v)} y_e^m = x_v^m \quad v \in \mathcal{V}, m \in \mathcal{M} \tag{15}$$

$$z_{ev}^m \geq y_e^m + X_v - 1 \quad v \in \mathcal{V}, e \in \mathcal{E}, m \in \mathcal{M} \tag{16}$$

$$z_{ev}^m \leq y_e^m, z_{ev}^m \leq X_v \quad v \in \mathcal{V}, e \in \mathcal{E}, m \in \mathcal{M} \tag{17}$$

$$N y_e^m + \sum_{v \in \mathcal{V} \setminus \{a(e)\}} P_{vb(e)} z_{ev}^m \leq \frac{1}{\gamma^m} P_{a(e)b(e)} y_e^m \qquad e \in \mathcal{E}, m \in \mathcal{M} \tag{18}$$

$$B_e = \sum_{m \in \mathcal{M}} B^m y_e^m \quad e \in \mathcal{E} \tag{19}$$

Each node $v \in \mathcal{V}$ and each link $e \in \mathcal{E}$ can use at most one MCS $m \in \mathcal{M}$ in the compatible set (12)–(13). At most one link $e \in \delta(v)$ incident to node v can be active (14) and exactly one link $e \in \delta^+(v)$ outgoing from node v is active and uses MCS m (15), provided the node is active and uses this MCS in the compatible set. The constraints (16)–(18) assure admissible SINR for link e using MCS m in the compatible set. The (raw) data rate B_e of link e in the compatible set is found by (19).

III. Fair aggregation operators

As stated before the basic operator used to preserve fairness among outcomes is max-min, regularized by lexicographic maximization of the second worst outcomes (provided that the worst one remains as large as possible) , third worst outcomes (provided that the two largest remain as large as possible), and so on (MMF). In the case of linear problem it is possible to carry out the MMF procedure based on simple algorithm that in each step uses the dual information to determine the outcomes that are blocked at their highest values possible. In the following steps, only the outcomes are optimized that have not been blocked before (for details see [15]).

The WOWA aggregation is based on the ordered weighted averaging operator introduced first by Yager [20]. In the OWA aggregation of outcomes $\mathbf{y} = (y_1, \ldots, y_m)$ weights $\mathbf{w} = (w_1, w_2, \ldots, w_m)$ are assigned to the ordered values rather than to the specific criteria:

$$A_w = \sum_{i=1}^{m} w_i \theta_i(\mathbf{y}) \tag{20}$$

where $(\theta_1(\mathbf{y}), \theta_2(\mathbf{y}), \ldots, \theta_m(\mathbf{y}) = \Theta(\mathbf{y})$ is the ordering map $R^m \to R^m$ with $\theta_1(\mathbf{y}) \leq \theta_2(\mathbf{y}) \leq \ldots \leq \theta_m(\mathbf{y})$ and there exists a permutation τ of set I such that $\theta_i(\mathbf{y}) = y_{\tau(i)}$ for $i = 1, 2, \ldots, m$.

If the weights are monotonic $w_1 > w_2 > \ldots > w_{m-1} > w_m$, the OWA aggregation has the property of equitability [13], that guarantees that an equitable transfer of an arbitrarily small amount from the larger outcome to a smaller outcome results in more preferred achievement vector. Every solution maximizing the OWA function is then an equitably efficient solution to the original multiple criteria problem. Moreover, for linear multiple criteria problems every equitably efficient solution can be found as an optimal solution to the OWA aggregation with appropriate weights.

For the maximization problem the OWA objective aggregation can be formulated as linear extension of the original problem, as follows. Let us apply linear cumulative map to the ordered achievement vectors $\Theta(\mathbf{y})$

$$\bar{\theta}_k(\mathbf{y}) = \sum_{i=1}^{k} \theta_i(\mathbf{y}) \quad k = 1, 2, \ldots m \tag{21}$$

