A Declarative Approach to Cost Estimation in Multi-Project Environment

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Abstract—The paper aims to present a cost evaluation for multi-project environment, taking into account imprecision in activity duration and cost. Data specification in the form of discrete α-cuts enables the connection of distinct and imprecise data, and the implementation of a constraints satisfaction problem with the use of constraint programming. Moreover, using α-cuts, optimistic, pessimistic, and several intermediate scenarios concerning the project scheduling and cost can be obtained and considered in terms of different risk levels. Each scenario can be assessed according to criteria such as time, cost, and risk level. A declarative form of the description of a multi-criteria decision problem allows its implementation in constraint programming languages and facilitates the development of a decision support system. The proposed methodology can be easily incorporated into available fuzzy project scheduling software to provide a better perception of risk that is usually obscured in the conventional approach.

I. INTRODUCTION

The traditional approach to project scheduling is the well-known CPM (Critical Path Method) and PERT (Program Evaluation and Review Technique). The hypothesis made in CPM that activity durations are deterministic and known is rarely satisfied in real life where tasks are often uncertain and variable [1]. The inherent uncertainty and imprecision in project scheduling has motivated the proposal of several fuzzy set theory based extensions of activity network scheduling techniques. Among these extensions can be found, for instance, resource-constrained fuzzy project-scheduling problem [2], criticality analysis of activity networks with uncertainty in task duration [3], fuzzy repetitive scheduling method [4], and fuzzy dependency structure matrix for project scheduling [5]. Considerable research effort has been recently focused also on the application of constraint programming frameworks in the context of project scheduling [6], [7].

The Constraint Programming (CP) environment seems to be particularly well suited to modelling real-life and day-to-day decision-making processes at an enterprise. CP is qualitatively different from the other programming paradigms, in terms of declarative, object-oriented and concurrent programming. Compared to these paradigms, constraint programming is much closer to the ideal of declarative programming: to state what we want without stating how to achieve it [8]. CP is an emergent software technology for a declarative Constraints Satisfaction Problem (CSP) description and can be considered as a pertinent framework for the development of decision support system software aims.

Declarative programming languages base on the idea that programs should be as close as possible to the problem specification and domain [9]. In the field of constraint-based scheduling, two strengths emerge: natural and flexible modelling of scheduling problems as CSP and powerful propagation of temporal and resource constraints. Thus, the scheduling problem is modelled as CSP at hand in the required real-life detail and it enables to avoid the classical drawbacks of being forced to discard degrees of freedom and side constraints. Discarding degrees of freedom may result in the elimination of interesting solutions, regardless of the solution method used. Discarding side constraints gives a simplified problem and solving this simplified problem may result in impractical solutions for the original problem [10]. The limitations of imperative languages provide the motivation to develop a reference model of project management in an enterprise and to implement it in declarative languages. The advantage of working with such a model is that users are driven by the system to produce the required results, whilst the manner in which the results are produced is dependent on the preferences of the users [11].

The model formulated in terms of CSP determines a single knowledge base and it enables effective implementation in constraint programming languages, as well as the development of a task-oriented decision support system (DSS) for project portfolio planning. As a result, the problem specification is closer to the original problem, obtaining solutions that are unavailable with imperative programming. Moreover, the descriptive approach enables the specification of decision problems according to deductive reasoning (a query about the results of proposed decisions) and abductive reasoning (a query about decisions ensuring the expected results).

Cost planning is crucial for the assessment of cash flow during project implementation. Moreover, an accurate cash flow is required in conducting project cost-benefit analysis, the determination of project financing requirements and in performing earned value analysis [12]. Several researchers have applied different approaches to fuzzy set theory or probability theory in project flow generation and analysis (e.g. [13], [14], [15]). However, the main focus in the research concerning fuzzy project scheduling is principally on the calculation of early/late start and finish times and the determination of activity and path criticality, whereas issues related to uncertainty in cost with the use of a declarative approach have not yet been comprehensively addressed.
The goal of this research is to present the use of constraint programming to fuzzy project scheduling and cost evaluation in multi-project environment whose durations and costs are in the imprecise form. The model of project portfolio planning is specified in terms of fuzzy CSP, using constraint programming to seek a solution to the problem, and enabling cost analysis at different \( \alpha \)-levels. An \( \alpha \)-cut is a crisp set consisting of elements of fuzzy set \( A \) which belong to the fuzzy set at least to a degree of \( \alpha \). The proposed methodology is relatively similar to what practitioners are using to generate project cost and cash flows but is considerably more effective and realistic in modelling uncertainty. The proposed DSS for project portfolio planning allows a decision-maker to obtain a set of project scenarios and to perform analysis of cost uncertainty at different \( \alpha \)-levels, which appears to be more intuitive than alternative methodologies that employ other fuzzy techniques.

