

# The Set of Time Structures for Economic Phenomena Description

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**Abstract**—The paper describes some possible time structures for describing and analyzing economic phenomena. Since a simple linear time structure is not enough for this task, use of more complex structures are proposed.

## I. INTRODUCTION

INTRODUCING the notion of time allows to perform inference about changing domains, including the economic one. It also allows a computer to simulate human inference, because people infer about time and change [3]. In particular, such notions as change, causality or actions are described in the context of time, therefore the proper representation of time, and proper temporal reasoning is so important in the field of e.g. artificial intelligence [9, 10].

In artificial intelligence domain, which is concerned about knowledge, the temporal aspects of this knowledge are particularly important, because they are connected with both representation and inference. For an intelligent system to simulate intelligent behavior, to adapt to changes in the environment, or to verify its beliefs it has to be able to acquire new knowledge, and to maintain knowledge in an up-to-date form. Knowledge changes due to two main reasons. The first reason is the passing of time, the second one is new information on objects in the knowledge, having temporal characteristics [2, 11].

In order to represent properly temporal phenomena (or temporal knowledge about them) it is necessary to establish – prior to choosing the temporal formalism – a proper time structure. It is so because time structure determines the representation. For example, if one chooses dense time, it is not possible to represent knowledge in the situation calculus, because it is for only discrete phenomena [8]. In turn, the structure of time depends on the characteristics of the domain to be modeled. As Kania writes [5] p. 62, determining the structure of time is necessary to properly operate on time values, to operate on database etc.

The structure of time has to depend on a phenomena being investigated and should enable the best way to model it.

## II. TIME STRUCTURES TO DESCRIBE ECONOMIC PHENOMENA – BASIC ASSUMPTIONS

We are convinced that for a temporal analysis of enterprise's environment it should be assumed that time is: dis-

crete, branching into the future, finite in the past, but infinite in the future. We have chosen this structure because of several reasons:

- a) Discrete time – there are several elements of the environment that change in a continuous manner, but some of the elements (e.g. barriers to entry) change discretely. From a practical point of view, it is not possible to provide information to the temporal intelligent system continuously. Changes have to be registered discretely. Moreover, assuming continuous time would be linked with introducing a second order axiomatization [1] p. 36.
- b) Time branching into the future – the enterprise's environment is nondeterministic. Linear time assumes deterministic domain, while time branching into the future assumes a nondeterministic one. Also, introducing time branching into the future, when present actions may develop into several future ones, would allow to deepen the analysis of the environment, allowing e.g. for „what-if” analyses. It is not the only possible structure. For example, if we take into account the differences of temporal aspects of different markets we may think of a parallel time structure, which enables e.g. analyzing different markets simultaneously. Also a right linear time structure (time branching into the past) could be adopted – in order to determine, which changes in the past on the markets are responsible for the present situation of an enterprise. Using nonlinear time structures for analyzing economic environment is surely an interesting research area.
- c) Time unbounded in the future – this assumption seems obvious: in a given moment, an enterprise is not able to define for how long it will be operating, therefore it is not possible to determine a moment in time, when the analysis will not be needed any more. However, as managerial practice shows, a time horizon longer than 5 years is not needed. For example, investment plans for more than 5 years are practically unreal. But it should be pointed out here, that a time point named „now” is different every day, moving into the future, and so moves the time horizon, even striving for infinity. Also obvious is assuming time bounded in the past: nor an enter-

prise, nor a temporal intelligent systems operate from „always”. Moreover, an analysis getting far into the past would not be useful, because the environment is changing and turbulent. It should be assumed a certain „past time horizon of analysis”, therefore bounding time in the past is justified.