As stated in [13], for any given vector $\mathbf{y} \in R^m$, the cumulated ordered coefficient $\bar{\theta}_k(\mathbf{y})$ can be found as the optimal value of the following LP problem:

$$\bar{\theta}_k(\mathbf{y}) = \max \; kt_k - \sum_{i=1}^{m} d_{ki} \tag{22}$$

subject to

$$t_k - y_i \le d_{ki}, \; d_{ki} \ge 0 \quad i = 1, 2, \ldots, m \tag{23}$$

The ordered outcomes can be expressed as differences $\theta_i(\mathbf{y}) = \bar{\theta}_i(\mathbf{y}) - \bar{\theta}_{i-1}(\mathbf{y})$ for $i = 2, \ldots, m$ and $\theta_1(\mathbf{y}) = \bar{\theta}_1(\mathbf{y})$. Hence, the maximization of the OWA operator (20) with weights w_i can be expressed in the form:

$$\max\{\sum_{i=1}^{m} w'_i \bar{\theta}_i(\mathbf{y}) : \mathbf{y} \in Y\} \tag{24}$$

where coefficients w'_i are defined as $w'_m = w_m$ and $w'_i = w_i - w_{i+1}$ for $i = 1, 2, \ldots, m-1$ and Y is the feasible set of outcome vectors $\mathbf{y}$. If the original weights w_i are strictly decreasing, then $w'_i > 0$ for $i = 1, 2, \ldots, m$.

For the WMN flow optimization problem (5)–(10) the final OWA aggregation of the outcomes f_d for all demands/routers can be stated as the following LP model:

$$\max \sum_{k=1}^{|D|} k w'_k t_k - \sum_{k=1}^{|D|} \sum_{d \in D} w'_k d_{dk} \tag{25}$$

subject to

$$d_{dk} \ge t_k - f_d, \; d_{dk} \ge 0 \quad k = 1, 2, \ldots, |D|, \; d \in D \tag{26}$$

$$\mathbf{f} \in F \tag{27}$$

where $\mathbf{f} = [f_d]_{d \in D}$ and F is a feasible set of flows/throughputs defined by (7)–(10).

The WOWA aggregation is a generalization of the OWA, that allows assigning importance weights to specific criteria [12]. Those weights could express, for example, relative importance of the routers. The weights assigned to ordered values will be further called preferential weights.

Let $\mathbf{p} = (p_1, \ldots, p_m)$ be an m-dimensional vector of importance weights such that $p_i \ge 0$ for $i = 1, \ldots, m$ and $\sum_{i=1}^{m} p_i = 1$. The corresponding Weighted OWA aggregation of vector $\mathbf{y}$ is defined [18] as follows

$$A_{w,p} = \sum_{i=1}^{m} \omega_i \theta_i(\mathbf{y}) \tag{28}$$

with

$$\omega_i = w^*(\sum_{k \le i} p_{\tau(k)}) - w^*(\sum_{k < i} p_{\tau(k)}), \tag{29}$$

where w^* is an increasing function interpolating points $(i/m, \sum_{k \le i} w_k)$ together with the point $(0,0)$ and τ representing the ordering permutation for $\mathbf{y}$ (i.e. $y_{\tau(i)} = \theta(\mathbf{y})$). Moreover, function w^* is required to be a straight line when the points can be interpolated in this way. We assume the piecewise linear interpolation function w^* which is the simplest form of the required interpolation.

Note, that the piecewise linear functions may be built with various number of breakpoints, not necessarily m [12]. Thus, any nonlinear function can be well approximated by a piecewise linear function with appropriate number of breakpoints. Therefore, we will consider weights vectors $\mathbf{w}$ of dimension n not necessarily equal to m. It is even possible to define a generalized WOWA aggregation where the preferential weights w_k are allocated to an arbitrarily defined grid of ordered outcomes defined by quantile breakpoints (see [12] and references therein).

As shown in [12], maximization of an equitable WOWA aggregation with decreasing preferential weights $w_1 \ge w_2 \ge \ldots \ge w_n$ may be implemented as the LP expansion of the original problem. In the case of the WMN flow optimization problem (7)–(10), this can be stated as follows:

$$\max \sum_{k=1}^{n} w'_k \left[\frac{k}{n} t_k - \sum_{d \in D} p_d d_{dk} \right] \tag{30}$$

subject to

$$d_{dk} \ge t_k - f_d, \; d_{dk} \ge 0 \quad k = 1, 2, \ldots, n, \; d \in D \tag{31}$$

$$\mathbf{f} \in F \tag{32}$$

If the importance weights are equal $p_d = 1/|D|$, the model reduces to the OWA aggregation.

