Failure Analysis and Estimation of the Healthcare System

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***Abstract*—The principal goal of information technologies application in medicine is improvement and conditioning of medical care. Modern healthcare systems have to perfect the care of a patient. Therefore, the healthcare system has to be characterized, first of all, by high reliability and reliability analysis of such a system is an important problem. The new method for estimation of system reliability is considered in this paper. This method permits to investigate the influence of any system component failure to the system functioning.**

I. Introduction

INITIATIVES for implementing healthcare systems based on the information technologies are now a principal part of the development in medicine. The development of these systems depends on organization of the healthcare provision in each country and the presence of the information and telecommunication technologies in the healthcare sector [1–3]. There is one principal characteristic for all healthcare systems. It is reliability that is defined as the probability that a system will perform its intended function during a period of running time without any failure. A fault is an erroneous state of the system. The system reliability is a complex characteristic that depends on the functioning of separate parts (components) of the system.

Based on bibliography in reliability analysis of the healthcare domain, we can show two principal approaches. The first of them is reliability estimation of medical equipment and devices that includes reliability quantification of hardware and software of the healthcare system [4–6]. The second approach agrees with examination of human errors [7, 8]. However, independent evaluation of these principal parts of the healthcare system does not allow providing detail and actual reliability analysis. In [9], new tendencies in reliability engineering are considered. According to [9], the reliability analysis has to be based on joint evaluation of all principal parts (components).

The typical healthcare system structure consists of some principal components from the point of view of reliability analysis [4, 7, 9]. In [4], two of them have been defined: equipment/device and human factors. We need to note that the human factor has been considered as errors of operators of medical equipment or devices in [4]. A detailed structure of the human factor and human errors for the healthcare system is presented in [7]. The healthcare system structure includes three components: technical, human and organization [9]. The technical component includes two types of medical devices/equipment that are based on special and standards-based technologies according to [10]. For example, the first type is the medical decision support system, the system for integrating electronic medical records or picture archiving communication systems. The second type is the special medical device and equipment that can be used for a special operation only (as magnetic resonance imaging scanners, for example). The human component of the healthcare system causes medical errors. The organization component of the system joins management aspects and maintenance of the healthcare system.

In this paper, we develop results that have been presented in [9] as well as methods proposed for estimation of system components based on a single approach. Particularly, we consider the Importance Analysis of the healthcare system. This analysis allows investigation of every system component functioning/failure into the system reliability. In Section II, the typical approach for the reliability examination from the step of mathematical modelling to the calculation of the reliability indices is considered basing on an example of the mathematical model of performance shaping factors for human errors in the healthcare system. The Direct Partial Logic Derivatives are also proposed in this section for description of the system behaviour. Section III presents most frequently used importance measures that allow defining the system component with minimal or maximal influence to the system reliability. The algorithms for calculation of these measures based on Direct Partial Logic Derivatives are developed in this section. The new algorithm for calculation of one of the possible importance measures is proposed in Section IV. The possible development of the proposed methodology is analysed in Conclusions.

II. Mathematical Background

The reliability analysis of a system includes three principal steps [11]:

- the quantification of the system model;
- the representation and modelling of the system;

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- the representation, propagation and quantification of the uncertainty in the system behaviour.

I. Quantification of the System

Quantification of the system is a principal step and it causes development of a mathematical model. There are two approaches to the quantification in reliability engineering.

The first of them defines only two states of the system reliability: the functioning and failure. The mathematical model for the representation of this quantification is called a Binary-State System (BSS). The system and its components are allowed to have only two possible states (completely failure and functioning) in BSS (Fig. 1). This approach is well known and widely used in reliability engineering. The system failure can be investigated in detail based on this quantification. However, the analysis of other performance levels, before the system failure, has some difficulties for BSS. In this case, the quantification of the system reliability to some performance levels is used. The mathematical model with some performance levels is called a Multi-State System (MSS).

