Proceedings of the Federated Conference on
Computer Science and Information Systems pp. 93–100

ISBN 978-83-60810-51-4

Artificial Reasoning in Relative Dilemmatic Logic

Juliusz L. Kulikowski
Nalecz Institute of
Biocybernetics and Biomedical
Engineering PAS,
4, ks Trojdena str.,
02-109 Warsaw, Poland,
juliusz.kulikowski@ibib.waw.pl

Abstract— In this paper principles of relative dilemmatic logic as a modification of conventional relative logic are formulated and based on them methods of logical reasoning are presented and illustrated by examples. It is shown that dilemmatic logic makes possible not only to relatively evaluate logical values of statements without using any numerical parameters but also it makes possible to eliminate from logical inference process premises and inductions whose relative value is lower than this of their negations. Graphical representation of logical inference processes by bi-partite and tri-partite graphs is proposed and the role of graph theory methods in solution of the logical inference tasks based on relative dilemmatic logic is indicated.

Keywords: artificial reasoning, relative logic, logical dilemmas, dilemmatic logic, Hasse diagrams

I. Introduction

SINCE the first works of J. Łukasiewicz concerning non-classical logics [1,2] various attempts to make logical systems more flexible and suitable to simulate the natural reasoning principles have been proposed. In fact, a binary (“true”, “false”) logical scale widely used in classical logical systems [3,4] in everyday human thinking is rather occasionally used. The last is based mostly on vague concepts and non-determined sharply inference rules based on experience, intuition, analogies, etc. This is invariance of the natural rules governing in the real world that for long decision sequences makes this type of reasoning effective. The main ideas in the logical inference “naturalization” took the form of multi-valued logic [5], modal logics [6], inductive logic [7], fuzzy sets logic [8], rough sets logic [9], possibility theory [10], etc. In all the above-mentioned cases, a common idea consists in an extension of the scales of logical values with respect to the classical, binary one. However, in all cases the scales remain linearly ordered, numerical. Even the widely known rough set approach, based on the categories “surely yes”, “possibly yes” and “surely no”, is in fact a sort of a three-valued logic [11]. On the other hand, in natural thinking numerical scales for logical evaluation of statements are rarely used. Nevertheless, the classical logic rules play an irreplaceable role in any exact non-classical logic description. And still, the problem: how to evaluate or to establish an intuitive probability, a membership function value, a logical value level, etc. of a statement has no satisfactory explanation. This is why a proposal of a relative logic (originally called topological logic, due to its graphical representation [12]) draw attention of some authors [13, 14, 15]. In relative logic any statement A may be less, more or comparably true with respect to some other one B or A and B may be mutually incomparable; however, no numerical values of their “trueness” or “falseness” are used. Instead, logical relationships among the statements by contour-free directed (not obviously compact) graphs can be represented. This approach makes us free of any logical values (membership levels, probabilities, etc.) calculation; the most and/or the less logically valuable statements can be found by analysis of the graphs. However, this approach has also some drawbacks. Knowing that “A *is more logically valuable than B”* tells nothing about A and B being true at all. Only adding that e.g. “A is as logically valuable as 2 + 2 = 4” makes the former statement anchored on a logical scale. It thus arises a problem whether it is possible and if so, how to extend the relative logic so as to establish a logical reference level for the considered sentences without taking into account any additional, reference statements. The answer is positive if taking into account that to each statement its negation in a natural way can be assigned and their logical validities can be compared. In other words, before asking whether a statement A is less or more logically valuable than a statement B, it is natural to ask whether A is less or more logically valuable than “*not A”*. A pair consisting of a statement and of its logical negation constitutes a logical dilemma. In [14] using a mode “rather A” has been proposed in the case if higher logical value to the statement A than to its negation is assigned. Moreover, the more different logical values to the components of a dilemma are assigned, the lower is the uncertainty that a real situation by the more logically valuable statement is described. This uncertainty does not concern the information carried by the lexical expression of the statement but a meta-information

This work was not supported by any organization

about the assigned to it truth level. In this paper a concept of using the comparative dilemmas' uncertainty assessment to decision making instead of the "conventional" relative logic approach is presented and illustrated by examples. The concept is based on the idea that if A has relatively higher logical value than B then this statement is more convincing if we know that B is of high certainty then if it is close to ambivalence. The paper is organized as follows. In Sec. 2 some basic notions concerning the relative logic of dilemmas are introduced. Sec 3 presents a method of logical inference based on dilemmas. In Sec. 4 application of the proposed approach to a pattern recognition problem is described. Conclusions are formulated in Sec. 5.

