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Bicriteria Fuzzy Optimization Location-Allocation Approach

Santiago García-Carbajal
Escuela Politécnica Superior de Ingeniería, Universidad de Oviedo, 33204 Gijón, Spain
Email: carbajal@lsi.uniovi.es

Belarmino Adenso-Díaz
Escuela Politécnica Superior de Ingeniería, Universidad de Oviedo, 33204 Gijón, Spain
Email: adenso@epsig.uniovi.es

Sebastián Lozano
Escuela Superior de Ingenieros, University of Seville, 41092 Seville, Spain
Email: slozano@us.es

Abstract—**Distribution network design deals with defining which elements will be part of the supply chain and how they will be interrelated. Many authors have studied this problem from a cost minimization point of view. Nowadays the sustainability factor is increasing its importance in the logistics operations and must be considered in the design process. We deal here with the problem of determining the location of the links in a supply chain and the assignment of the final customers considering at the same time cost and environmental objectives. We use a fuzzy bicriteria model for solving the problem, embedded in a genetic algorithm that looks for the best trade-off solution. A set of experiments have been carried out to check the performance of the procedure, using some instances for which we know a priori a good reference solution.**

I. Introduction

THE fierce competition between the different supply chains makes it necessary that efficiency be continuously pursued. One of the most important strategic decisions, and one that has a long-term impact in the economic results of the logistics operations, is the design of the distribution network.

Distribution network design is the process of determining the structure of a supply chain, defining which elements will be part of it (i.e., where locate the facilities), and what will be the interrelationships between them (i.e., the allocation of customers to facilities and how the material and products will flow in the network between the nodes in the network). For that reason the problem is often called location-allocation (e.g. [1]).

In [2], Akkerman *et al.* consider, from a hierarchical point of view, a second level called distribution network planning that includes the decisions related to fulfilling the aggregate demand (i.e., aggregate product flows and delivery frequencies)

Many authors have studied these problems, most of them (around two thirds according to [3]) by considering as the objective the minimization of the costs involved in the process. However, for different reasons (legal pressure, customers demand, ethical consciousness, etc) nowadays the sustainability factor is increasing its importance in the business management and specifically in the logistics operations, where transportation of goods is a high pollutant activity. It is therefore necessary to consider the operations impact when defining the distribution network.

The aim of this study is the formulation of a model and a solution procedure for the location-allocation problem when two criteria (cost and environmental impact of transportation) are considered at the same time.

II. Problem Setting

Let us suppose that there is an uncapacitated central plant that must distribute a single product among many customers. Those customers have uncertain (i.e. fuzzy) demands. We need to define the distribution network, choosing the capacitated intermediate warehouses to set up, and allocating each customer to one of warehouses (or to the central facility). There are two types of vehicles: large trucks (used for high demand customers and for serving the warehouses from the central plant) and smaller trucks.

We are going to consider two objective functions. One is the minimization of the logistics costs (transportation and warehouses set-up). The transportation costs will be proportional to distances and depend on type of truck used. The second objective function is the minimization of the environmental impact of the Greenhouse Gases (GHG) emissions (e.g. CO_2) due to transportation.

Note that, in principle, every customer could be served from the central facility, but if the demand is small, the cost and environmental impact of such direct shipments would be very high, likely bigger that delivering the goods from a near warehouse.

Our problem consists in deciding which of the potential warehouse locations will be opened and from which warehouse should each customer be served warehouse each customer will be allocated (considering the limited capacities of the warehouses) in such a way that total cost and GHG emissions are minimized.

III. Model Formulation

Table I shows the notation used for modeling the problem. Note that the set of potential warehouse locations are given together with the distance, unit transport cost and unit GHG emissions factor from the central plant to each potential warehouse location j. From each potential warehouse loca-