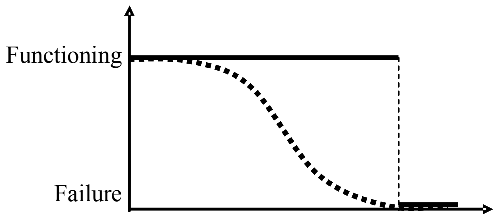

Fig. 1 System reliability interpretations for BSS

MSS reliability analysis is a more flexible approach to evaluating system reliability, as it can be used when both the system and its components may experience more than two states, including, for example, completely failed, partially failed, partially functioning and perfect functioning (Fig. 2). The MSS scientific achievement has been documented in [11 – 13]. However, a mathematical approach to analysing such a system is complex. In many applications, the definition of the system failure is the principal problem. Therefore, the system quantification can be simplified and considered as the BSS (Fig. 1).

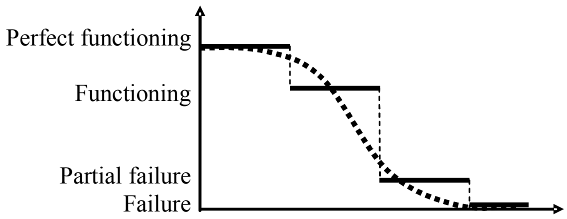

Fig. 2 System reliability interpretations for MSS

The BSS permits to investigate the system failure that has the high priority in reliability engineering. Therefore, mathematical models and methods for the estimation of the system failure are developed dominantly based on the system representation by BSS, because MSS complicates the system failure investigation.

In this paper, the analysis of the healthcare system failure is considered on the basis of the use of BSS.

II. Modelling of the System

The next step, after the definition of quantification, is the mathematical model development. There are some types of the BSS representation as the mathematical model. These representations (mathematical models) correlate with the mathematical methods for the calculation of the system reliability indices and measures. One of these representations is the structure function. This function allows the mathematical description of a system with any complexity [12, 13].

The system reliability in the stationary state depending on component states is defined by *structure function* [14]:

$$\phi(x_1,...,x_n) = \phi(x) : \{0,1\}^n \rightarrow \{0,1\}. \qquad (1)$$

A coherent system is considered in the paper below. The important assumptions for this system [12, 14] are as follows: (a) the structure function (1) is monotone, and (b) the system component failure does not improve the system reliability.

In the considered mathematical model, every system component x_i is characterized by probability of the reliability:

$$p_i = \Pr\{x_i = 1\}. \qquad (2)$$

The system component unreliability is defined as:

$$q_i = \Pr\{x_i = 0\} = 1 - p_i. \qquad (3)$$

For example, the structure function of the human sub-system (component) can be defined on the basis of the mathematical model of performance shaping factors for human errors in the healthcare system that is proposed in [10]. According to the model in [10], the analysis of the human error has to include social, personal, organization and technological aspects (Fig. 3). We can interpret this model as the structure function:

$$\varphi(x) = x_1 \wedge \left((x_2 \wedge x_3) \vee (x_2 \wedge x_4) \vee (x_3 \wedge x_4)\right), \qquad (4)$$

where $\wedge$ and $\vee$ are symbols of the operations AND and OR accordingly.

The structure function (4) defines correlation of the social, technical and organizational aspects as the system 2-out-of-3, that is to say, the combination of these aspects is dependable if two or more of these aspects are reliable. These aspects and the personal aspect are correlated as the series system. We use the term "component" for any of these aspects and indicate as x_i in this paper in the examples below.

The BSS behavior specified by the structure function is described by the Direct Partial Logic Derivative. In this case, the structure function variables are interpreted as the component state, and the function value is agreed with the system state (reliability).

