

Adaptive Conjoint Analysis. Training Data: Knowledge or Beliefs?

A Logical Perspective of Preferences as Beliefs

Adrian Giurca¹, Ingo Schmitt¹, Daniel Baier²

¹Dept. of Databases and Information Technology,

²Dept. of Marketing and Innovation Management

Brandenburg University of Technology

P.O. 101344, 03013 Cottbus, Germany

Email: {giurca, schmitt, daniel.baier}@tu-cottbus.de

Abstract—The foundational model of conjoint analysis is to model consumer purchase preferences by means of utility functions. Analysts run surveys and interviews to obtain a basic set of training data, typically user preferences on which the utility function is mapped. The utility theory trust the training data as knowledge while there is large literature emphasizing that users preference may change, may be incomplete and sometimes inconsistent. This paper argues on a logic-based model of conjoint analysis, particularly by proposing an alternative model of preferences as belief instead as fully trust knowledge. We adopt the categorical beliefs approach but the quantitative, probabilistic approach may be considered too. In the context of adaptive conjoint analysis, we identified three kinds of beliefs, describe a mechanism of mapping answers to beliefs and provide the basis on belief update when new information occurs. Future work on our logic-based framework will focus obtaining an optimal logic-based preference aggregation including by relaxing Pareto efficiency in Arrow’s aggregation framework as well as researching on non-prioritized belief revision in adaptive conjoint analysis.

I. INTRODUCTION AND MOTIVATION

DURING the last forty years many mathematical models to capture and aggregate preference were developed. However, due to the psychological, economical and mathematical controversies, there is not yet a common agreement on a minimal set of preference properties nor on their aggregation. Processing preferences towards understanding how individuals evaluate products/services and as well as on predicting behavioral outcomes is a continuously research subjects for philosophers, mathematicians, psychologists, sociologists and marketers. Significant results are obtained when processing complete, transitive acyclic and consistent preferences but, as many communities mention, such models are quite far from the real life. When asking people about thing they like, then they may not answer (*incompleteness*), or they may change

This research is supported by (1)DFG Project SQ-System: Entwicklung von Konzepten für ein quantenlogikbasiertes Retrieval-Datenbank-Anfragesystem: Anfragesprache, interaktive Suchformulierung sowie effiziente Anfrageauswertung and (2) German Federal Ministry of Education and Research, ForMaT project (Forschung für den Markt im Team), Phase II, Innovationslabor: Multimediale Ähnlichkeitssuche zum Matchen, Typologisieren und Segmentieren

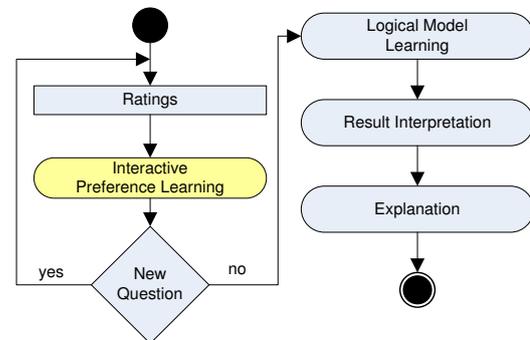


Fig. 1. Basic Logic-Based Conjoint Analysis Chain

their initial preferences due to reception of new information (*preference change*). In addition, while it seems that the preference system of one respondent must be non contradictory, when processing preferences from many respondents this may not remain valid.

When using statistical models to process preferences the usual guidelines are offered in terms of utility/importance of specific features or entire products. However, there is evidence that non-researchers does not really understand utilities. Answers such as “*The importance of attribute screen-size is 0.65*” are less understood than sentences like “*If the smartphone has no WIFI and a low price then it should have OS Android*”. When performing explanations it is obvious that decision makers better understand rules than plain numbers (*choose 5 inch screen because the importance is 0.67 and don’t choose 4 inch screen because importance is 0.56 ...*). As such, there were proposed many mathematical models to handle what the philosophers called *preference* and *indifference* relations. The usual process chain of a conjoint analysis test is shown by Figure 1.

Our previous work [18], introduced a logic-based utility solution to conjoint analysis but the approach was limited by a number of assumptions such as ignorance of neutral

rated questions, transitivity assumption and the restriction of using only 2 stimuli choice pair comparisons. Moreover, it does not consider in any way preference change and interview (data collection) adaptation. Many of these restrictions were influenced by the approach of computing the logic-based utility, basically adaptation of the weighted majority learning algorithm allowing only binary preference as input. While a non-adaptive conjoint solution may consider that expressed preferences does not change, when adaptively collecting training data one may experience preference change. In addition, on long term data collection cases i.e., over days and not by a standard 15-20 minutes survey in a contiguous manner, respondents may remake-up their mind therefore change is frequently expected.