Formally speaking, a time structure for modeling economic realm is a structure fulfilling the following conditions::

- a)  $\forall t_1, t_2 \in T$ 
  - $t_1 < t_2 \Rightarrow \exists t_3: (t_1 < t_3) \wedge \neg(\exists t_4: t_1 < t_4 \wedge t_4 < t_3)$ ,
  - $t_2 < t_1 \Rightarrow \exists t_3: (t_3 < t_1) \wedge \neg(\exists t_4: t_3 < t_4 \wedge t_4 < t_1)$ ,
- b)  $\forall t_1, t_2, t_3 \in T (t_2 < t_1 \wedge t_3 < t_1) \Rightarrow (t_2 < t_3) \vee (t_2 = t_3) \vee (t_3 < t_2)$
- c)  $\neg(\forall t_1, \exists t_2: t_2 < t_1)$ .

It is a general assumption, but in specific situations the model may be broadened, because – as it has been already pointed out – the time structure has to be adjusted to the phenomenon being analyzed. In the next section different time structures are presented and discussed.

### III. A SET OF TIME STRUCTURES

The most commonly adopted model is the one of linear time, which can be graphically depicted as a straight line, while formally a time structure  $T$  is linear, if [6] p. 20:

$$\forall t_1, t_2 \in T: (t_1 < t_2) \vee (t_1 = t_2) \vee (t_2 < t_1).$$

The models of nonlinear time are: time branching into the future, time branching into the past, time branching in both directions (parallel time), cyclic time. A motivation for adopting the branching time structure is as follows: many different pasts („routes”) may have led to the present time, and from „now” may arise many different „routes” into the future. The formal definitions are: ([6] p. 21 and next):

A time structure  $T$  is branching into the future (left-linear), if

$$\forall t_1, t_2, t_3 \in T (t_2 < t_1 \wedge t_3 < t_1) \Rightarrow (t_2 < t_3) \vee (t_2 = t_3) \vee (t_3 < t_2).$$

A time structure  $T$  is branching into the past (right-linear) if

$$\forall t_1, t_2, t_3 \in T (t_1 < t_2 \wedge t_1 < t_3) \Rightarrow (t_2 < t_3) \vee (t_2 = t_3) \vee (t_3 < t_2).$$

A time structure  $T$  is parallel, if it is left- and right-linear, that is branching into both directions.

One more structure, discussed rarely in the literature, but interesting, is a cyclic time structure. A metric point time structure  $T$  is an ordered tuple  $\langle T, C, <, <^*, \delta, S \rangle$ , where:  $T$  – set of time points,  $C$  – set of distances between points,  $<$  – a global order over  $T$ ,  $<^*$  – local order over  $T$ ,  $\delta$  – metrics over  $T$ ,  $S$  – length of a semicircle.

For each time point  $x \in T$  there exists exactly one point  $x^* \in T$  that  $\delta(x, x^*) = S$ . These two points divide the time circle into two semicircles. The characteristics of a cyclic time structure are as follows (after [4], p. 30):

- completeness:  $\forall x, y (x < y)$ ,

- local antisymmetry:  $\forall x, y (x <^* y \rightarrow \neg(y <^* x))$ ,
- local linearity:  $\forall x, y ((x \neq y \ \& \ x \neq y^*) \rightarrow (x <^* y \vee y <^* x))$ ,
- local transitivity:  $\forall x, y, z ((\delta(x, y) + \delta(y, z) < S) \rightarrow (x <^* y \ \& \ y <^* z \rightarrow x <^* z))$ ,
- coherence:  $\forall x, y, z ((\delta(x, y) + \delta(y, z) < S) \rightarrow (x <^* y \ \& \ y <^* z \rightarrow \delta(x, y) + \delta(y, z) = \delta(x, z)))$ .

One may imagine such situations, in which classic time structures would be not enough. Consider a situation, when two enterprises join (perform a fusion), operate as one enterprise for a certain period of time, and divide again into two enterprises, but cooperating together (so it is justified to analyze them together, but not on one time axis). In this case we deal subsequently with time structures: right-linear one, linear one, and left-linear one. Formally this situation may be written as:

