

InterCriteria Analysis of Crossover and Mutation Rates Relations in Simple Genetic Algorithm

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Abstract—In this investigation recently developed InterCriteria Analysis (ICA) is applied to examine the influences of two main genetic algorithms parameters – crossover and mutation rates during the model parameter identification of *S. cerevisiae* and *E. coli* fermentation processes. The apparatuses of index matrices and intuitionistic fuzzy sets, which are the core of ICA, are used to establish the relations between investigated genetic algorithms parameters, from one hand, and fermentation process model parameters, from the other hand. The obtained results after ICA application are analysed towards convergence time and model accuracy and some conclusions about derived interactions are reported.

I. INTRODUCTION

INTERCRITERIA Analysis (ICA), given in details in [3], is a contemporary approach for multi-criteria decision making. ICA implements the apparatuses of index matrices (IM) and intuitionistic fuzzy sets (IFS) in order to compare some criteria reflecting the behaviour of considered objects. Recently ICA has been successfully applied for EU Member States competitiveness analysis [8], thus provoking the search for further ICA applications. The idea to implement ICA in the field of tuning of the optimization techniques parameters has intuitively appeared.

Fermentation processes (FP) are objects of increased research interest because of their widespread use in different branches of industry. The FP modeling and optimization are a real challenge for the investigators due to the fact that FP models have complex structures based on systems of non-linear differential equations with several specific growth rates [9]. The choice of appropriate model parameter identification procedure is the most important problem for FP adequate modeling. Among others biologically inspired optimization techniques, genetic algorithms (GA) [12] has been proved as a global search method [10] for solving different engineering and optimization problems [18], among that for parameter identification of FP [1], [16], [17], [19]. GA efficiency strongly depends on the tuning of different operators, functions, and parameters. These settings are specifically implemented to different problems. Current investigation is focused on the examining the impact of two of the main GA parameters, namely crossover (*xovr*) and mutation (*mutr*) rates. Simple GA (SGA) is applied for the purposes of model parameter identification of two fed-batch FP – *S. cerevisiae* and *E. coli*. Both yeast and

bacteria have numerous applications in food and pharmaceutical industries. Also both microorganisms are widely used as model organisms in genetic engineering and cell biology due to their well known metabolic pathways [11], [14].

In this investigation the obtained results from SGA parameter identification of considered here FP models are used to determine some dependencies between some criteria preliminary defined as of significant importance. The establishment of the influences and relations between criteria – model parameters, from one hand, and GA parameters crossover and mutation rates, from the other hand, is performed by the ICA implementation. This is expected to lead to additional exploring of the models or the relation between models and optimization algorithm outcomes, which will be valuable especially in the case of modelling of living systems, such as FP.

II. PROBLEM FORMULATION

A. Mathematical models of fermentation processes

Two Case studies are going to be presented here – for the fermentation processes of *S. cerevisiae* (Case study 1) and of *E. coli* (Case study 2).

Case study 1. *S. cerevisiae* fed-batch fermentation model

The mathematical model of *S. cerevisiae* fed-batch process is presented by the following non-linear differential equations system [16]:

$$\frac{dX}{dt} = (\mu_{2S} \frac{S}{S + k_S} + \mu_{2E} \frac{E}{E + k_E})X - \frac{F_{in}}{V}X \quad (1)$$

$$\frac{dS}{dt} = -\frac{\mu_{2S}}{Y_{S/X}} \frac{S}{S + k_S}X + \frac{F_{in}}{V}(S_{in} - S) \quad (2)$$

$$\frac{dV}{dt} = F_{in} \quad (3)$$

where X is the biomass concentration, [g/l]; S – substrate concentration, [g/l]; E – ethanol concentration, [g/l]; F_{in} – feeding rate, [l/h]; V – bioreactor volume, [l]; S_{in} – substrate concentration in the feeding solution, [g/l]; μ_{2S} , μ_{2E} – the maximum values of the specific growth rates, [1/h]; k_S , k_E – saturation constants, [g/l]; $Y_{S/X}$ – yield coefficient, [-].

For the considered here model (Eqs. (1-3)), the vector of parameters to be identified is as follows:

$$p_1 = [\mu_{2S} \ \mu_{2E} \ k_S \ k_E \ Y_{S/X}].$$

Case study 2. *E. coli* fed-batch fermentation model

The mathematical model of *E. coli* fed-batch process is presented by the following non-linear differential equations system [9], [16]:

$$\frac{dX}{dt} = \mu_{max} \frac{S}{k_S + S} X - \frac{F_{in}}{V} X \quad (4)$$

$$\frac{dS}{dt} = -\frac{\mu_{max}}{Y_{S/X}} \frac{S}{S + k_S} X + \frac{F_{in}}{V} (S_{in} - S) \quad (5)$$

$$\frac{dV}{dt} = F_{in} \quad (6)$$

where all notations keep their meaning as described above, and, additionally, μ_{max} is the maximum value of the specific growth rate, [1/h].

For the considered here model (Eqs. (4-6)), the vector of parameters to be identified is as follows:

$$p_2 = [\mu_{max} \ k_S \ Y_{S/X}].$$

Model parameters identification of both fed-batch FP is performed based on experimental data for biomass, glucose and ethanol concentrations. The detailed description of the process conditions and experimental data can be found in [16].