Another study [39], focused on a mathematical optimization approach by translating ratings into algebraic constraints (as such working under the preference transitivity assumption and under the hypothesis that preferences do not change) and looking for a logic-based utility fulfilling a core of these constraints. New debates on solution proposed by [39] were discussed by [24] in the context of non-additive utility aggregations such as Choquet integral. However, none of these approaches consider non-transitive and/or cyclic preferences, [40].

As [18], focused on computing preferences from ratings, this work is looking much deeply on the logical nature of the users ratings and rankings, otherwise linking to the same concept of logic-based utility. We argue that computing beliefs from ratings and rankings is much close to the mental expectations of respondents.

II. USER'S RATINGS. DO WE GET KNOWLEDGE OR BELIEFS?

Many studies by the economics community argued that a wide range of phenomena such as addiction, fashion, advertising can be explained by stable and unchangeable preferences and accordingly, regression-based methods, random utility models, as well as hierarchical Bayes estimations, remain very popular [5].

However theory of social choice [4], [41] studying how the preferences of a group of agents can be combined in a rational way, as well as other more recent trends [43], showed evidence that preference change is very significant with respect on analytic and reasoning models. Preference change was reported by many research [44], [6] and is subject of many dimensions [12] but, with respect of conjoint analysis, the most significant is that *preference may change in due to occurrence of new information*. This case is little discussed by the traditional conjoint models mostly because of mutual exclusivity assumption [34]. However, there is research reporting that new revealed beliefs may influence an agent to change its preferences. For example an agent prefers *meat to vegetables* but she may change this preference when information such as *eating too much meat poses a risk on heart attack*. A significant model, dealing with this use case is based on *conditional preferences* [27], [46].

The artificial intelligence perspective is looking to understand if phenomena such as *preference change* and how *preference inconsistency* can be captured by considering logical formalization of preferences as beliefs and therefore to be able to take advantage of the research performed on designing rational agents, [15], [43].

We consider belief revision and social choice theory as being similar while both combine a set of outcomes into one, although they differ on how these orderings are interpreted: preferences by social choice theory; plausibilities [31], [13], [17], [19], similarity [38], [45], possibility [10],[11],[7], by belief revision.

Assuming preference change as social and psychological phenomena then a logic-based model of conjoint analysis should assume a mapping from user's ratings on questioning (trade-off matrix, pair comparison) to beliefs rather than a mapping to knowledge. Moreover, while most of the mathematical models (notable exceptions being paraconsistent logics [36] and defeasible logic [2]) deal with consistent constraint sets, the conjoint analysis case is clearly opposite: it is unlikely that a preference set collected from a group of users will be logically consistent.

From the logical perspective the distinction between knowledge and belief is not a dichotomy since both typically share the same representation language and typically knowledge is considered as a form of belief. We follow the Plato understanding of *knows* by assuming that "P *knows* A if and only if A is *true* and P *believes* A and P is *justified* believing A", that is we consider knowledge as true evidence (justified and true belief) [14]. Moreover, following Bayesian analysis approach, we consider ratings as very close to degrees of plausibility towards measuring respondent beliefs [19], [20].

III. ON TRADE-OFF MATRIX AND PAIR COMPARISON DATA COLLECTION TECHNIQUES

In recent years, the use of ratings to encode respondent preferences has become increasingly common. Traditionally, conjoint analysis models request attributes and values rating by following a well understood natural numbers scales encoding meaning from definitive unimportant (the smallest number in the scale) to really important (the biggest number in the scale), sometimes called Likert scales [32]. While for non-metric/qualitative attributes, the respondents are typically required to provide rank-order evaluations, metric attributes are requested to be rated rather than ranked.

A trade-off matrix ([28]) asks a respondent to consider a pair of attributes. It displays all combinations of values for those attributes, asking the respondents to provide a ranking for the combinations. The main advantage of such solution is that they can handle a relatively large number of attributes. However, there are also arguments ([29], [30]) that such solution is somehow artificial and many respondents may not understand what they really have to do. The Table I show an example of a trade-off matrix related to attributes *Operating System* and *Battery life*.

TABLE I
A TRADE-OFF MATRIX WITH RESPONDENT RANKING

	12 hours	6 hours	4 hours	2 hours
And	1	2	7	5
WinPhone	3	4	6	11
other	8	9	10	12

TABLE II
PAIR COMPARISONS AND RATINGS

id	Left side	OR (weight)	Right side
1	And,... AND greater than 500EUR, ...	(4)	Windows Phone,... AND less than 3.5" screen, ...
2	Greater than 4" screen,... AND , Battery life 6h	(5)	And,... AND WIFI, ...
3	Greater than 4" screen,... AND , oth OS	(1)	Battery life 10h,... AND no WIFI, ...