A special case of the generalized WOWA aggregation is defined for single breakpoint and corresponds to optimization of the predefined quantile of worst outcomes and in finance is known as the CVaR (Conditional Value at Risk). It can be computed as a standard linear extension of the original problem [12]:

$$\max \; t - 1/\beta \sum_{d \in D} p_d d_d \tag{33}$$

subject to

$$d_d \geq t - f_d,\, d_d \geq 0 \quad d \in D \tag{34}$$

$$\mathbf{f} \in F \tag{35}$$

IV. Numerical experiments

We analyzed the performance of the CVaR and WOWA aggregation operators together with the performance of the classic max-min and lexicographic max-min (MMF) operators. The aggregation operators were applied to the problem defined by the constraints (7)–(10) with the network flows f_d as the optimization criteria.

For the pricing problem (11)–(19) we applied the Simulated Annealing (SA) algorithm. SA was first introduced by Kirkpatrick [7], while Černý [5] pointed out the analogy between the annealing process of solids and solving combinatorial problems. Researchers have been studying the application of the SA algorithm in various fields of optimization problems [9], [17], [6] and to the WMN problem, as well [10].

The process of Simulated Annealing adapted to the pricing problem, i.e. the dynamic allocation of MCSs to nodes, can be described as follows. First, an initial solution - a compatible set with no active nodes ($\mathcal{A} = \emptyset$) is specified as a starting point. Then, repeatedly, a candidate solution is randomly chosen from the neighborhood of the current one. The procedure for creating a neighboring solution (i.e. a compatible set in this case), known as the perturbation scheme, consists in activating/deactivating nodes and changing the MCS of randomly selected link, see Algorithm 1. If the newly generated solution is better than the current one, it is accepted and becomes the new current solution. Otherwise, the candidate solution still has a chance to be accepted with, so called, acceptance probability. This probability is determined by the difference between the *energy* denoted by E of the current and the candidate solution, and depends on the *temperature* denoted by τ, a control parameter taken from the thermodynamics. The *energy* of the compatible set is found by:

$$E(CS) = \sum_{e \in \mathcal{E}} \sum_{m \in \mathcal{M}} \pi_e^* B^m y_e^m \tag{36}$$

The acceptance probability for the compatible set is calculated as follows:

$$p\,(\delta, \tau) = e^{-\delta/k\tau} \tag{37}$$

where δ is the difference between the current (CS) and the candidate (CS') solution:

$$\delta = E(CS) - E(CS'), \tag{38}$$

and k is the Boltzmann constant found by:

$$k = \frac{\delta^0}{\log \frac{p^0}{\tau^0}}. \tag{39}$$

where δ^0 is an estimated minimal difference between the two solutions, p^0 is the initial value of the acceptance probability, and τ^0 is the initial temperature.

TABLE I
Initial values of Simulated Annealing parameters

Param.	Description	Value
α	Reduce factor	$1 - \frac{5}{N}$
τ^0	Initial temperature	0.999
δ^0	Minimal diff. between solutions	0.001
p^0	Initial acceptance probability	1
T	Iterations number at each temp.	10
N	Number of SA iterations	300000

TABLE II
IEEE 802.11a MCS, FER 61%, 1500 Byte payload.

MCS m	Raw rate B^m (Mbps)	SINR threshold $\hat{\gamma}^m$ (dB)	Max. link length d^m (m)
BPSK 1/2	6	3.5	273.5
BPSK 3/4	9	6.5	230.0
QPSK 1/2	12	6.6	228.0
QPSK 3/4	18	9.5	193.7
16-QAM 1/2	24	12.8	160.2
16-QAM 3/4	36	16.2	131.7
64-QAM 2/3	48	20.3	103.8
64-QAM 3/4	54	22.1	93.5

After a number of iterations in a constant temperature, the colling takes place, i.e. the temperature is decreased by a factor denoted by α, known as the reduce factor, and the process is continued as described above. The annealing scheme can be represented as the following recursive function:

$$\tau^{i+1} = \alpha\, \tau^i, \tag{40}$$

where i is the number of current iteration in which the cooling schedule takes place. The algorithm is stopped when a maximum number of iterations is reached. The best solution found during the whole process is considered a final. Initial values of SA parameters are collected in Table I.