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TABLE I.
NOTATION

i	Index on customers (i=1..N)
j	Index on potential warehouse locations (j=1..A)
$I(j)$, $\hat{I}$	Subsets of customers that can be served from warehouse j and from the central plant, respectively
$\widetilde{D}_i$	Fuzzy demand of customer i. A Triangular Fuzzy Number membership function is assumed. $\mu_{D_i}(x)=\begin{cases} 0 & \text{if } x \le D_i^- \\ \frac{x-D_i^-}{D_i^0-D_i^-} & \text{if } D_i^- \le x \le D_i^0 \\ \frac{D_i^+-x}{D_i^+-D_i^0} & \text{if } D_i^0 \le x \le D_i^+ \\ 0 & \text{if } x \ge D_i^+ \end{cases}$
$\widetilde{U}_j$	Fuzzy capacity of warehouse j. A decreasing linear membership function is assumed $\mu_{U_j}(x)=\begin{cases} 1 & \text{if } x \le U_j^- \\ \frac{U_j^+-x}{U_j^+-U_j^-} & \text{if } U_i^- \le x \le U_j^+ \\ 0 & \text{if } x \ge U_j^+ \end{cases}$
$\widetilde{L}_j$	Fuzzy minimum flow of warehouse j. An increasing linear membership function is assumed (with parameter $L_i^+ << U_j^-$). $\mu_{L_j}(x)=\begin{cases} 0 & \text{if } x \le L_j^- \\ \frac{x-L_j^-}{L_j^+-L_j^-} & \text{if } L_i^- \le x \le L_j^+ \\ 1 & \text{if } x \ge L_j^+ \end{cases}$
f_j	Fixed cost of warehouse j
c_{ji}	Unit transport cost between warehouse j and customer i
$\hat{c}_i$	Unit transport cost between central plant and customer i
$\hat{\hat{c}}_j$	Unit transport cost between central plant and warehouse j
e_{ji}	Unit GHG emissions factor for transport between warehouse j and customer i
$\hat{e}_i$	Unit GHG emissions factor for transport between central plant and customer i
$\hat{\hat{e}}_j$	Unit GHG emissions factor for transport between central plant and warehouse j
t_{ji}	Distance between warehouse j and customer i
$\hat{t}_i$	Distance between central plant and customer i
$\hat{\hat{t}}_j$	Distance between central plant and warehouse j
x_{ji}	Amount of product shipped from warehouse j to customer $i \in I(j)$
$\hat{x}_i$	Amount of product shipped from central plant to customer
y_j	Amount of product shipped from central plant to warehouse j

tion j only a subset of customers I(j) can be served. The distance, unit transport cost and unit GHG emissions factors from each warehouse location to each customer $i \in I(j)$ are given.

The membership function of the demand of each customer i is given by a Triangular Fuzzy Number (TFN) with parameters $\left(D_i^-, D_i^0, D_i^+\right)$. Each warehouse has a capacity, i.e. an upper bound on the flow of goods that it can convey from the central plant to its allocated customers. The membership function of the capacity of warehouse j is given by a linear decreasing function with parameters $\left(U_j^-, U_j^+\right)$. Each warehouse also has a lower bound on the flow that it should handle in case it is selected. This minimum flow is imposed to guarantee an economic operation of the warehouse. The membership function of the minimum flow of warehouse j is given by a linear increasing function with parameters $\left(L_j^-, L_j^+\right)$.

The proposed bicriteria optimization model consist in the minimization of both cost and GHG emissions:

$$\text{Min} \quad \sum_j f_j\, y_j + \sum_{i\in\hat{I}} \hat{c}_i\, \hat{t}_i\, \hat{x}_i + \sum_j \sum_{i\in I(j)} \left(\hat{\hat{c}}_j\, \hat{\hat{t}}_j + c_{ji}\, t_{ji}\right)\cdot x_{ji} \tag{1}$$

$$\text{Min} \quad \sum_{i\in\hat{I}} \hat{e}_i \cdot \hat{t}_i \cdot \hat{x}_i + \sum_j \sum_{i\in I(j)} \left(\hat{\hat{e}}_j \cdot \hat{\hat{t}}_j + e_{ji} \cdot t_{ji}\right)\cdot x_{ji} \tag{2}$$

subject to

$$\hat{x}_i + \sum_{\{j: i\in I(j)\}} x_{ji} = \tilde{D}_i \qquad \forall i \in \hat{I} \tag{3}$$

$$\sum_{\{j: i\in I(j)\}} x_{ji} = \tilde{D}_i \qquad \forall i \notin \hat{I} \tag{3'}$$

$$\tilde{L}_j y_j \le \sum_{i\in I(j)} x_{ji} \le \tilde{U}_j y_j \tag{4}$$

$$\hat{x}_i \ge 0 \quad \forall i \in \hat{I} \quad x_{ji} \ge 0 \quad \forall j\, \forall i \in I(j) \quad y_j \in \{0,1\} \quad \forall j \tag{5}$$

In order to solve this model, a Fuzzy Multiobjective Optimization approach based on the additive model of Tiwari [4] is proposed. Thus, the new objective function, to be maximized, will be the sum of the membership functions of the fuzzy constraints and of the two objective functions. The latter are fuzzified using decreasing linear membership functions, between the thresholds (C^-,C^+) and (E^-,E^+), respectively. These total cost and total emissions thresholds are evaluated in the following way. For C^+, model (1),(3)-(5) is solved maximizing transportation costs and assuming all the potential warehouses are closed. Let Ψ be the resulting maximum transportation cost, then $C^+=\Psi^+ + \Sigma f_i$.