Technically the trade-off matrices corresponds to respondent ranking of some particular partial profiles (the example from Table I corresponds to respondent ranking of 12 binary stimuli, i.e., partial profiles with only two attribute values) and consequently a learner can easily derive preferences on top of them. However, while they are quite efficient on ranking binary stimuli, trade-off matrices cannot be used if we consider stimuli with more than two attributes. A solution to these limitations is to use pair comparisons. Each pair comparison (see Table II for examples) contains two stimuli and is rated on a Likert scale, typically from 1 (strongly prefer "left side") to 9 (strongly prefer "right side") with 5 as "neutral".

As discussed above, information derived from user's ratings is (a) subject of significant uncertainty, (b) subject of change and (c) sometimes non-transitive. To solve inherent difficulties coming from these issues some researchers claim that questioning is most efficient if the two stimuli in the pair are chosen to be nearly equal in importance ([30]). Inline with this approach [23] introduces two main principles when designing interviews:

- 1) *Minimal overlap* – the probability that an attribute value repeats itself in each interview should be as small as possible and
- 2) *Utility balance* – the stimuli utilities in a pair comparison should be approximately equal.

According with these principles a number of methodologies on creating interviews were developed, including so called D-efficient designs ([25], [26]). Other research used Bayesian methods to create more efficient designs. The reader may consider appropriate literature such as [37], [8], [9] and [5].

IV. MODELING ANSWERS AS BELIEFS

After thirty year from the seminal work (the AGM model) reported by [1], research in belief modeling has already a lot of significant results and, many major concepts and constructions have been subject of significant elaboration and development. Although this paper does not intend to review these results

TABLE III
PAIR COMPARISONS: REGIONS OF BELIEF BY DIFFERENT RESPONDENTS

	Scale 1..9				
	Strong + Left	Regular Left	Neutral	Regular Right	Strong + Right
U_1	1	2.4	5	6.8	9
U_2	1..2	3..4	5	6..7	8..9
U_3	1..2	3	4..6	7	8..9

(see [33], [16] for details) we find useful to recall some foundational concepts.

The beliefs held by an agent are represented by a set of belief representation sentences. Most of the time these sentences are defined with respect of a logical language and usually, it is assumed that the belief set of an agent is closed under the logical consequence of the language (agent rationality). In practice that does not require the belief set to contain all these beliefs (closure) but that a logical consequence is also a valid belief. The reader must notice that artificial intelligence belief representational principles are very close with respondent preferences in conjoint analysis: *respondents form beliefs and, mentally, according with their reasoning rules (respondent profile) they form new beliefs (related beliefs)*.

Recall, the AGM model [1] considers three types of belief change:

- A specified belief B is removed (*contraction*), i.e., the belief base is updated and is a subset of the initial belief base not containing B . Such change may not just involve only B removal;
- A belief B is added (*expansion*) to the belief base, i.e. the base is replaced by a set that is the smallest logically closed set that contains both the initial base and B ;
- A belief B is added and at the same time other beliefs are removed (*revision*) if this is needed to ensure that the resulting belief set is consistent with respect of the logical language.

Below we analyze how to obtain beliefs from respondent answers and what should be belief change with respect of interactive interviews in adaptive conjoint analysis [22], [28], [29]. While conjoint analysis considers many kinds of interview questions (see [5], for a survey), this work focuses only on trade-off matrices and pair comparisons. Getting beliefs from question answers is to understand *what is the logical information that can be asserted behind information* shown in Table I and Table II.

As the Likert scales identifies four regions of preference: strong positive, strong negative regular and neutral, mapping from such scales to beliefs should follow the same logic. Of course the mapping is different from one user to another, depending on her profile. The Table III shows examples of how a Likert scale map to regions of beliefs in the context of pair comparison ratings. Mapping ranks provided by trade-off matrices is a very similar process illustrated by Table IV.

Next we describe how ratings and rankings are mapped to weighted beliefs. When interviewing for conjoint analysis the economics community do not think terms of negative

TABLE IV
TRADE-OFF MATRICES: REGIONS OF BELIEF BY DIFFERENT RESPONDENTS

	Scale 1..12				
	Strong +	Regular	Neutral	Regular	Strong -
U_1	1	2..5	6..7	8..11	12
U_2	1..2	3..5	6..7	8..11	12
U_3	1..2	3..5	6..7	8..10	11..12

information but only on *levels* of attributes, i.e., the pair comparisons do not contain negative information.

While the AGM model does not make any ontological distinction between beliefs with respect of conjoint analysis we can assume a beliefs base obtained from an interview as containing the following kinds of beliefs:

Definition 1 (Beliefs in Conjoint Analysis):

We allow weighted beliefs with a weight parameter coming from $(0, 1]$ where 1 means full truth degree (complete certainty, the perfect belief), while a value $\alpha \in (0, 1)$ describe a regular belief that can be doubted.