The Direct Partial Logic Derivative with respect to variable x_i for the BSS structure function (1) permits to

The mathematical model of performance shaping factors for human errors

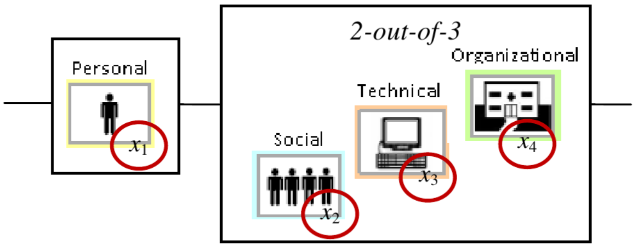

Fig. 3 The structure function for the healthcare system human component

analyse the system reliability change from j to $\bar{j}$ when the i-th component state changes from a to $\bar{a}$ [14]:

$$\partial\varphi(j\to\bar{j})/\partial x_i(a\to\bar{a})= \\ =\begin{cases}1, & if\ \varphi(a_i,x)=j\wedge\varphi(\bar{a}_i,x)=\bar{j}\\ 0, & otherwise\end{cases} \quad (5)$$

where $\phi(a_i, \boldsymbol{x}) = \phi(x_1,\ldots, x_{i-1}, a, x_{i+1},\ldots, x_n)$; $\phi(\bar{a}, \boldsymbol{x}) = \phi(x_1, \ldots, x_{i-1}, \bar{a}, x_{i+1},\ldots, x_n)$; $a, j \in \{0, 1\}$ and $\bar{a} = 1-a$, $\bar{j} = 1-j$.

Let us consider the system failure in the Direct Partial Logic Derivative terminology. The system failure is represented as a change of the structure function value $\phi(\boldsymbol{x})$ from state 1 into 0. This change can be caused by the i-th variable change from 1 to 0 if we consider a coherent system. Therefore the Direct Partial Logic Derivative for the BSS failure analysis is defined by the equation

$$\partial\varphi(1\to0)/\partial x_i(1\to0)= \\ =\begin{cases}1, & if\ \varphi(1_i,x)=1\ and\ \varphi(0_i,x)=0,\\ 0, & otherwise.\end{cases} \quad (6)$$

The Direct Partial Logic Derivative (6) $\partial\varphi(1\to0)/\partial x_i(1\to0)$ allows investigating boundary states of this system for which failure of one component x_i causes the system breakdown. For example, these states for the system in Fig. 3 are shown in Table 1. Therefore, there are 4 boundary states for the first component and 2 states for every other component of the mathematical model of performance shaping factors for human errors in the healthcare system.

The Direct Partial Logic Derivative can be used for the investigation of the influence of the system components failure to the failure of BSS. This investigation is a subject of the Importance Analysis in the reliability engineering [12–15].

III. Mathematical Method

Importance analysis allows examining different aspects of reliability changes and the uncertainty in the system failure. In particular, the importance analysis is used for BSS reliability estimation depending on the system structure and its component states. The various evaluations of the BSS component importance are called Importance Measures (IMs). IM quantifies the criticality of a particular component within BSS. They have been widely used as tools for identifying system weaknesses, and to prioritise reliability improvement activities.

The most frequently used IMs as Structural Importance (SI), Birnbaum importance (BI), Fussell-Vesely Importance (FVI) are shown in Table II [12, 13].

TABLE I.
BOUNDARY STATES FOR THE HEALTHCARE SYSTEM HUMAN COMPONENT (FIG.3)

$\partial\phi(1\to0)/\partial x_1(1\to0)$	$\partial\phi(1\to0)/\partial x_2(1\to0)$	$\partial\phi(1\to0)/\partial x_3(1\to0)$	$\partial\phi(1\to0)/\partial x_4(1\to0)$
(1→0, 0, 1, 1)	(1, 1→0, 0, 1)	(1, 0, 1→0, 1)	(1, 0, 1, 1→0)
(1→0, 1, 0, 1)	(1, 1→0, 1, 0)	(1, 1, 1→0, 0)	(1, 1, 0, 1→0)
(1→0, 1, 1, 0)			
(1→0, 1, 1, 1			

TABLE II.
IMPORTANCE MEASURES

Short name	Description
SI	SI concentrates on the topological structure of the system and determines the proportion of working states of the system in which the working of the i-th component makes the difference between system failure and working state.
BI	BI of a given component is defined as the probability that such a component is critical to MSS functioning and represents loss in MSS when the i-th component fails.
FVI	FVI quantifies the maximum decrement in MSS reliability caused by the i-th system component state deterioration and if $a = 0$, the measure allows estimating system performance level decrease for full unreliability of the i-th system component.