For the calculation of C^-, model (1),(3)-(5) is solved minimizing transportation costs and assuming that all the warehouses are open. Let Ψ^- be the resulting minimum transportation cost, then $C^-=\Psi^- - \Sigma f_i$. For the calculation of E^+, model (2)-(5) is solved maximizing total emissions and assuming that all the warehouses are closed. Finally, for calculating E^-, model (2)-(5) is solved minimizing total emissions and assuming that all the warehouses open.

We assume that both objectives (minimizing total costs and total emissions) are equally important. As regards the constraints we shall request that their membership function values should be higher than a lower bound μ_{min} (see [5]). The model to solve is, thus, the following

$$\text{Max} \quad \lambda_1 + \lambda_2 \tag{6}$$

subject to

$$C = \sum_j f_j\, y_j + \sum_{i\in\hat{I}} \hat{c}_i\, \hat{t}_i\, \hat{x}_i + \sum_j \sum_{i\in I(j)} \left(\hat{\hat{c}}_j\, \hat{\hat{t}}_j + c_{ji}\, t_{ji}\right) x_{ji} \tag{7}$$

$$E = \sum_{i\in\hat{I}} \hat{e}_i\, \hat{t}_i\, \hat{x}_i + \sum_j \sum_{i\in I(j)} \left(\hat{\hat{e}}_j\, \hat{\hat{t}}_j + e_{ji}\, t_{ji}\right)\cdot x_{ji} \tag{8}$$

$$\lambda_1 \le \frac{C^+ - C}{C^+ - C^-} \tag{9}$$

$$\lambda_2 \le \frac{E^+ - E}{E^+ - E^-} \tag{10}$$

$$D_i^- + \mu\,(D_i^0 - D_i^-) \le \hat{x}_i + \sum_{\{j: i\in I(j)\}} x_{ji} \le D_i^+ - \mu\,(D_i^+ - D_i^0) \quad \forall i \in \hat{I} \tag{11}$$

$$D_i^- + \mu\,(D_i^0 - D_i^-) \le \sum_{\{j: i\in I(j)\}} x_{ji} \le D_i^+ - \mu\,(D_i^+ - D_i^0) \quad \forall i \notin \hat{I} \tag{11'}$$

$$\mu \cdot y_j \le \frac{U_j^+ - \sum_{i\in I(j)} x_{ji}}{U_j^+ - U_j^-} \cdot y_j \qquad \forall j \tag{12}$$

$$\mu \cdot y_j \le \frac{\sum_{i\in I(j)} x_{ji} - L_j^-}{L_j^+ - L_j^-} \cdot y_j \qquad \forall j \tag{12'}$$

$$\sum_{i\in I(j)} x_{ji} \le U_j^+ \cdot y_j \qquad \forall j \tag{12''}$$

$$0 \le \lambda_1 \le 1; \quad 0 \le \lambda_2 \le 1; \quad \mu_{min} \le \mu \le 1 \tag{13}$$

$$x_{ji}, \hat{x}_i \ge 0 \quad \forall i \in \hat{I}\ \forall j \in I(j) \quad y_j \in \{0,1\} \quad \forall j \tag{14}$$

IV. Solution procedure

In order to solve the above model a Genetic Algorithm (GA) will be used. The GA explores which warehouses are to be opened (binary variables y_j) and, for each individual, a Linear Programming (LP) solver is used to compute the corresponding fitness function selecting is the best customer allocation, using model (6)-(14) with variables y_j fixed (see Fig. 2). Note that, in principle, not every subset of warehouses is feasible, i.e., there is not always enough demand in the area of influence of the warehouses I(j) to cover the minimum flow required to open the facilities as per constraints (12'). Therefore, a check needs to be done previous to calling the optimization software that solves the LP model. In case the candidate warehouses to be opened are seen to lead to an infeasible solution, changes in the