1) *regular beliefs* such as

$$(A_1(a_1) \wedge \dots \wedge A_t(a_t)) : \alpha$$

2) *indifference beliefs* such as

$$(L \leftrightarrow R) : 1$$

Indifference beliefs are always have full truth because we claim that if the respondent would distinguish degrees of truth then she is able to express preference.

3) *negative beliefs*

$$(\neg F) : 1$$

where A_i are attribute predicates and L, R, F are regular atom conjunctions. Again, it is obvious in conjoint to don't ask user to express thoughts on negative information. As such there are no real negative beliefs such as $F : 0$. Moreover, the reader may notice that we adopt the intuitionistic logic approach i.e., there is no assumption on any kind of law of excluded middle, as we don't necessarily assume $F : 0 \leftrightarrow (\neg F) : 1$.

A. Modeling Pair Comparisons to Beliefs

Let us simplifying notations and consider a pair comparison similar with the first one of Table II. Let, for example,

$$q = A(a) \text{ and } B(b) \text{ or } A(a') \text{ and } C(c)$$

where $A(a)$ means attribute A takes value a , e.g., attribute OS (operating system) takes value *Android*. Let r be the respondent's rating of q and let us denote $L = A(a) \text{ and } B(b)$ the *left side* of the question and $R = A(a') \text{ and } C(c)$ its *right side*.

A natural representation of the sides of this question is by using logical connective \wedge . As such both the left side and the right side becomes ground logical formulas $L = A(a) \wedge B(b)$ (or similarly in another syntax $L = A \text{ is } a \wedge B \text{ is } b$) and $R = A(a') \wedge C(c)$.

Example 1 (Question as pair of logical formulas):

Translation from pair comparison to belief formula is straightforward, e.g., question 1 from Table II encodes to the belief formulas:

$$OS(\text{"And"}) \wedge Price(\text{"gt500"})$$

and

$$OS(\text{"Win"}) \wedge Scr(\text{"lt3.5"})$$

1) *Strong Positive Ratings:*

Rating on the strong positive left region (e.g. 1), the respondent meant that she certainly belief L and do not believe R . Therefore the beliefs base updates with two completely certain beliefs:

$$L : 1 \text{ and } (\neg R) : 1$$

When rating into the strong positive right region:

$$(\neg L) : 1 \text{ and } R : 1.$$

Example 2 (Rating to Beliefs):

The question 2 in Table II is a neutral rated question therefore will translate into a fully certain belief:

$$(Scr(\text{"lt4"}) \wedge Batt(\text{"6h"}) \leftrightarrow OS(\text{"And"}) \wedge WIFI(\text{"yes"})) : 1$$

Question 3 in the same Table is rated "at extreme" (1, "strongly prefer left side") therefore the following two beliefs are generated:

$$(Scr(\text{"lt4"}) \wedge OS(\text{"oth"})) : 1$$

and

$$(\neg(Batt(\text{"6h"}) \wedge WIFI(\text{"yes"}))) : 1$$

2) *Regular Ratings:*

When the rating is in the regular left region (for example, less than 5 but not 1) then the respondent believe more a L -based solution rather than an R -based one: However, there is no evidence on excluding a R -based approach. Therefore we cannot assume falsity of any of L or R . This happens conversely when the rating is in the regular right region(e.g., greater than 5 but not 9). As the respondent cannot exclude any of L and R it comes naturally that is preferably to use a multi-valued logic to encode the meaning of the rating value. As such, after answering such question, the belief base updates two more beliefs namely $L : \alpha$ and $R : \beta$ where $\alpha, \beta \in (0, 1)$ are their degrees of truth, obtained by processing the question rating.

The reader should notice that actually we don't make any assumption on how these α, β are computed. This is related to the specification of the entailment of the logic representing the beliefs. There are many possible logics such as but this work does not intend to analyze and recommend one of them.

3) *Indifference (neutral) Ratings:*

When rating into the neutral region the respondent keep believing both L and R but she cannot express a degree of belief (that is, indifference). Recall that indifference cannot have degrees of truth because if the respondent would distinguish such degrees then she is able to express preference (the user is fully confident that L and R are the same for her, because cannot express an option between one of them.). By consequence we obtain a perfect belief $(L \leftrightarrow R) : 1$.

B. Modeling Ranks to Beliefs

The trade-off matrices are shown to the respondent with the goal of getting a rank of preferences. For example, the one depicted in Table I requests ranking from 1 to 12 (3×4 matrix).