Calculation of IMs is based on different mathematical approaches and the Direct Partial Logic Derivative is one of them. This approach has been proposed in [14]. According to [14], the Direct Partial Logic Derivative has been used for calculation of SI and BI. The FVI definition is based on the minimal cuts of BSS. In this paper, a new algorithm for calculation of the minimal cuts of the system based on the Direct Partial Logic Derivative is proposed.

III. IMPORTANCE MEASURES

IV. Structural Importance

SI is one of the simplest measures of the component importance and this measure focuses on the topological aspects of the system. According to the definition in [16], this measure determines the proportion of working states of the system in which the working of the i-th component makes the difference between system failure and its working:

$$IS_i=\frac{\rho_i}{2^{n-1}} \quad (7)$$

where ρ_i is a number of system states when the change component state results in the system failure.

For example, calculated IMs (7) based on Direct Partial Boolean Derivatives for the system are shown in Fig. 3. Values of SI (8) and intermediate values of ρ_i are shown in Table III. According to this table, the first component has maximal influence to the system reliability from the point of view of the system topology.

TABLE III.
STRUCTURAL IMPORTANCE FOR THE SYSTEM IN FIG.3

i	ρ_i	IS_i
1	4	4/8 = 0.500
2	2	2/8 = 0.250
3	2	2/8 = 0.250
4	2	2/8 = 0.250

V. Birnbaum Importance

BI of a given component is defined as the probability that the system is sensitive to inoperative of the *i*-th system component [17]. Let us consider the Direct Partial Logical Derivatives for calculation of BI. In [14], BI has been defined as

$$IB_i = \Pr\{\partial\varphi(1\to 0)/\partial x_i(1\to 0)=1\} \tag{8}$$

For example, let us consider the system shown in Fig. 3. Probabilities of the system element reliability and unreliability are shown in Table IV. According to the data in Table I, elements of this system have the following values of BIs:

$$IB_1=\Pr\{\partial\phi(1\to 0)/\partial x_1(1\to 0)=1\}=$$
$$=q_2q_3p_4+q_2p_3q_4+q_2p_3p_4+p_2q_3q_4+p_2q_3p_4+p_2p_3q_4+p_2p_3p_4=0.964;$$

$$IB_2=\Pr\{\partial\phi(1\to 0)/\partial x_2(1\to 0)=1\}=p_1q_3q_4=0.096;$$

$$IB_3=\Pr\{\partial\phi(1\to 0)/\partial x_3(1\to 0)=1\}=p_1q_2q_4=0.072.$$

$$IB_4=\Pr\{\partial\phi(1\to 0)/\partial x_4(1\to 0)=1\}=p_1q_2q_3=0.096.$$

BI for the first component has the maximal value. The BIs, as the SIs, show that the first system component is more important for reliability.

TABLE IV.
PROBABILITIES OF ELEMENTS FOR THE SYSTEM IN FIG.3

	x_1	x_2	x_3	x_4
p_i	0.80	0.70	0.60	0.70
q_i	0.20	0.30	0.40	0.30

VI. Fussell-Vesely Importance

FVI represents the contribution of each component to the system failure probability and for BSS it is calculated by the following equation [17]:

$$I_{FV}(x_i)=\frac{F_{\text{min cut}}(x_i)}{Q} \tag{9}$$

where $F_{\text{min cut}}$ (x_i) is the system minimal cut that includes the *i*-th system component, Q is the function of the system unreliability [14, 17]:

$$Q=\Pr\{\phi(\boldsymbol{x})=0\}. \tag{10}$$

Therefore, for calculation of this measure, the minimal cut set is needed. In the next section, we propose a new algorithm for calculation of the minimal cut set for BSS by Direct Partial Logic Derivatives.