II. Basic Notions

We will consider a non-empty set U of simple assertive propositions u called statements. It is assumed that no statement in U concerns direct or hidden assessment of its proper logical value. A proposition is called simple if it contains a single subject and a single predicate and, thus, it cannot be presented by several propositions connected by conjunctions. It is assumed that in U a first-order propositional calculus with logical operators of negation $\neg$, disjunction $\cup$ and conjunction $\cap$ of statements has been defined. Then, it will be defined a set D of ordered pairs (u | $\neg$ u) called dilemmas. The propositional calculus can easily be on the dilemmas extended:

Definition 1 . For a given set D of dilemmas it is defined:

a) A negation of a dilemma d = (u | $\neg$u) as a dilemma $\neg$ d = ($\neg$u | u);
b) A disjunction of the dilemmas $d' = (u' \mid \neg u')$, $d'' = (u'' \mid \neg u'')$ as a dilemma $d' \vee d'' = [u' \vee u'' \mid \neg(u' \vee u'')]$;
c) A conjunction of the dilemmas $d' = (u' \mid \neg u')$, $d'' = (u'' \mid \neg u'')$ as a dilemma $d' \wedge d'' = [(u' \wedge u'' \mid \neg(u' \wedge u'')]$ •

From the Definition 1 and the de Morgan laws [3] it follows:

Corollary 1. The following identities hold:

a) $\neg(\neg d) \equiv d$;
b) $d' \vee d'' \equiv [u' \vee u'' \mid (\neg u') \wedge (\neg u'')]$;
c) $d' \wedge d'' \equiv [(u' \wedge u'' \mid (\neg u') \vee (\neg u'')]$ •

In the set of dilemmas a semi-ordering relation of their certainty can be defined in the below described way.

Definition 2. Let D be a set of dilemmas. Then in D a binary relation $\approx^c$ satisfying the conditions of:

a) reciprocity: for each d $\in$ D it holds $d \approx^c d$;
b) symmetry: for any $d', d'' \in$ D if $d' \approx^c d''$ then also $d'' \approx^c d'$ holds;
c) reflexivity: for any $d', d'' \in$ D if $d' \approx^c d''$ then also $\neg d' \approx^c \neg d''$ holds;
d) transitivity: for any $d', d'', d''' \in$ D if $d' \approx^c d''$ and $d'' \approx^c d'''$ then also $d' \approx^c d'''$ holds;
e) fixation: for any $d', d'' \in$ D if $d' \approx^c \neg d'$ and $d'' \approx^c \neg d''$ then also $d' \approx^c d''$ holds,

will be called an equal certainty relation. Any dilemma satisfying the condition $d \approx^c \neg d$ will be called an ambivalent dilemma; any dilemmas such that $d' \approx^c d''$ holds will be called equivalent dilemmas; any dilemmas such that $d' \approx^c \neg d''$ holds will be called anti-equivalent dilemmas •

It thus follows from the Definition that all ambivalent dilemmas are both mutually equivalent and mutually anti-equivalent. As such, they can be established as a reference level for other dilemmas' certainty assessment.

Definition 3. Let D be a set of dilemmas with established equal certainty relation. Then a binary relation $\preceq^c$ described in D and satisfying the conditions of:

a) reciprocity: for each d $\in$ D it holds $d \preceq^c d$;
b) symmetry: for any $d', d'' \in$ D $d' \preceq^c d''$ and $d'' \preceq^c d'$ hold if and only if $d' \approx^c d''$ holds;
c) anti-reflexivity: for any $d', d'' \in$ D if $d' \preceq^c d''$ then $\neg d'' \prec^c \neg d'$ holds;
d) transitivity: for any $d', d'', d''' \in$ D if $d' \preceq^c d''$ and $d'' \preceq^c d'''$ then also $d' \preceq^c d'''$ holds,

will be called a certainty ranking •

If for any two dilemmas $d' \preceq^c d''$ holds and $d'' \preceq^c d'$ does not hold then it will be called that *the certainty of d' is* dominated by this of d "; if $\neg$ d is dominated by d then d will be called a correct dilemma, in the opposite case it will be called incorrect; any two dilemmas such that neither $d' \preceq^c d''$ nor $d'' \preceq^c d'$ holds will be called mutually incomparable and denoted by $d' ?^c d''$.

It remains to establish the rules of assigning certainty relations between algebraic compositions of dilemmas described by Definition 1 and their components.