The numerical experiments were performed on a number of randomly generated problem instances of different sizes. The algorithm of generating network topology instances can be described as follows. A grid of length 25m of 30×30 points is created. Each of the grid points can be chosen to be a mesh router or a mesh gateway. First, the location of each gateway $g \in \mathcal{G}$ is chosen at random. Then, for each router $r \in \mathcal{R}$ a location is chosen at random that satisfies condition (1) for at least one link $e = (g, r),\ g \in \mathcal{G}$ and MCS $m \in \mathcal{M}$. This condition is equivalent to that $d_{gr} \leq d^m$, where d_{gr} is the distance between gateway g and router r and d^m is the maximum distance for selected MCS m. Finally, paths rooted in the gateways are established by iteratively connecting the neighboring routers that are reachable with the highest link rate and, if possible, with the lowest hop count. The specific data for different MCSs are presented in Table II.

Although weights determination is an important issue in the theory of Ordered Weighted Averaging [19], [1], [2], for the performance check simple generation methodology has been

Algorithm 1 Compatible set perturbation scheme

Require: Current solution compatible set CS
Ensure: CS' = neighbor(CS)
Choose at random $v \in \mathcal{V}$ and $e \in \delta^+(v)$ that satisfy (12)–(15).
if $v \in \mathcal{A}$ **then**
  if $random(0,1) < 1/|\mathcal{M}(e)|$ **then**
    $\mathcal{A} \leftarrow \mathcal{A} \setminus \{v\}$ [deactivate node]
  else
    $m \leftarrow random(\mathcal{M}(e) \setminus \{m\})$ [switch MCS]
  end if
else
  $\mathcal{A} \leftarrow \mathcal{A} \cup \{v\}$ [activate node]
  $m \leftarrow random(\mathcal{M}(e))$ [select MCS]
end if

TABLE III
COMPUTING TIMES [S]

Problem size		Aggregation operator			
$\lvert D\rvert, \lvert E\rvert$	$\lvert G\rvert$	Max-Min	MMF	CVaR	WOWA
10	2	15.1	16.8	12.9	9.3
10	4	15.2	17.4	13.1	7.9
10	8	14.1	18.8	12.4	7.0
20	2	53.1	63.0	44.0	36.6
20	4	50.4	63.0	39.2	32.3
20	8	46.9	63.3	44.1	27.3
50	2	313.2	335.3	254.3	226.5
50	4	258.0	325.7	191.3	185.8
50	8	253.0	329.2	199.5	180.7

chosen. All the weights, except two, are strictly decreasing numbers with the step 0.1, while the two selected weights ($k = \lfloor n/3 \rfloor$ and $k = \lfloor 2n/3 \rfloor$) differ from the previous ones by 0.5.

All the experiments were performed on the Intel Core i7 2.9GHz microprocessor using CPLEX 12.1 optimization library for the linear master problem. The results are the average of 10 randomly generated problems of a given size. Computing times are presented in Table III and the total number of the columns (compatible sets) generated with SA in Table IV.

One can notice the advanced aggregation operators generally perform not only better than the MMF but also better than the max-min operator. Additionally, problems with increased number of gateways can also be computed more efficiently.

V. CONCLUSION

The problem of fair resource allocation is of considerable importance in network optimization. Advanced aggregation operators based on the Ordered Weighted Averaging allow to model diverse preferences with regard to fairness and efficiency. We have shown that application of the advanced aggregation operators for the flow optimization in Wireless Mesh Networks can be effectively modeled and its computation times can be shorter than of the traditional lexicographic max-min approach.

TABLE IV
NUMBER OF COMPATIBLE SETS GENERATED

Problem size		Aggregation operator			
$\lvert D\rvert, \lvert E\rvert$	$\lvert G\rvert$	Max-Min	MMF	CVaR	WOWA
10	2	15.2	17.3	13.7	10.9
10	4	16.6	19.1	15.1	10.4
10	8	16.3	21.7	15.1	9.3
20	2	38.7	48.9	35.4	32.4
20	4	40.9	53.7	34.9	32.5
20	8	42.8	60.1	41.0	29.5
50	2	145.5	158.8	132.8	124.6
50	4	134.2	178.7	109.8	112.2
50	8	141.8	193.3	118.6	117.3

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