A comprehensive solution would be to build all possible preferences coming from the rank but then this would not be convenient with the goal of keeping the beliefs base as smaller as possible. Assuming transitivity we would need to keep only few preferences but as transitivity is quite controversial [40], we are looking to solutions not necessary assuming it.

We map rankings close to the pair comparison approach. Each stimuli is interpreted as a conjunction of its attributes such as $S = A_1(a_1) \wedge A_2(a_2)$ (corresponding to a cell in Table I).

1) Strong Positive and Strong Negative Ranking:

As soon as a stimuli S is ranked into the strong positive region then, similarly with pair comparisons, we get the belief $S : 1$.

Whenever the rank is into the strong negative region we obtain the belief $(\neg S) : 1$.

Example 3: According with the ranking shown in Table I we have the beliefs:

$$(OS("And") \wedge Batt("12h")) : 1$$

and

$$(\neg(OS("oth") \wedge Batt("2h")))) : 1$$

2) Regular Ranking:

Regular rankings, the ones with are not extreme now middle are translated into regular beliefs such as $(A_1(a_1) \wedge A_2(a_2)) : \alpha$, $\alpha \in (0, 1)$.

3) Indifference (neutral) Ranking:

When ranking into the neutral region (e.g., 6 and 7 as for the user 3 in Table IV) then indifference should be modeled, therefore if S_1, \dots, S_k are the neutral ranked stimuli then the set

$$\{(S_{i_1} \leftrightarrow S_{i_2}) : 1 | i_1, i_2 \in \{1, \dots, k\}, i_1 \neq i_2\}$$

is derived.

Example 4: According with the ranking shown in Table I we obtain the indifference belief:

$$(OS("Win") \wedge Batt("4h")) \leftrightarrow OS("And") \wedge Batt("4h")) : 1$$

C. On Maintaining a Belief Set

When a new question is answered, the decision maker has to perform what is sometimes called as *beliefs update*: the new information is about the situation at present, while the old beliefs refer to the past therefore we may have to change the old beliefs to take into account the new information. In addition, as both the old beliefs and the new information refer to the same situation, an inconsistency between the new and old information may occur and as such the decision maker should perform a specific update (including beliefs revision) to capture the new information into the current set of beliefs without generating an inconsistency (as we assume that the respondent do not have an inconsistent behavior). A common

assumption of belief revision is that of minimal change: the knowledge before and after the change should be as similar as possible i.e., the change should preserve as much information as possible.

What makes this update non-trivial is that several different ways for performing this operation may be possible.

Example 5 (Beliefs Update):

For instance, if the current beliefs include the three completely certain beliefs:

$$A(a) : 1$$

$$B(b) : 1$$

and

$$(A(a) \wedge B(b) \leftrightarrow C(c)) : 1$$

then the assertion of the new belief $(\neg C(c)) : 1$ can be done only by removing/changing at least one of the three existent ones (consistency preservation). Such kind of treatment is particularly very significant in adaptive conjoint analysis which emphasizes an interactive interview and new coming questions may introduce preference change.

As discussed in the beginning of Section IV the main model of belief revision has three kinds of operations (see [15] for details). However, in the context of conjoint analysis we see beliefs revisions only in presence of new information therefore contraction cannot take place directly, while expansion and revision takes various forms.

To be more precise on defining *beliefs update* lets introduce some notations widely used in model theory. Let $\mathcal{A} = A_1, \dots, A_n$ be a set of database attributes with $dom(A_i)$ the domain of values. Each attribute corresponds either to (a) a *standard logic predicate* i.e., a unary predicate with only 1(*true*) and 0(*false*) as truth values) or to a *multi-valued logic predicate* or *similarity predicate* i.e., a unary predicate with many truth values from $[0, 1]$ following the similarity approach interpretation.

Let \mathcal{L} be a, possibly non-classical, logic over the vocabulary \mathcal{A} and logical connectives $\neg, \wedge, \vee, \rightarrow, \leftrightarrow$ ¹.

Recall that predicate logic model theory defines an interpretation as a subset of Herbrand-base i.e., a subset of ground atoms defined over the Herbrand universe. In standard logic when A is a formula and \mathcal{I} is an interpretation, if A evaluates to 1(*true*) with respect of \mathcal{I} then we say \mathcal{I} is a *model* of A and denote $\mathcal{I} \models A$. Similarly we consider a model relation \models associated with \mathcal{L} . Finally let \sqsubset be a preference relation between the models of \mathcal{L} . To keep concise we denote $\mathcal{L} = (\mathcal{L}, \models, \sqsubset)$ this preference logic as described by [42].

Let KB be a belief base each $B : \beta \in KB$ being one of the three kinds of beliefs introduced by Definition 1 and based on logic \mathcal{L} .