Definition 4. Let D be a set of dilemmas with established equal certainty and certainty ranking relations. Then for any $d', d'' \in$ D

a) *if* $d' \preceq^c d''$ and not $d'' \preceq^c d'$ then $d' \vee d'' \approx^c d''$;
b) *if* $d' \preceq^c d''$ and not $d'' \preceq^c d'$ then $d' \wedge d'' \approx^c d'$;
c) *if* $d' \approx^c d''$ then $d' \vee d'' \approx^c d' \wedge d'' \approx^c d', \approx^c d''$;
d) *if* $d' ?^c d''$ then $d' \wedge d'' \preceq^c d', d''$;
e) *if* $d' ?^c d''$ then $d', d'' \preceq^c d' \vee d''$ •

Logical value of a dilemma is thus characterized by two parameters: its certainty related to the level of ambivalence and its correctness (a binary parameter). The certainty ranking of a set D of dilemmas can be presented by a labeled contour-free directed graph (the term contour is used here for an uniformly directed cycle) with a single input node representing all ambivalent (and mutually equivalent) dilemmas. Any other node of the digraph represents a single or a group of equivalent correct dilemmas. Arcs (arrows) directed from *d'* to *d"* correspond to situations when *d'* is dominated by *d"*. Parallel to directed paths (by-passing) arcs in the digraph are omitted. An example of a digraph describing a certainty ranking of a set of dilemmas is presented in Fig. 1. It in two versions is shown. Version a) corresponds to an initial state of the digraph construction when contours corresponding to mutually equivalent dilemmas, a surplus arc and (marked in black) nodes representing incorrect dilemmas yet exist.

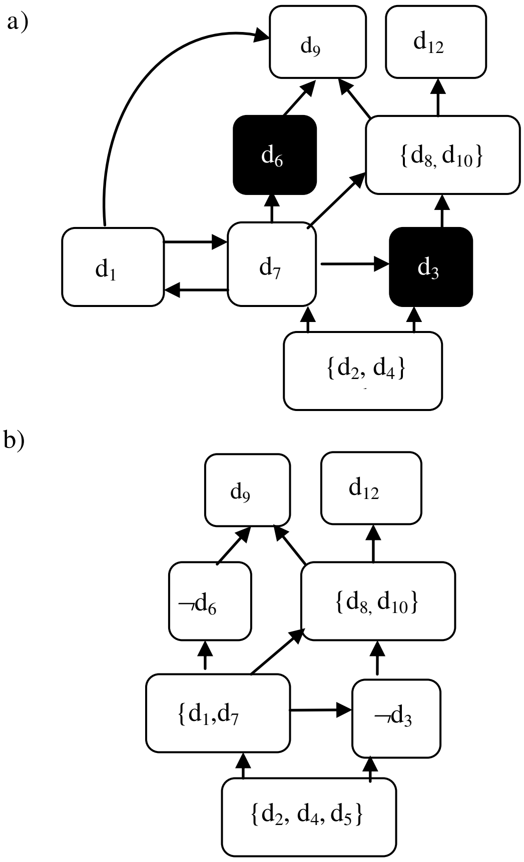

Fig. 1. Digraph representation of dilemmas' certainty ranking: a) initial version, b) simplified version.

This version of the digraph then has been modified (simplified), as shown in Fig. 1 b. In this version the nodes of the equivalent dilemmas d_1 and d_7 have been merged so that the corresponding contour disappeared; the surplus arc from d_1 to d_9 has been removed; the incorrect dilemmas d_3 and d_6 by the correct ones, $\neg d_6$ and $\neg d_6$ have been replaced. So, the digraph took the form of a Hasse diagram [16]. A difference to the Hasse diagrams used in conventional relative logic systems consists in a) nodes being assigned to dilemmas, not to single statements and b) the diagram being anchored to a single input node assigned to a collection of all ambivalent dilemmas. The difference between the conventional and dilemmatic relative logic can be illustrated by the following example.

Example 1. Fig. 2 presents a microscopic image of a stained human blood specimen containing granulocytes affected by different states of necrosis. The most advanced states are characterized by fuzziness of cells' contours and large amount of vacuoles in the cytoplasm surrounding the nuclei. A practical problem consists in selection and counting of the most damaged cells. For this purpose, there have been distinguished circular dark objects in the image reminding cells' nuclei and they have been denoted by the letters *a, b, c,…,k*. We try to assign logical values to the statements:

u_x = *"Object x represents a cell in advanced necrotic state"*

for $x \in \{a, b, c, ..., k\}$ as a criterion for final selection of the objects of interest. However, it can be observed that a quite unambiguous selection in this case is not possible. If any objective methods (spectral analysis, morphological analysis, etc.) are used to characterize the selected objects they give at most some premises for inferring about the state of the necrotic process. That is why a relative logic seems to be more suitable than the classical logic to decide about the classification of the objects. According to the conventional relative logic methodology, for some pairs of statements (u_x, u_y) their logical equivalence ($\approx^c$) or order ($\preceq^c$) was established and the corresponding Hasse diagram shown in Fig. 3a has been constructed.