- $(t_1, t_2, t_3 < t_F) \Rightarrow (\forall t_1, t_2, t_3 \in T: (t_1 < t_2 \wedge t_1 < t_3) \Rightarrow (t_2 < t_3) \vee (t_2 = t_3) \vee (t_3 < t_2))$ ,
- $((t_1, t_2 > t_F) \wedge (t_1, t_2 < t_P)) \Rightarrow (\forall t_1, t_2 \in T: (t_1 < t_2) \vee (t_1 = t_2) \vee (t_2 < t_1))$ ,
- $(t_1, t_2, t_3 > t_P) \Rightarrow (\forall t_1, t_2, t_3 \in T (t_2 < t_1 \wedge t_3 < t_1) \Rightarrow (t_2 < t_3) \vee (t_2 = t_3) \vee (t_3 < t_2))$ ,

where:  $t_F$  – the moment of fusion,  $t_P$  – the moment in which enterprise divides again.

This structure has the following properties:

- transitivity:  $\forall x, y (x < y \ \& \ y < z \rightarrow x < z)$ ,
  - anti-reflexivity:  $\forall x \neg(x < x)$ ,
  - antisymmetry:  $\forall x, y (x < y \rightarrow \neg(x < y))$ ,
  - discreteness:  $\forall x, y (x < y \rightarrow \exists z(x < z \ \& \ \neg\exists u(x < u \ \& \ u < z)))$
- $$\forall x, y (x < y \rightarrow \exists z(z < y \ \& \ \neg\exists u(u < u \ \& \ u < y)))$$

Of course, different structures may be combined together in different ways, according to the analytical needs. It seems that the easiest is combining branching structures with linear one. The combinations similar to the one shown above may be numerous, one can imagine e.g. chains composed of branching and linear time structures. On the other hand, combining the cyclic time structure with branching and/or linear ones seems difficult, or even impossible, because the cyclic structure is a closed one.

It is necessary to discuss, how many time structures can arise from basic ones? Before we answer this question, we have to make some assumptions:

- for simplicity, we consider only a basic structure  $\langle T, < \rangle$ , other axioms of this structure are omitted;
- we assume representation in 1st order predicate calculus; in the calculus generally the linearity axiom is manipulated, therefore we deal with a finite set: linearity axiom, left-linearity axiom, right-linearity axiom;
- we omit the question of time metrics, because this does not affect the linear or non-linear property of time structure.

Having the above assumptions, we may say that the set of possible time structures arising from combining basic time structures is a combination with repetitions of those struc-

tures. As it is commonly known, the number of  $k$ -element combinations with repetitions of a  $n$ -element set is given by a formula:

$$C_n^k = \frac{(k+n-1)!}{k!(n-1)!}$$

Where

$k$  – number of sequence elements,

$n$  – number of set elements.

In the considered case,  $k = 1, 2, 3$  or  $4$ , and  $n = 4$  (linear structure, left-linear structure, right-linear structure, parallel structure).

Therefore, the set of possible time structures is computed as:

$$S = 1 + \frac{(2+4-1)!}{2!(4-1)!} + \frac{(3+4-1)!}{3!(4-1)!} + \frac{(4+4-1)!}{4!(4-1)!} = 66$$

It should be noted here that in case of using multiple time structures for economic realm description and in case of using a temporal intelligent system, we have to deal with a heterogenic time structure in the knowledge base of the system. Thus, we face a problem of unifying the structure. Is it necessary for performing reasoning by an intelligent system? This problem is similar to the one of the heterogeneous knowledge in a temporal intelligent system, described in detail in [7], but it is beyond the scope of this paper.

#### IV. CONCLUDING REMARKS

In the paper the motivation for temporal representation of economic knowledge has been presented, the possible time structures have been pointed out, and the possible combinations of them for economic realm description have been shown. The main conclusion stresses out the variety of possibilities given by only 4 time structures, combined in different ways. Leaving linear time axiomatization, in order to

take into account richer structures, will enable to better depict the economic realm, e.g. for building a knowledge base of a temporal intelligent system.

The problem of time structures heterogeneity in a temporal knowledge base arises while using more than one structure. It has to be discussed and checked, whether to perform reasoning in a temporal intelligent system it is necessary to unify these structures, and if so, how it should be done. This will be the topic of future research studies.

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