Let $B : \beta$ be a new belief. We say $B : \beta$ *does not contradict* KB iff all models of beliefs in KB are not models of $\neg(B : \beta)$ ($\forall \mathcal{I}, B, \mathcal{I} \models B : \beta \Rightarrow \mathcal{I} \not\models \neg(B : \beta)$).

¹Actually, we don't need any assumption on how these connectives are defined and what are the relations between them.

When introducing an update operation one must consider at least the following update rules:

- 1) If the new belief $B : \beta$ does not contradict with KB , then updating KB with respect of $B : \beta$ is

$$Upd(KB, B : \beta) = Cn(KB \cup \{B : \beta\})$$

where Cn denotes the regular deductive closure of a belief set. That is, any non-contradictory belief which does not have a similar one in KB is simply added to the belief base. Semantically, the models of the updated beliefs base consist of the intersection between the models of KB and the models of $B : \beta$.

- 2) If the new belief $B : \beta$ contradicts with KB then, according with [15], the main idea is that the new updated belief set $Upd(KB, B : \beta)$ should contain the new information $B : \beta$ and as many of the old beliefs in KB by preserving that the new belief set must be consistent and deductively closed. [15] introduces the concept of contraction \div , then

$$Upd(KB, B : \beta) = Cn((KB \div \neg(B : \beta)) \cup \{B : \beta\})$$

V. TOWARDS CONJOINT ANALYSIS BY BELIEFS AGGREGATION

Again, all above discussion assumes that respondents do not form inconsistent beliefs. However, this perspective is subject of criticism as some researchers consider that is was considering that people have inconsistent beliefs and they may even doing this in a rational way [35].

As we consider that the most of such cases comes from an apparent overlap between something and its negation and as AGM model is based on classical logic (including law of excluded middle) we did not made any such assumption therefore we consider that, using a multi-valued preference logic (see Section IV-C) which may not consider the law of excluded middle is appropriate to overcome such cases.

In addition, while the entire framework of belief updates considers the creation of beliefs by a specified respondent the conjoint analysis aims to aggregate such beliefs from many respondents. Therefore belief aggregation is unlikely to be modeled as simple union because of consistency assumption. While is agreed that most people keep consistent beliefs we cannot make a similar assumption with respect of beliefs coming from various respondents. As such this work, also inline with the Arrow's framework [3], keep the following two assumptions:

- 1) *Individual beliefs are consistent*
- 2) *Collective beliefs may not be consistent.* While the AGM model considered *consolidation* as a maintenance operation of removing some dispensable beliefs resulting in a consistent knowledge base, we would like to avoid such approach due to missing of motivated criteria with respect of belief elimination, as inconsistency comes from joining various respondents beliefs.

As such we propose an updated process chain of adaptive logic-based conjoint analysis as depicted by Figure 2.

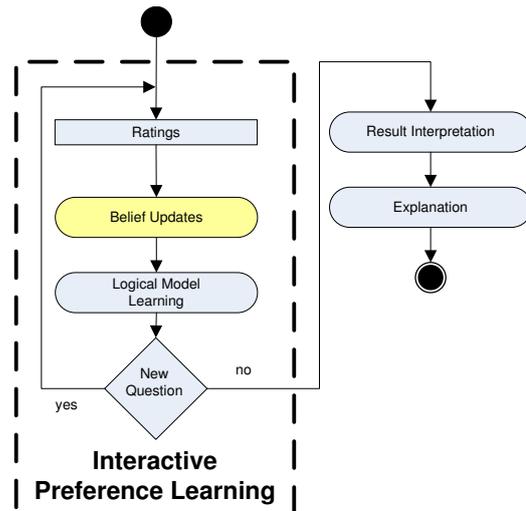


Fig. 2. Logic-Based Adaptive Conjoint Analysis Chain

VI. CONCLUSION

This work argued towards a logic-based model on conjoint analysis. We consider that respondent preferences can be better captured in a preference logic based on weighted beliefs rather than simple partial orderings requiring acyclicity and transitivity. While a belief set of one respondent is closed to deduction this does not necessary imply transitivity of its beliefs as reasoning in a non-monotonic logic might be chosen.

In the context of conjoint analysis surveys we identified three kinds of beliefs that can be obtained from question answers. The proposed framework considers consistent respondent belief sets but on belief sets aggregation there is no need to require consistency: moreover this is inline with the Arrow's seminal result that for sets of more than 3 objects there are not Pareto efficient² and independent preferences aggregations that are not dictatorial³. As most of the work around this result was concerned with relaxation of the theorem conditions, the future work on our logic-based framework will study how we can obtain an optimal logic-based preference aggregation including by relaxing Pareto efficiency.

In addition, the reader may noticed that, as in standard belief theory, described belief update rules always consider the new belief as part the new updated belief set. However there is research that does not impose this assumption – non-prioritized belief revision [21] relaxes this requirement and we plan to address the impact of such assumption to conjoint analysis in future work.