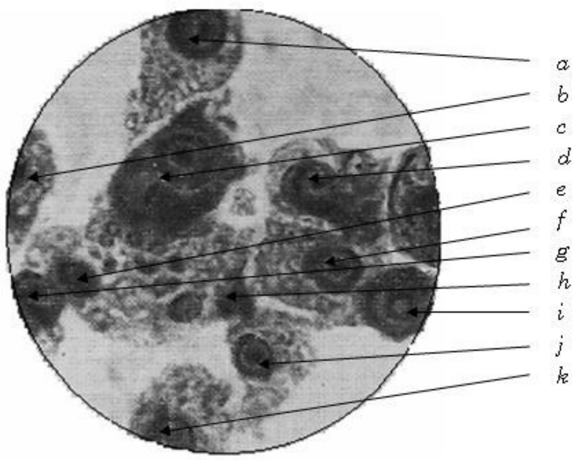

Fig. 2. Microscopic image of blood specimen with objects selected for analysis.

Similarly, according to the dilemmatic logic approach the dilemmas $d_x = (u_x, \neg u_x)$ are taken into consideration and some pairs of them are compared in order to establish their logical equivalence or dominations. Then a corresponding Hasse diagram is constructed as shown in Fig. 3 b.

Definition 5. A minimal node of a Hasse diagram is a node not preceded by any other node of the diagram •

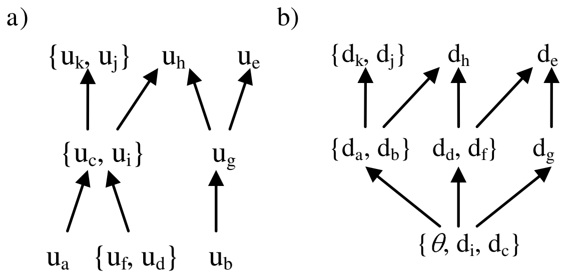

Fig. 3. Hasse diagrams of logical values: a) of statements in conventional relative logic, b) of dilemmas in dilemmatic relative logic.

Let us remark that a Hasse diagram contains at least one minimal node, however, it may contain more than one of them. The Definition 5 can easily be extended on the maximal node of a Hasse diagram. The dilemmas represented by a minimal (maximal) node of the Hasse diagram are called, respectively, minimal (maximal). Ambivalent dilemmas are thus mandatory minimal dilemmas in a Hasse diagram of a set D of dilemmas; if no such dilemma really exists, the diagram should be completed by a hypothetical ambivalent dilemma θ, as shown in Fig. 3b.

III. DILEMMATIC LOGICAL INFERENCE

3.1. Basic logical induction rule.

A **c**lassical logical inference is based on a modus ponens scheme [3]:

$$\begin{array}{l} \text{If A then B} \\ \underline{\text{A}\qquad\qquad} \\ \text{B} \end{array} \qquad (1)$$

A multi-step logical inference consists of a network of modus ponens acts schemes ordered so that some conclusions (B) of foregoing schemes are used as the premises (A) in the succeeding schemes. In relative logic it arises the problem of relative logical evaluation of the conclusion B with respect to the premise A and the induction *If... then* , the last also being logically relatively evaluated. Like in classical logic, the induction "*If A then B"* (symbolically denoted by A$\Rightarrow$B) can be interpreted as "Not A or B". Therefore, the modus ponens scheme in relative dilemmatic logic takes the form:

$$\begin{array}{l} [(\neg A \vee B), (A \wedge \neg B)] \\ \underline{(A, \neg A)\qquad\qquad\qquad} \\ (B \mid \neg B) \end{array} \qquad (2)$$

and the problem consists in evaluation of the concluding dilemma's (B | ¬B) sign and certainty level if the relative certainty levels of dilemmatic premise *d'* = (A, ¬A) and induction *d"*= [(¬A ∨ B), (A ∧ ¬B)] are given. First, it will be assumed that both *d'* and *d"* are correct (or at least ambivalent) dilemmas, otherwise, they should be replaced by the corresponding negative dilemmas. Then, the conclusion d = (B | ¬B) can be evaluated as a conjunction *d'* ∧ *d"* . Taking into account the correctness of *d'* and Definition 4 the following situations thus can be taken into consideration:

1^0, if $d' \preceq^c d''$ and not the reverse then it should be d $\approx^c d'$;

2^0, if $d' \approx^c d''$ then it should be $d' \approx^c d \approx^c d''$;

3^0 , if $d'' \preceq^c d'$ and not the reverse then it should be d $\approx^c d''$;

4^0 , if *d' ?* ^{c} *d"* then it should be d $\preceq^c d'$ as well as d $\preceq^c d''$.