The remaining sections of this paper are organised as follows: Section 2 presents a problem formulation in terms of fuzzy project portfolio planning allowing a decision-maker to obtain a set of project scenarios and to perform analysis of cost uncertainty at different \( \alpha \)-levels, which appears to be more intuitive than alternative methodologies that employ other fuzzy techniques.

Finally, some concluding remarks are contained in Section 5.

II. FUZZY CONSTRAINT SATISFACTION PROBLEM FOR PROJECT PORTFOLIO SCHEDULING

The specification of project portfolio scheduling encompasses technical parameters, expert’s experiences and user expectations in the form of a knowledge base, i.e. as a set of variables, their domains, and a set of relations (constraints) that restrict and link variables. In this context, it seems natural to classify some decision problems as CSP. The problem formulation in terms of CSP enables a simplified description of actuality, i.e. a description encompasses the assumptions of object, implementing therein tasks, and a set of routine queries - the instances of decision problems [6].

In a classical form, the structure of the constraints satisfaction problem may be described as follows [10]: CSP = \((\tilde{V}, D, C)\), where: \( \tilde{V} \) - a set of variables, \( D \) - a set of discrete domains of variables, \( C \) - a set of constraints. In turn, for the imprecise description of variables, the Fuzzy Constraints Satisfaction Problem (FCSP) takes the following form:

\[
FCSP = ((\tilde{V}, D), C)
\]

where:
- \( \tilde{V} = \tilde{v}_1, \tilde{v}_2, ..., \tilde{v}_n \) - a finite set of \( n \) fuzzy variables that are described in the form of fuzzy number (a finite set of discrete \( \alpha \)-cuts);
- \( D = d_1, d_2, ..., d_n \) - a set of domains for \( n \) fuzzy variables;
- \( C = c_1, c_2, ..., c_m \) - a finite set of \( m \) constraints limiting and linking decision variables.

Given a set of projects \( P = \{P_1, P_2, ..., P_j\} \), where the project \( P_i \) consists of \( J \) activities: \( P_i = \{A_{i,1}, ..., A_{i,j}, ..., A_{i,J}\} \). The \( j \)-th activity of \( i \)-th project is specified as follows: \( A_{i,j} = \{s_{i,j}, z_{i,j}, t_{i,j}, d_{p_{i,j}}\} \), where:
- \( s_{i,j} \) - the starting time of the activity \( A_{i,j} \), i.e., the time counted from the beginning of the time horizon \( H \);
- \( z_{i,j} \) - the completion time of the activity \( A_{i,j} \);
- \( t_{i,j} \) - the duration of the activity \( A_{i,j} \);
- \( d_{p_{i,j}} \) - the financial means allocated to the activity \( A_{i,j} \).

The project \( P_i \) is described as an activity-on-node network, where nodes represent the activities and the arcs represent the precedence constraints between activities. According to this, the precedence constraints are as follows:

- the \( k \)-th activity follows the \( i \)-th one:
  \[
s_{i,j} + t_{i,j} \leq s_{i,k}
  \]

- the \( k \)-th activity follows other activities:
  \[
s_{i,j} + t_{i,j} \leq s_{i,k},
  \]

- the \( k \)-th activity is followed by other activities:
  \[
s_{i,k} + t_{i,k} \leq s_{i,j},
  \]

CSP can be considered as a knowledge base that is a platform for query formulation as well as for obtaining answers, and it comprises of facts and rules that are characteristic of the system’s properties and the relations between its different parts. As a consequence, a single knowledge base facilitates the implementation of a decision support system.

The distinction of decision variables that are embedded in the knowledge base as an input-output variable permits to formulate the standard routine queries concerning project cost analysis such as:

- is there a cost scenario at given \( \alpha \)-level and constraints (e.g. project deadline, budget, precedence constraints)?
- is there a cost scenario for a given cost limit, and if yes, what starting times of project portfolio activities \( s_{i,j} \) ensure that the cost allocation \( d_{p_{i,j}} \) does not exceed the cost limit and other constraints?