²A preference aggregation \succ is Pareto efficient if for all $o_1, o_2 \in O$ and for all respondents p , $o_1 \succ_p o_2 \Rightarrow o_1 \succ o_2$. In other words, if all respondents agree between o_1 and o_2 then the aggregate preference must agree the same.

³A preference aggregation \succ does not have a dictator iff $\nexists p \forall o_1, o_2 (o_1 \succ_p o_2 \Rightarrow o_1 \succ o_2)$. Non-dictatorship means that there does not exist one respondent whose preferences always determine the preference aggregation

REFERENCES

- [1] C.E. Alchourron, P. Gärdenfors and D. Makinson. On the Logic of Theory Change: Partial Meet Contraction and Revision Functions. *Journal of Symbolic Logic*, 50: 510-530, 1985.
- [2] G. Antoniou. Defeasible logic with dynamic priorities. *Int. J. Intell. Syst.* 19(5): 463-472, 2004.
- [3] K.J. Arrow. A Difficulty in the Concept of Social Welfare. *Journal of Political Economy* 58(4) (August, 1950), pp. 328-346.
- [4] K. J. Arrow. *Social Choice and Individual Values*. 2nd ed., 1963.
- [5] D. Baier and M. Bruschi (Eds.) *Conjointanalyse, Methoden - Anwendungen - Praxisbeispiele*, Springer, Berlin, 2009.
- [6] P. Bennett and N. Howard. Rationality, emotion and preference change: Drama-theoretic models of choice. *European Journal of Operational Research*, Volume 92, Issue 3, 9 August 1996, pp. 603-614, Elsevier.
- [7] J. Beringer, E. Hüllermeier. Case-based learning in a bipolar possibilistic framework. *Int. J. Intell. Syst.* 23(10): 1119-1134, 2008.
- [8] L. Burgess, and D. Street. Optimal Designs for 2^ok Choice Experiments. *Communications in Statistics: Theory and Methods*, 32 (11), 2003, pp. 2185-2206.
- [9] L. Burgess, and D. Street. Optimal Designs for Choice Experiments with Asymmetric Attributes. *Journal of Statistical Planning and Inference*, 134, 2005, pp. 288-301.
- [10] D. Dubois and H. Prade. Possibility Theory, Probability Theory and Multiple-valued Logics: A Clarification. *Annals of Mathematics and Artificial Intelligence* 32:35-66, 2001.
- [11] D. Dubois, E. Hüllermeier. Comparing probability measures using possibility theory: A notion of relative peakedness. *Int. J. Approx. Reasoning* 45(2): 364-385, 2007.
- [12] J.N. Druckman and A. Lupia. Preference Formation. *Annu. Rev. Polit. Sci.* 2000. 3:1-24, <http://www.annualreviews.org/doi/pdf/10.1146/annurev.polisci.3.1.1>.
- [13] N. Friedman and J. Y. Halpern. Plausibility Measures: A User's Guide. In *Proc. Eleventh Conference on Uncertainty in Artificial Intelligence (UAI95)*, pp. 175-184, 1995.
- [14] E. L. Gettier. Is justified true belief knowledge?, *Analysis* 23, pp.121-123, 1963, see also <http://www.ditext.com/gettier/gettier.html>.
- [15] P. Gärdenfors and H. Rott. Belief revision. In *Handbook of Logic in Artificial Intelligence and Logic Programming*, Volume 4, pp. 35-132. Oxford University Press, 1995.
- [16] P. Gärdenfors. Notes on the history of ideas behind AGM. *Journal of Philosophical Logic*, 40: 115-120, 2011.
- [17] A. Giurca. A Logic with Plausibility. *Annales of Craiova University, Mathematics and Computer Science Series*, XXVII, pp.105-115, 2000.
- [18] A. Giurca, I. Schmitt, and D. Baier. Performing Conjoint Analysis within a Logic-based Framework. *Proc of IEEE Federated Conference on Computer Science and Information Systems, (FedCSIS2011)*, Szczecin, Poland, 18-21 September, 2011.
- [19] J. Y. Halpern. Conditional Plausibility Measures and Bayesian Networks. *Journal of Artificial Intelligence Research*, Volume 14, pages 359-389, 2001, <http://www.jair.org/papers/paper817.html>.
- [20] J. Y. Halpern. *Reasoning about Uncertainty*, Cambridge, MA: MIT Press, 2003.
- [21] S. O. Hansson. A Survey of Non-Prioritized Belief Revision. *Erkenntnis*, 50: 413-427, 1999.