In other words, **the certainty level of a dilemmatic conclusion following from the correct dilemmatic premise and induction does not transcend the lower certainty level of the premise and the induction.**

The multi-step logical inference processes can be classified according to their topological properties. The processes consist of logical induction steps in which some premises are transformed into local conclusions. Then, algebraic combinations of initial premises and local conclusions are used as premises of next induction steps up to the steps leading to final conclusions. The variety of possible topological structures is very large, however, they can be classified within several below described typical schemes.

3.2. Single-way reasoning scheme.

The simplest type of logical inference process starts from a single initial premise dilemma d_1 and it leads to a conclusion d_k through a final number (at least one) linearly ordered mediate dilemmas. Any pair of consecutive dilemmas constitutes a logical induction $d_i \Rightarrow d_j$. The scheme can be presented in the form of a bi-partite digraph whose nodes assigned to dilemmas (premises and conclusions) and to inductions constitute two disjoint subsets as shown in Fig. 4.

According to Definition 4b and conclusion following from the dilemmatic modus ponens properties, the uncertainty of the final conclusion d_k of the single-way reasoning process, assuming that no pair of inductions is incomparable, is given by the expression:

$$d_k \approx^c \min(d_1, d_1 \Rightarrow d_2, d_2 \Rightarrow d_3, \ldots, d_{k-1} \Rightarrow d_k) . \qquad (3)$$

3.3. Confluence-tree reasoning scheme.

In this reasoning scheme the inductions, like before, are sequentially ordered but more than one premise is taken into consideration; additional premises are included into the process at consecutive reasoning steps as shown in Fig. 5.

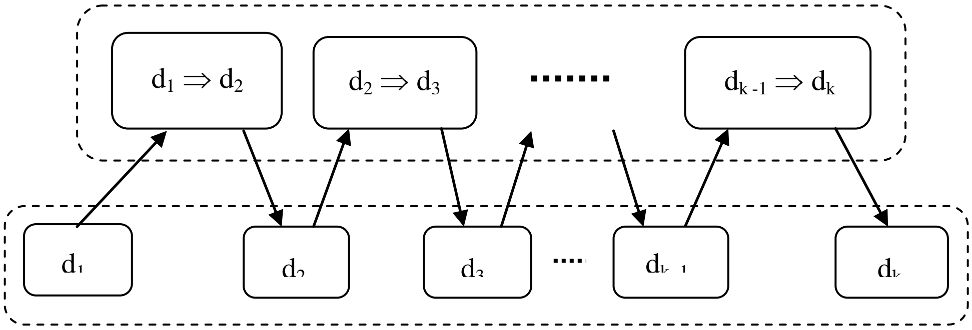

Fig. 4. A bi-partite digraph illustrating the structure of a single-way reasoning process.

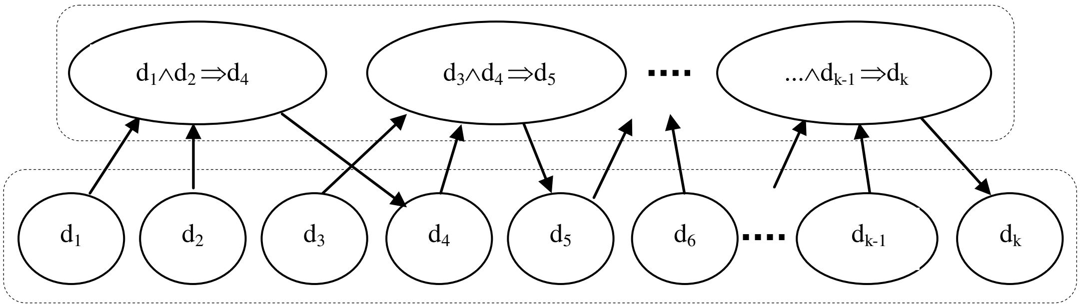

Fig. 5. A bi-partite digraph illustrating the structure of a confluence-tree reasoning process.

In this case, like before, the final conclusion is given by a conjunction of inductions. Moreover, the premises of inductions are also given as conjunctions of dilemmas. However, it may happen that some pairs of premises are incomparable. Therefore, according to Definition 4, the relative uncertainty of final conclusion is in general given by the formula

$$d_k \preceq^c \min (d_1, d_2, \ldots, d_{k-1}, d^*_1 \Rightarrow d_4, d^*_2 \Rightarrow d_5, \ldots, d^*_{k-1} \Rightarrow d_k) \quad (4)$$

where $d^*_1, d^*_2, \ldots, d^*_{k-1}$ denote the composed premises of inductions.