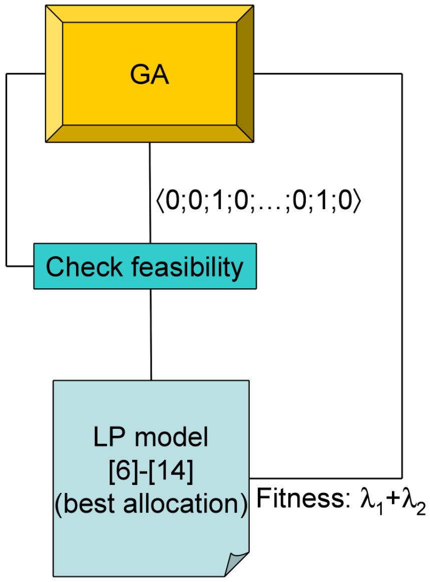

Fig. 1 GA solution procedure solves LP model for fitness evaluation

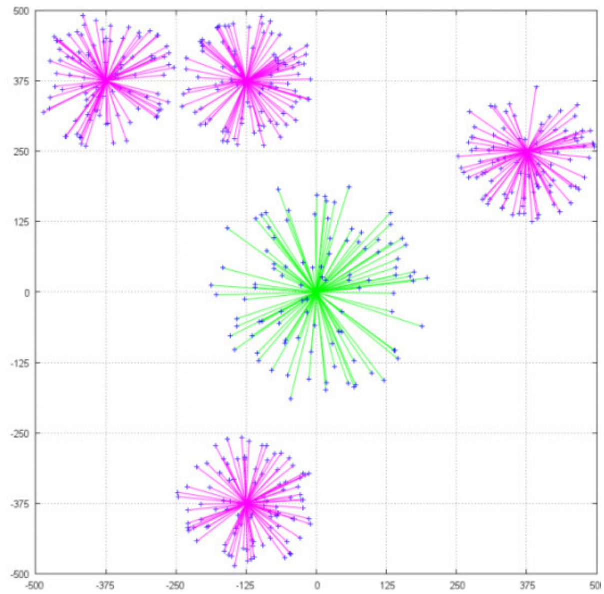

Fig. 2 Example of a priori solution with 4 warehouses

warehouses subset are made until it can be assured there that the LP optimization software will return a feasible solution. This can be seen as a repair operator, which is one of the possible ways of handling constraints in GA.

Since the solution space explored by the GA corresponds to binary variables (y_j) a binary codification of the solution is used, i.e. each chromosome is just a vector of as many components as potential warehouse locations. Each component encode whether a warehouse is open or not. In order to assign a fitness value to an individual a linear solver is used to solve model (6)-(14) also obtaining the complete specification of the solution, including the flows between the central plant and the open warehouses and from these to their allocated customers.

About the crossover and mutation operators, standard binary coding operators have been used, namely the 1-point crossover (1X crossover) and the bitwise mutation. Fitness-proportional selection (i.e. roulette wheel) is used to choose the individuals to cross over. A generational GA is used with a maximum number of generations. An additional stopping criterion consists in a limit on the number of generations without improving the best solution found.

As regards the implementation of the GA, an efficient parallel Python code has been programmed. Although the details of the parallelization strategy is out of the scope of this paper, let us just say that parallel python allows for calculating in parallel of the fitness of all the individuals in initial population as well as of the new individuals created in each generation.

V. Computational Experiments and Results

For testing the good performance of the proposed approach, we have created a testbed of instances, each one with a 7x7 square grid of potential warehouses locations and with the central plant in the middle of the grid. The size of each of the grid cells is 100 km×100 km. The data were created in such a way that we have a clue about which could be the best possible solution, and them we shall check if our procedure is able to find a solution at least as good as that. With that purpose, customers were created locating them around a specific warehouse, forming a kind of cluster. Thus, for example, Fig. 2 shows an instance with four clusters of customers generated around four chosen warehouses. An additional cluster of customers, not in the vicinity of the four chosen warehouses, is also generated, with the expectation that these customers will likely be allocated to the central plant.

Two sets of 20 instances each were created. In the first set 2 warehouses are opened and 4 in the other 4. Therefore, 40 instances were solved and compared with the corresponding a priori "cluster" solution.