- [22] J. R. Hauser, and G. L. Urban. Assessment of Attribute Importances and Consumer Utility Functions: von Neumann-Morgenstern Theory Applied to Consumer Behavior. *Journal of Consumer Research*, Vol. 5, (March), 1979, pp.251-262.
- [23] J. Huber, and K. Zwerina. On the Importance of Utility Balance in Efficient Designs. *Journal of Marketing Research*, vol. 33, 1996, pp. 307-317.
- [24] E. Hüllermeier and I. Schmitt. Non-Additive Utility Functions: Choquet Integral versus Weighted DNF Formulas, The 4th Japanese-German Symposium on Classification (JGSC2012), March 9-10, 2012, Kyoto, Japan (to appear).
- [25] W.F. Kuhfeld, R.D. Tobias, and M. Garratt. Efficient Experimental Designs with Marketing Research Applications. *Journal of Marketing Research*, 31 (November), 1994, pp. 545-557.
- [26] W.F. Kuhfeld. *Marketing Research Methods in SAS: Experimental Design, Choice, Conjoint, and Graphical Techniques*. SAS 9.1. Edition, TS-222, SAS Institute Inc.
- [27] R.C. Jeffrey. A Note on the Kinematics of Preference. *Erkenntnis*, Vol. 11, No. 1, May, 1977 *Social Ethics*, Part 1, Springer, 1977, <http://www.jstor.org/pss/20010537>.
- [28] R. M. Johnson. Tradeoff Analysis of Consumer Values. *Journal of Marketing Research*, 1974, pp. 121-127.
- [29] R. M. Johnson. Comment on Adaptive Conjoint Analysis: Some Caveats and Suggestions. *Journal of Marketing Research*, 28, 1991, pp. 223-225.
- [30] R. M. Johnson. Comments on Studies Dealing With ACA Validity and Accuracy, With Suggestions for Future Research, 1991 published by Sawtooth Software.
- [31] Daniel Lehmann. Plausibility logic. *LNCS*, Volume 626/1992, pp.227-241, 1992, <http://www.springerlink.com/content/m46xq54qr4n04737/>
- [32] R. Likert. A Technique for the Measurement of Attitudes. *Archives of Psychology* 140, 1932, pp.1-55.
- [33] D. Makinson. Ways of doing logic: what was new about AGM 1985, *Journal of Logic and Computation*, 13: 5-15, 2003.
- [34] B. Orme. *Getting Started with Conjoint Analysis: Strategies for Product Design and Pricing Research*. 2nd Edition, Madison, Wis.: Research Publishers LLC, 2010.
- [35] G. Priest. Paraconsistent Belief Revision. *Theoria*, 67: 214-228, 2001.
- [36] G. Priest. Paraconsistent Logic. *Handbook of Philosophical Logic* (2nd Ed.), D. Gabbay and F. Guenther (eds.), Dordrecht: Kluwer Academic Publishers, Vol. 6, pp. 287-393, 2002.
- [37] Z. Sandor, and M. Wedel. Designing Conjoint Choice Experiments using Manager's Beliefs. *Journal of Marketing Research*, 38 (November), pp. 430-444, 2001.
- [38] I. Schmitt. QQL: A DB&IR Query Language. *VLDB Journal*, 17(1), pp.39-56, 2008.
- [39] I. Schmitt, and D. Baier. Logic Based Conjoint Analysis using the Commuting Quantum Query Language, *Proc. of Conference of the German Classification Society (GfK12011)*, August 31 to September 2, 2011, Frankfurt am Main, Germany.
- [40] G.F. Schumm. Transitivity, Preference and Indifference. *Philosophical Studies*, 52: 435-437, 1987.
- [41] A. K. Sen. *Collective Choice and Social Welfare*, Holden-Day, 1970.
- [42] Y. Shoham. Nonmonotonic Logics: meaning and utility. *Proceedings of 10th IJCAI*, pp.388-393, Milan, 1987.
- [43] Y. Shoham and K. Leyton-Brown. *Multiagent Systems: Algorithmic, Game-Theoretic, and Logical Foundations*. New York: Cambridge University Press. 2009, Chapter 9: Aggregating Preferences: Social Choice see <http://www.masfoundations.org/mas.pdf>.
- [44] R. B. Zajonc, H. Markus. Affective and Cognitive Factors in Preferences. *Journal of Consumer Research*, Vol. 9, No. 2, Sep., 1982 pp. 123-131.
- [45] D. Zellhöfer and I. Schmitt. A preference-based approach for interactive weight learning: learning weights within a logic-based query language. *Distributed and Parallel Databases* 27, pp.31-51, 2010, DOI 10.1007/s10619-009-7049-4.
- [46] T. Wang. Conditional preferences and updating. *Journal of Economic Theory*, Vol. 108, Issue 2, February 2003, pp. 286-321.