3.4. Reasoning schemes based on composed inductions.

The above-considered reasoning scheme cannot easily be extended on the situations when the premises and/or conclusions take the form of sophisticated logical formulae consisting of several logical terms. The following Example illustrates the problem.

Example 2. Let us take into consideration the following statement: *"If somebody has studied computer science* (d_1) or mathematics (d_2) and logics (d_3) then he is familiar with backgrounds of graph theory (d_4) and propositional calculus (d_5) or propositional calculus and the theory of finite automata (d_6)*".* More formally, it can be written as:

$$\text{If } [d_1 \vee (d_2 \wedge d_3)] \text{ then } [(d_4 \wedge d_5) \vee (d_5 \wedge d_6)] \quad (5)$$

The problem consists in relative evaluation of the conclusions d_5 and d_6 in this type of dilemmatic induction. Taking into account that each logical formula can be presented in a disjunctive normal form (the form of disjunction of conjunctions of single statements or of their negations [3]), a composed logical formula like this given by (5) can be presented in the form of a disjunction of simple formulae:

$$[d_1 \Rightarrow (d_4 \wedge d_5)] \vee [d_1 \Rightarrow (d_5 \wedge d_6)] \vee [(d_2 \wedge d_3) \Rightarrow (d_4 \wedge d_5)] \vee [(d_2 \wedge d_3) \Rightarrow (d_5 \wedge d_6)]. \quad (6)$$

For a graphical representation of this formula a tri-partite directed graph can be used as shown in Fig. 6.

Suppose, a problem consists in evaluation of the dilemma:

d_5 = [*"N is familiar with propositional calculus"* | *"N is not familiar* with propositional *calculus"*]

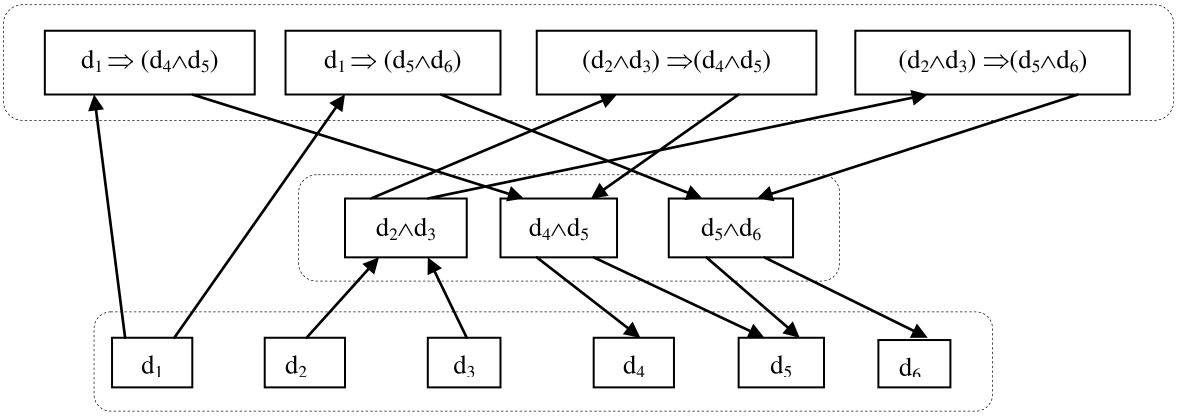

Fig. 6. Graphical representation of a composed induction .

as a conclusion following from the premises:

d_1 = [*"N has studied computer sciences"* | *"N h*as not *studied computer sciences"*]

or $d_2 \wedge d_3$ where:

d_2 = ["*N has studied mathematics"* | *"N has not studied mathematics"*],

d_3 = ["*N has studied logics"* | *"N has not studied logics"*],

First, it should be proven whether all the above-mentioned dilemmas are correct; otherwise, the incorrect dilemmas should be replaced by their negations. Then, the following paths in the graph in Fig. 6 should be taken into consideration:

a) $[d_1] \rightarrow [d_1 \Rightarrow (d_4 \wedge d_5)] \rightarrow [d_4 \wedge d_5] \rightarrow [d_5]$,
b) $[d_1] \rightarrow [d_1 \Rightarrow (d_5 \wedge d_6)] \rightarrow [d_5 \wedge d_6] \rightarrow [d_5]$,
c) $[d_2 \wedge d_3] \rightarrow [(d_2 \wedge d_3) \Rightarrow (d_4 \wedge d_5)] \rightarrow [d_4 \wedge d_5] \rightarrow [d_5]$,
d) $[d_2 \wedge d_3] \rightarrow [(d_2 \wedge d_3) \Rightarrow (d_5 \wedge d_6)] \rightarrow [d_5 \wedge d_6] \rightarrow [d_5]$.