IV. MINIMAL CUT SET IN IMPORTANCE ANALYSIS

VII. Minimal Cut Set and Minimal Cut Vector

Let us consider the conception of the cut set. The cut set is the set of the system components whose simultaneous failure results in the system failure (if the system has been functioning). As a rule, the number of the cut set components k is changed from 1 to n. The system failure is caused by one component reduction only if k = 1 and all components have to fail that to cause the system failure if $k = n$. The minimal cut set is a cut set in which any subset remaining after the removal of any of its components is no longer the cut set.

Let $\boldsymbol{a} = (a_1\ldots a_n)$ and $\boldsymbol{b} = (b_1\ldots b_n)$ be two state vectors for system component states or values of structure function (1). The vector $\boldsymbol{a} < \boldsymbol{b}$ if $a_i < b_i$ for $i = 1, \ldots, n$.

The state vector $\boldsymbol{a} = (a_1\ldots a_n)$ is a cut set vector if $\phi(\boldsymbol{a}) = 0$.

The cut set vector $\boldsymbol{a}$ is minimal, if $\phi(\boldsymbol{b}) = 1$ for any $\boldsymbol{b} > \boldsymbol{a}$.

For example, the system shown in Fig. 3 has 12 cut set vectors (if $\phi(\boldsymbol{x}) = 0$, $\phi(\boldsymbol{x})$ is defined by (4)) and 4 minimal cut set vectors:

$$\{(x_1), (x_2\, x_3), (x_2\, x_4), (x_3\, x_4)\}. \tag{11}$$

Therefore, FVI for this system, according to (9), is calculated as:

$$IFV_1=\Pr\{(x_1)\} / Q = q_1 / Q = 0.492$$

$$IFV_2=\Pr\{(x_2\, x_3), (x_2\, x_4)\} / Q = q_2q_3 + q_2q_4 / Q = 0.517$$

$$IFV_3=\Pr\{(x_2\, x_3), (x_3\, x_4)\} / Q = q_2q_3 + q_3q_4 / Q = 0.591$$

$$IFV_4=\Pr\{(x_2\, x_4), (x_3\, x_4)\} / Q = q_2q_4 + q_3q_4 / Q = 0.517$$

where the system unreliability Q (10) is calculated as:

$$Q = \Pr\{\phi(\boldsymbol{x})=0\} = q_1 + p_1(q_2q_3 + q_2q_4 + q_3q_4) = 0.4064.$$

FVI of the first component has the minimal value. Therefore, the first component does not have most significant influence to the system reliability if the component combination failure is considered. According to value IFV_3, the third component refuse causes the system failure in combination with other component refuses predominantly.

FVI is an alternative measure of Importance Analysis that allows estimating influence of the particular component to the system reliability and functioning. However, the minimal cut sets for the calculation of this measure are needed.

VIII. Minimal Cut Set Vectors and Direct Partial Logic Derivatives

Let us compare two definitions of the Direct Partial Logic Derivatives and minimal cut set vectors. Let the Direct Partial Logic Derivative be $\partial\phi(1\to0)/\partial x_i(1\to0)$. This derivative permits to determine the structure function state vectors that are boundary for the structure function value with respect to the variable x_i. The minimal cut set vector is the boundary state vector too, but for some variables (components). Therefore, the set of Direct Partial Logic Derivatives with respect to some variables can be defined by the minimal cut set vector. This supposition has been verified and tested. The result of testing confirms the supposition.

The Direct Partial Logic Derivative $\partial\phi(1\to0)/\partial x_i(1\to0)$ indicates state vectors $(0_i, \boldsymbol{x})$, in which improvement of component i results in the system improvement. To identify minimal state vectors, i.e., state vectors for which improvement of any broken component results in the improvement of the whole system, we have to compute $\partial\phi(1\to0)/\partial x_i(1\to0)$ (5) for every component and then compute the intersection of these derivatives. To compute the intersection, the modified type of derivative has to be used that is defined as:

$$\partial\varphi(j\to\bar{j})/\partial x_i(s\to\bar{s})= \\ =\begin{cases}1 & \text{if } x_i=s \text{ and } \varphi(s_i,x)=j \text{ and } \varphi(\bar{s}_i,x)=\bar{j}\\ 0 & \text{if } x_i=s \text{ and } \varphi(s_i,x)=\varphi(\bar{s}_i,x)\\ * & \text{if } x_i\neq s\end{cases} \qquad (12)$$

The rule for intersection of two modified derivatives (11) is defined in Table V. This intersection identifies state vectors, in which improvement of both components (if the component can be repaired) results in improvement of the system.