The method of generation of admissible solutions for the above-described problem is presented in the next section.

III. METHOD OF FUZZY PROJECT SCHEDULING AND COST GENERATION

Imprecise variables determined by convex membership function \( \mu(t) \) (e.g. a triangular fuzzy number \( t = \langle a, b, c \rangle \)) can be specified as \( \alpha \)-cuts. An \( \alpha \)-cut is a crisp set consisting of elements belong to the fuzzy set at least to a degree of \( \alpha \) (\( 0 < \alpha \leq 1 \)). An \( \alpha \)-cut is a method of defuzzifying a fuzzy set to a crisp set at desired \( \alpha \)-levels that correspond to
the perceived risk ($\alpha=1$ meaning no risk, $\alpha=0$ meaning the lowest risk, $\alpha=0+$ meaning the highest risk). Additionally, the low ($\alpha=0-$) and high ($\alpha=0+$) values of every $\alpha$-cut represent the optimistic and pessimistic outcomes of that risk level. The main objective of fuzzy project scheduling is to apply fuzzy set theory concepts to the scheduling of real world projects where task duration can be specified as fuzzy numbers instead of crisp numbers [12].

If in the fuzzy project scheduling algorithm, the start and completion times are in fuzzy form, then this usually leads to difficulties with the interpretation, since the fuzzy starting time of the activity can be greater than the fuzzy completion time. In the order to avoid this situation, it is assumed that the starting time of the activity is in a distinct form, whereas the completion time of the activity can be specified as a fuzzy number. The fuzzy completion time is the sum of the activity start with the fuzzy activity duration (see Fig. 1).

It is noteworthy that using the presented methodology, the duration of some activities ($A_1$, $A_2$, $A_3$, $A_4$, $A_5$, $A_6$, $A_7$) is specified in the imprecise form. An example of the interval of minimum and maximum duration intervals ($\min t_{\alpha}$, $\max t_{\alpha}$) are defined for each $\alpha$-cut.

The uncertainties of the duration and cost of an activity are positively correlated, so the minimum ($\min h_0$) and maximum ($\max h_0$) cost distribution per unit of time $h$ of the $j$-th activity at the level $\alpha$ depict the best and the worst scenario respectively. The presented approach can be expanded for several $\alpha$-cuts (e.g. 0.1, 0.2, ..., 1) for $\min h_{\alpha}$ and $\max h_{\alpha}$, generating activity cost accordingly. A number of scenarios depend on a number of $\alpha$-cuts. For instance, if a fuzzy number is described at 3 $\alpha$-cuts, then there are 5 scenarios ($\min t_0$, $\min t_{0.5}$, $t_1$, $\max t_{0.5}$, $\max t_0$). In turn, the use of 5 $\alpha$-cuts results in 9 scenarios ($\min t_0$, $\min t_{0.25}$, $\min t_{0.5}$, $\min t_{0.75}$, $t_1$, $\max t_{0.75}$, $\max t_{0.5}$, $\max t_{0.25}$, $\max t_0$).

An example of the use of the presented methodology in constraint programming environment is presented in the next section.

IV. ILLUSTRATIVE EXAMPLE

The example consists of three subsections: the description of the project portfolio, the analysis of the first admissible solution of the fuzzy scheduling problem, and the what-if analysis (the fuzzy scheduling problem for a given cost limitation). Both analyses contain the examination of fuzzy project Gantt charts and fuzzy project cost distribution.

A. Project portfolio description

It is assumed that the time horizon for the project portfolio ($P = \{P_1, P_2, P_3\}$) equals 34 months ($H = 0, 1, ..., 34$) and the budget of the project portfolio is fixed at 950 m.u. The network diagrams of the activities in the project portfolio are shown in Fig. 3-5.

The duration of some activities ($A_{1.7}$, $A_{1.10}$, $A_{2.4}$, $A_{2.7}$, $A_{2.9}$, $A_{3.4}$, $A_{3.5}$, $A_{3.6}$, $A_{3.7}$) is specified in the imprecise form. The sequences of activity duration for the considered projects can be described as follows: $T_1 = (2, 1, 1, 6, 2, 2, "about 6", 0)$, $T_2 = (2, 1, 1, 6, 2, 2, "about 6", 0)$, $T_3 = (2, 1, 1, 6, 2, 2, "about 6", 0)$.
1, 4, "about 6"), $T_2 = (2, 2, 1, "about 9", 6, 4, "about 6", 4, "about 4"), T_3 = (1, 1, 1, "about 6", "about 6", "about 4", "about 9"). For instance, the duration of the activity $A_{1,7}$ is "about 6", i.e. the activity can be completed within the time period of 4 to 8 units of time.