(symbols $\rightarrow$ denote connections of the reasoning steps represented by arcs in the tri-partite graph). Reaching d_5 is possible by at least one of the above-presented paths, therefore, its certainty level is given as maximum of certainties assigned to the paths. It also follows from the Definition 4 b, c, d that the certainty levels of the final dilemma d_5 can not be dominated by the certainty levels of the preceding conjunctions of dilemmas. Therefore, the solution of the problem is given formally by the following expression:

$$d_5 \preceq^c \max\{\min[d_1, d_1 \Rightarrow (d_4 \wedge d_5)], \min[d_1, d_1 \Rightarrow (d_5 \wedge d_6)], \min[(d_2 \wedge d_3), d_2 \wedge d_3) \Rightarrow (d_4 \wedge d_5)], \min[(d_2 \wedge d_3), (d_2 \wedge d_3) \Rightarrow (d_5 \wedge d_6)]\}. \quad (7)$$

A final result depends on the relationships established between 6 dilemmas d_1, $d_1 \Rightarrow (d_4 \wedge d_5)$, $d_1 \Rightarrow (d_5 \wedge d_6)$, $(d_2 \wedge d_3)$, $(d_2 \wedge d_3) \Rightarrow (d_4 \wedge d_5)$ and $(d_2 \wedge d_3) \Rightarrow (d_5 \wedge d_6)$. The number of possibilities is quite enormous: it corresponds to the number of different Hasse diagrams consisting of at most 6 nodes labeled by 6 elements (some labels can be assigned to the same node). For example, 6 linearly ordered elements can be in 6! ways represented by linear Hasse diagrams whose arcs correspond to the $\preceq^c$ signs. Each of such sequences generates next Hasse diagrams if some of the $\preceq^c$ signs are replaced by the $\approx^c$ or $?^c$ signs, etc. Let the following situation be assumed:

$$[d_1] \approx^c [d_1 \Rightarrow (d_4 \wedge d_5)] \preceq^c [d_1 \Rightarrow (d_5 \wedge d_6)] \preceq^c [(d_2 \wedge d_3)] \preceq^c [(d_2 \wedge d_3) \Rightarrow (d_4 \wedge d_5)] \approx^c [(d_2 \wedge d_3) \Rightarrow (d_5 \wedge d_6)].$$

Then, it can be found that:

$$d_5 \preceq^c \max\{d_1, d_1, (d_2 \wedge d_3), (d_2 \wedge d_3)\} \approx^c (d_2 \wedge d_3) \bullet$$

A general reasoning scheme can be presented as a network consisting of simple or composed premises and inductions. Like before, the structure of such scheme can be presented by a tri-partite digraph whose nodes constitute three mutually disjoint subsets representing inductions, their composite premises or conclusions and simple premises or conclusions. The network is correctly constructed if the following conditions are satisfied:

i. All its components are correct dilemmas;
ii. Composite premises and conclusions are presented in disjunctive canonical forms;
iii. The corresponding tri-partite digraph is compact and contour-free;
iv. The subset of simple premises contains a non-empty sub-subset of initial premises which all are the elements of composite premises or are directly used as single premises of inductions;

v. The subset of simple premises contains a non-empty sub-subset of terminal premises which are not initial and are not used as components of premises of any inductions.

For using a correctly designed scheme in logical reasoning the subsets of its initial premises and inductions should be relatively evaluated according to the Definitions 2 and 3. Unlike other logical systems, relative logic does not force an user to assign logical values to all premises, however, in the case of a lack of some relative assessment the final conclusions may be undefined, like it happens in natural thinking. At the next step, as it was illustrated above, for relative evaluation of a conclusion:

I. There should be found in the network all alternative paths from the initial premises to the conclusion;
II. There should be found the relative certainty levels of the paths as minima of the certainty levels of their components;
III. It should be found the global certainty level of the conclusion as a maximum of the certainty levels of the leading to it alternative paths.

3.5. Remark about a solution method.

Formula (7), consisting of a maximum of minima, is typical for finding the most certain conclusion in a reasoning network. In the case of a large number of terms that should be compared for finding a minimum and a large number of paths connecting the initial assumptions with final conclusions a solution of the problem may be a very large time consuming one. However, it can be simplified if taking into account that if d* is a minimal term found in a path by pair-wise comparison of its consecutive terms then no other path containing d* can contribute to reaching a higher maximum and all such paths can be removed from further considerations. This remark may lead to a significant reduction of the number of paths that should be analyzed in order to find a final solution of the problem.