Let us continue the hand calculation example for the mathematical model of performance shaping factors for human errors in the healthcare system (Fig. 3) that is defined by the structure function (4). The Direct Partial Logic Derivatives $\partial\phi(1\to0)/\partial x_i(1\to0)$ for system components have been calculated and shown in Table I. The intersection of these derivatives, according to the rule in Table V, allows getting 4 cut set vectors:

{0***, *00*, *0*0, **00},

which are consistent with the minimal cut sets (12).

TABLE V.
DEFINING THE INTERSECTION OF TWO MODIFIED DPLD

		$\partial\varphi(j\to\bar{j})/\partial x_i(s\to\bar{s})$		
		*	0	1
$\partial\varphi(j\to\bar{j})/\partial x_i(s\to\bar{s})$	*	*	0	1
	0	0	0	0
	1	1	0	1

The test of the proposed algorithm has been implemented on the basis of the sets of the benchmarks LGSynth91 [18]. Testing characteristic is a number of cut set vectors and time for computation (Fig. 4). The numbers in the left part of the graphs indicate the time and the numbers in the right part are numbers of the cut set vectors for the system. There is the proportional correlation between the number of cut set vectors in the system and time for the computation.

V. CONCLUSION

In this paper, the problem of calculation of IMs is considered. The IM definitions based on the Direct Partial Logic Derivatives [4] are provided. A new algorithm for calculation of FVI by the Direct Partial Logic Derivatives is presented in this paper. The experimental investigation corroborates the possible application of this algorithm for investigation of the large dimension system and computation of the IMs. The development of the presented result in future investigation will be adaptation of this algorithm for analysis of Multi-State System. This system allows the analysis of some (more than two) states in the reliability [12].

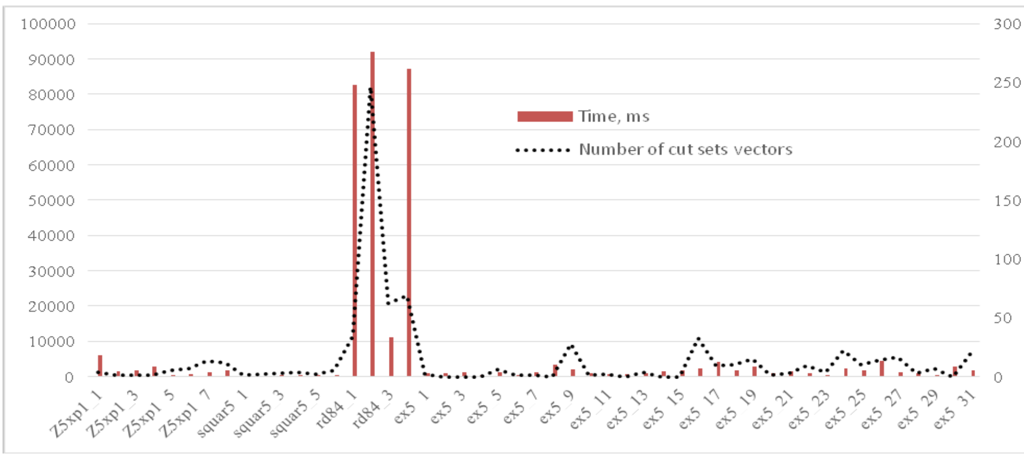

Fig. 4 Computational complexities

Reliability analysis of the health care system is an important issue. The principal problem in this analysis is the development of methodology that permits to investigate every system component and the system based on united approaches. This conception has been presented in details in [9, 15]. We propose and develop one of possible approaches that allow investigating the healthcare system component importance. The mathematical background of this approach is Direct Partial Logic Derivatives. The advantage of this approach is the possibility to use it for estimation of every system that is defined by the structure function.

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