B. Fuzzy scheduling and cost distribution: first admissible solution

Fuzzy project scheduling and cost generation problem can be reduced to the following questions: is there a portfolio schedule (and if yes, what are its parameters) that follows from the given project constraints specified by the activity duration times, the deadline and budget of project portfolio? What risk levels are there for the different fuzzy cost scenarios? The answer to the questions is connected with the determination of the starting $s_{i,j}$ and completion $z_{i,j}$ time of project portfolio activities and the allocation of financial means to the activities by different $\alpha$-level $d_{p_{i,j},\alpha}$. For the considered project portfolio, and $\alpha$-level equal to 1, the following sequences are sought: $S_1 = (s_{1,1,1}, ..., s_{1,10,1}), S_2 = (s_{2,1,1}, ..., s_{2,9,1}), S_3 = (s_{3,1,1}, ..., s_{3,7,1}), Z_1 = (z_{1,1,1}, ..., z_{1,10,1}), Z_2 = (z_{2,1,1}, ..., z_{2,9,1}), Z_3 = (z_{3,1,1}, ..., z_{3,7,1}), D_{p_1} = (d_{p_1,1,1}, ..., d_{p_1,10,1}), D_{p_2} = (d_{p_2,1,1}, ..., d_{p_2,9,1}), D_{p_3} = (d_{p_3,1,1}, ..., d_{p_3,7,1}).$

Fig. 6 presents the first admissible solution (project portfolio schedule), in which the sequences of activity starting and completion time are as follows: $S_1 = (0, 2, 3, 4, 4, 4, 10, 10, 16, 20), S_2 = (0, 2, 4, 5, 5, 14, 18, 18, 24), S_3 = (0, 1, 2, 3, 3, 9, 13), Z_1 = (2, 3, 4, 10, 6, 6, "about 16", 11, 20, "about 26"), Z_2 = (2, 4, 5, "about 14", 11, 18, "about 24", 22, "about 28"), Z_3 = (1, 2, 3, "about 9", "about 9", "about 13", "about 22"). The completion time of project $P_1, P_2, P_3$ equals "about 26", "about 28", and "about 22" months, respectively.

Fig. 7 presents five different cost scenarios for project portfolio (cumulative cost for project $P_1, P_2, P_3$). At $\mu=1$, the cost (dotted line) is equivalent to that generated from deterministic analysis. At $\mu=0.5$, there is an optimistic scenario below and a pessimistic one above (dashed line). In turn at $\mu=0$, the optimistic and pessimistic cost scenarios (solid line) have a wider spread indicating a higher degree of uncertainty. In the best case (min($d_{\alpha}$)), the project portfolio will be completed in 22 months with the total cost of 632 m.u., whereas in the worst (max($d_{\alpha}$)) in 34 months with the total cost of 920 m.u.

The above-presented S-curves are the basis for analyzing cost scenarios in project portfolio. However, S-curves are simply an edge of an S-surface that in practice is plotted by connecting S-curves from some possibility levels from 0 to 1 at selected time periods. Fig. 8 shows the project S-surfaces for the best and worst scenarios. The selection of specific possibility levels and time intervals determines the size of the rectangular patches that form the S-surface and consequently the overall plot quality.

Compared with conventional 2 dimensional S-curves, the S-surface shows how both uncertainty levels and time affect the
Thus, the surface steepness in terms of possibility and time provides additional insight about the project cost. At \( \mu = 1 \), the best and worst S-surface intersect each other. The presented approach also allows the decision-maker to examine surface cross section at specific times. Fig. 9 illustrates cross section at 30th time unit in which the cost variance of the best and worst case at the possibility level of \( \mu = 0 \) is 632 m.u. and 888 m.u., respectively.

The above-presented examples concern the cumulative cost for project portfolio. Further detailed analyses can include the cost distribution in the horizon of project portfolio (Fig. 10), as well as the analyses in the context of a single project, instead of a set of projects.

Let us assume that the cost distribution should be not greater than 40 m.u. in each time unit. This constraint is not fulfilled in the 6th and 11th time units (see Fig. 10). A possibility of searching solution for the additional constraint is presented in the next subsection.