IV. CONCLUSIONS

The dilemmatic logic is a modification of conventional relative logics; it consists in replacing the statements by dilemmas defined as pairs (A,¬ A) where A is a statement and ¬ A is its logical negation. Dilemmas are logically evaluated by their correctness (or ambivalence) and relative certainty levels; in logical inference correct and ambivalent dilemmas only are used. Due to this, the problem of non-admittance of rather false premises or inductions (represented by incorrect dilemmas) to be included into the reasoning process can be solved. The way of logical reasoning based on relative dilemmatic logic has been illustrated by examples. Relative approach makes the logical evaluation and reasoning free of using any numerical characteristics of statements (logical values in multi-valued logics, probabilities, membership values in fuzzy logics, etc.). In fact, it is easier in practice to relatively compare logical values of any two statements then to assign to them rationally founded membership values or probabilities. Graph representation of the reasoning processes makes them easy for a direct, manual analysis in simpler cases (say, up to few dozens of nodes). However, due to easily accessible effective programs of computer operations on graphs (e.g., detection of contours, paths, minimal and maximal nodes, etc., [16, 17]) a computer implementation of relative reasoning methods in more complicated cases seems to be not a serious computational problem. In particular, an important problem of analysis of the influence of relative certainty assessment of premises and inductions on the certainty level of conclusions can be effectively used on the basis of typical graph theory methods and algorithms.

ACKNOWLEDGMENT

The author would like to express his deep thanks to the (unknown to him) Reviewers for their substantial critical remarks and/or advices. Most of them have been used in the final version of the paper. Some, for technical reasons, will be taken into consideration in future works on the topic.

V. REFERENCES

[1]. J. Łukasiewicz. "About Three-valued Logic" (in Polish). Ruch Filozoficzny, **5**, 1920, pp. 169-170.
[2]. J. Łukasiewicz. "Elements of Mathematical Logic" (in Polish). Warsaw, 1929.
[3]. S.C. Kleene. Mathematical Logic. John Wiley & Sons, Inc., New York, 1967.
[4]. H. Rasiowa, R. Sikorski. "The Mathematics of Metamathematics". PWN, Warsaw, 1970.
[5]. J.B. Rosser, A.R. Turquette. "Many-valued Logics". North-Holland, Amsterdam, 1952.
[6]. C.I. Lewis, C.H. Langford. "Symbolic Logic". New York, 1932.
[7]. H. Mortimer. "Inductive Logic" (in Polish). PWN, Warsaw, 1982.
[8]. L.A. Zadeh. "Fuzzy Logic and Approximate Reasoning". Synthese, **30**, 1975, pp. 407-428.
[9]. Z. Pawlak. "Rough Sets – Theoretical Aspects of Reasoning About Data". Kluwer Academic Publishers, Dordrecht, 1991.
[10]. S.T. Wierzchoń. "Knowledge Representation and Manipulation within Dempster-Shafer Framework "(in Polish). IPI PAN Publisher, Warsaw, 1996.
[11]. J.L. Kulikowski. "Topological Logic versus Fuzzy Sets. A Comparison of Methods" (in Polish). In: J. Chojcan, J. Łęski, Eds. Zbiory rozmyte i ich zastosowania. Wydawnictwo Politechniki Śląskiej, Gliwice, 2001, pp.195-216.
[12]. C.G. Hempel. "A Purely Topological Form of Non-Aristotelian Logic". JSL, **2**(3), 1937.
[13]. H.A. Vessel. "About Topological Logic" (in Russian). In: *Neklassićeskaya* logika. Izd. Nauka, Moscow, 1970, pp. 238-261.
[14]. J.L. Kulikowski. "Decision Making in a Modified Version of Topological Logic". In: Proc. of the Seminar on Non-conventional Problems of Optimization, Part 1. Prace IBS PAN, No 134, Warsaw, 1986.
[15]. J.L. Kulikowski. "Using Relative Logic for Pattern Recognition". In: M. Branner, V. Devedyic (Eds.). Artificial Intelligence Applications and Innovations. Kluwer Academic Publishers, Boston, 2004, pp. 223-230.

[16]. N. Christofides. “Graph Theory. An Algorithmic Approach”. Academic Press, New York, 1975.

[17]. M.N.S. Swamy, K. Thulasiraman. “Graphs, Networks and Algorithms”. John Wiley & Sons, New York, 1981.