For each of the N selected warehouses, a set of (500/N)·4/5 clients in a radius distance of 125 km, all with the same demand, are randomly generated. The other fifth of the warehouse customers were generated out of that neighborhood. Note that there is always a feasible solution since we assume that the central facility can always deliver goods to any client (although at a higher cost). Capacity is assigned to each warehouse in such a way that the defined solution is feasible.

For the two types of vehicles (trucks and vans) cost and emission factors are shown in Table II and include the corresponding corrections to deal with non-full truckloads. The emission factors used correspond to those computed by the LIPASTO model developed by the Technical Research Centre of Finland (VTT) ([6]).

For the GA a population size of 100 was used, mutation probability was set to 0.001, maximum number of generations was 100 but stopping before reaching that limit if 10 generations pass without improving the best solution found.

Comparing the results obtained with the clustered solution from which the instance customer data were generated, it can be seen in Fig. 3 that the GA procedure has been suc-

TABLE III.
COSTS AND EMISSIONS OF TRUCKS AND VANS DEPENDING ON DISTANCE AND LOAD

	Truck	Van	Non-full truck, from central depot, >125km	From warehouse,>125km
c_{ji}	0.00004 €/kg/km	0.00030 €/kg/km		0.00045 €/kg/km
$\hat{c}_i$	0.00004 €/kg/km	0.00030 €/kg/km	0.00006 €/kg/km	
$\hat{\hat{c}}_j$	0.00004 €/kg/km	0.00030 €/kg/km		
e_{ji}	0.0621 gr Eq-CO_2/kg/km	0.0950 gr Eq-CO_2/kg/km		0.1425 gr Eq-CO_2/kg/km
$\hat{e}_i$	0.0621 gr Eq-CO_2/kg/km	0.0950 gr Eq-CO_2/kg/km	0.1425 gr Eq-CO_2/kg/km	
$\hat{\hat{e}}_j$	0.0621 gr Eq-CO_2/kg/km	0.0950 gr Eq-CO_2/kg/km		

cessful in 27 out of the 40 instances (two thirds of the cases) location the warehouses according to the corresponding a priori clustered solution considered. Overall, the fitness of the GA solution (measured by $\lambda_1+\lambda_2$) is 2.2% below that of the a priori clustered solution. Note that as the problem complexity increases (as the number of clusters in the instance increases), it occurs more often (0% in the case of two clusters, 45% in 4 clusters case) that the GA does not find the a priori clustered solution. Different ways to compensate this effect are being studied to make the GA more robust.

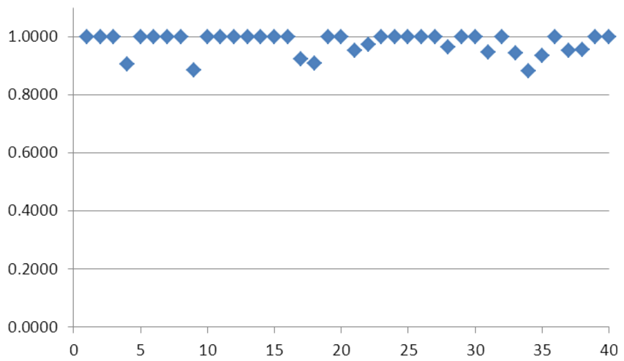

Fig. 3 Ratio between the fitness of the final GA solution and the fitness of the original clustered solution

VI. CONCLUSIONS

This research has proposed a new network design approach that aims not only at cost minimization but also at minimizing GHG emissions from goods transportation. This second objective function will contribute to the sustainability of logistics operations. The decision variables are the selection of the warehouse location (from a set of discrete potential locations) and the allocation of customers to the selected warehouses.

A fuzzy bicriteria optimization model for solving the problem has been formulated and a GA solution procedure has been implemented. The GA explores the space of solutions corresponding to the selection of warehouses to open. A binary codification has been used so that if a potential location is opened the corresponding gene is one and zero otherwise. Standard crossover and mutation operator are used. Since there are both lower and upper bounds on the capacity of the open warehouses, a repair mechanism is needed to guarantee that these constraints hold and that the individual whose fitness is to be evaluated leads to a feasible solution.

A set of experiments have been carried out to check the performance of the procedure, using some instances for which a good reference solution is known a priori. The results indicate that the proposed approach generally finds (or gets close to) this reference solution.

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[6] LIPASTO Unit emissions of vehicles, Freight transport - road traffic, http://lipasto.vtt.fi/yksikkopaastot/tavaraliikennee/tieliikennee/tavara_tiee.htm