C. Fuzzy scheduling for a given cost limitation

Table 1 presents the results of solution seeking for the different strategies of variable distribution and the two cases: for a fuzzy number described at 3 and 5 \( \alpha \)-cuts, respectively. The example was implemented in the Oz Mozart programming environment and tested on an AMD Turion(tm) II Ultra Dual-Core M600 2.40GHz, RAM 2 GB platform. The results show that the Naive and Split distribution strategy outperforms the First-fail ones.

The size of the instance equals 6,000; in turn the number of solutions equals 700 and 800 at 3 and 5 \( \alpha \)-cuts, respectively. The sequences of activity starting and completion time are as follows: \( S_1 = (0, 2, 3, 4, 6, 10, 11, 16, 20) \), \( S_2 = (0, 2, 4, 5, 8, 14, 18, 24) \), \( S_3 = (0, 1, 2, 3, 3, 9, 9, 13) \), \( Z_1 = (2, 3, 4, 10, 6, 8, "about 16" , 12, 20, "about 26") \), \( Z_2 = (2, 4, 5, "about 14", 14, 18, "about 24", 22, "about 28") \), \( Z_3 = (1, 2, 3, "about 9", "about 9", "about 13", "about 22") \). The first admissible solution of project portfolio completion for the minimal total duration of project portfolio is presented in Fig. 11.

The re-scheduling implies the reallocation of financial means in project portfolio that is presented in Fig. 12. It is noteworthy that the cost distribution fulfills the constraint \( (d_{p_{ij}} \leq 40 \text{ m.u.}) \). Moreover, the cost allocation is more even than for the case in subsection 4.2 (see Fig. 10). The presented approach allows the decision-maker to consider a wide range of further analyses. For instance, a risk level for cost scenario can be treated as an additional criterion for reducing a set of admissible solutions. The obtained schedules and cost scenarios provide a plan for project portfolio execution and are a basis for further adjustment aimed at fitting to real live execution.

V. Conclusions

The activity of a present enterprise comprises turbulent changes concerning technology, economics, and society [16]. Most projects are executed in the presence of uncertainty and are difficult to manage, due to comprising of many activities linked in a complex way. Hence, there is an increase in demand for new knowledge that enables the solution of problems encountered during complex project portfolio execution. In this case, knowledge concerning project management, especially fuzzy project scheduling, is particularly significant. In the current project implementation environment, a pure deterministic approach for the study of project cost is inadequate.
The proposed approach takes into account several elements, such as the fuzzy activity cost and duration estimations, project S-surfaces, and cost distribution analysis. Data specification in the form of $\alpha$-cuts enables the generation of a set of scenarios concerning the project scheduling and cost that can be assessed according to risk level. Moreover, the use of discrete $\alpha$-cuts facilitates the merger of distinct and imprecise data, and implementation of a constraints satisfaction problem in the constraint programming environment that solves CSP with a significant reduction of the amount of search space. As a result, a task-oriented decision support system has been effectively developed. This system can support a decision-maker in obtaining answers to the following questions: is there a portfolio schedule (and if yes, what are its parameters, e.g. starting time of activities, risk level), and what starting times of project portfolio activities can ensure the specified level of project risk and the required cost allocation?

The limitations of existing commercially available tools (e.g. lack of possibility for data specification in an imprecise form, lack of abilities to solve problems defined in the multi-project environment) was the motivation to develop a design methodology for task oriented decision support systems aimed at project portfolio scheduling and fuzzy project cost generation. In that context, the presented approach can be considered as a new contribution to project management.

The number of $\alpha$-levels can be modified according to the decision-maker’s requirements. As a result, it can assist project managers to gain deeper insight into the sources and extents of uncertainty, which may in turn lead to the avoidance of troubles during project implementation. Also, as the methodology is useful in the assessment of financial requirements during project realization, it may prove practical in evaluating alternative project proposals during the feasibility stage. Moreover, it tends to achieve a balance between complexity of methodology and an intuitive, effective decision support system that is realistic in modelling uncertainty. Finally, its application in performing earned value analysis during project monitoring can also obtain useful results.

The subject of future research includes an extension of the proposed model to other fields of project management where the decision problems are semi-structured, such as project communications management and project team building. Moreover, further research will be aimed at developing a decision support system towards real-life verification. Future research also includes a determination of membership functions of fuzzy numbers by using e.g. fuzzy neural system, and their description in the discretized $\alpha$-cuts.

### REFERENCES


### TABLE 1

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