B. Optimization criterion

The objective function is designed aiming at identification of parameter vectors p_1 and p_2 in order to obtain the best fit to a data set and is defined as:

$$J = \sum_{i=1}^m (X_{\text{exp}}(i) - X_{\text{mod}}(i))^2 + \sum_{i=1}^n (S_{\text{exp}}(i) - S_{\text{mod}}(i))^2 \rightarrow \min \quad (7)$$

where m and n are the experimental data dimensions; X_{exp} and S_{exp} – available experimental data for biomass and substrate; X_{mod} and S_{mod} – model predictions for biomass and substrate with a given model parameter vector.

C. Simple genetic algorithms for parameter identification

Simple genetic algorithm, initially presented in Goldberg [12], searches a global optimal solution using three main genetic operators in a sequence selection, crossover and mutation. SGA starts with a creation of a randomly generated initial population. Each solution is then evaluated and assigned a fitness value. According to the fitness function, the most suitable solutions are selected. After that, crossover proceeds to form a new offspring. Mutation is then applied with determinate probability aiming to prevent falling of all solutions into a local optimum. The execution of GA has been repeated until the termination criterion (i.e. reached number of populations, or found solution with a specified tolerance, etc.) is satisfied.

Crossover and mutation are among of the most important operators that can increase the efficiency of GA. The crossover operator is used to generate offspring by exchanging bits in a pair of parents chromosomes chosen from the population.

Crossover occurs with a crossover probability (crossover rate, $xovr$), that indicates a ratio of how many couples will be picked for mating. The mutation operator changes some elements in selected chromosomes with a mutation probability (mutation rate, $mutr$). As such, the operator introduces genetic diversity and helps GA to escape the local optimum. It is well known that optimal crossover and mutation rates vary for different problems and the success of GA depends on their choice [13]. Usually, determining what rates of crossover and mutation should be used is doing on the trial-and-error basis. In the literature there exist a number of guidelines how crossover and mutation rates to be tuned [12], [13], [15]. Recommended values of crossover rate are high, usually in the range 0.5-1.0 [13], [15]. On the other hand, low mutation rate values for preventing search process to be turn into a simple random search are commonly adopted in GA. Typical values of mutation rate are in the range 0.001-0.1 [13], [15].

In this investigation the impact of crossover and mutation rates is going to be examined choosing different values of the both GA parameters. In Case study 1 SGA is applied with the following values of crossover rate: $xovr = \{0.65; 0.75; 0.85; 0.95\}$, while in Case study 2 – with $xovr = \{0.5; 0.6; 0.7; 0.8; 0.9; 1\}$. Due to the specific peculiarities of two fed-batch FP, again different strategies were applied for mutation rates in both Case studies. In Case study 1 SGA is applied with the following values of mutation rate: $mutr = \{0.02; 0.04; 0.06; 0.08; 0.1\}$, while in Case study 2 – with $mutr = \{0.001; 0.01; 0.1; 0.5; 1\}$. The selected values of $xovr$ and $mutr$ are chosen based on the following prerequisites: i) concerning the recommended by the literature values and trying to comprise different values in the ranges for both Case studies [12], [13], [15]; ii) concerning the previous authors' experience of modelling of FP using GA [1], [16], [17], [18], [19]. All other GA operators and parameters are tuned as presented in [1], [19].

III. INTERCRITERIA ANALYSIS

InterCriteria analysis, based on the apparatuses of index matrices and intuitionistic fuzzy sets, is given in details in [3]. Here, for a completeness, the proposed idea is briefly presented.

An intuitionistic fuzzy pair (IFP) [4] is an ordered pair of real non-negative numbers $\langle a, b \rangle$, where $a, b \in [0, 1]$ and $a + b \leq 1$, that is used as an evaluation of some object or process. According to [4], the components (a and b) of IFP might be interpreted as degrees of “membership” and “non-membership” to a given set, degrees of “agreement” and “disagreement”, degrees of “validity” and “non-validity”, degrees of “correctness” and “non-correctness”, etc.

The apparatus of index matrices (IM) is presented initially in [5] and discussed in more details in [6], [7]. For the purposes of ICA application, the initial index set consists of the criteria (for rows) and objects (for columns) with the IM elements assumed to be real numbers. Further, an IM with index sets consisting of the criteria (for rows and for columns) with IFP elements determining the degrees of correspondence between

the respective criteria is constructed, as it is doing to be briefly presented below.

Let the initial IM is presented in the form of Eq. (8), where, for every p, q , ($1 \leq p \leq m, 1 \leq q \leq n$), C_p is a criterion, taking part in the evaluation; O_q – an object to be evaluated; a_{C_p, O_q} – a real number or another object, that is comparable about relation R with the other a -objects, so that for each i, j, k : $R(a_{C_k, O_i}, a_{C_k, O_j})$ is defined. Let \bar{R} be the dual relation of R in the sense that if R is satisfied, then \bar{R} is not satisfied, and vice versa. For example, if “ R ” is the relation “ $<$ ”, then \bar{R} is the relation “ $>$ ”, and vice versa. If $S_{k,l}^\mu$ is the number of cases in which $R(a_{C_k, O_i}, a_{C_k, O_j})$ and $R(a_{C_l, O_i}, a_{C_l, O_j})$ are simultaneously satisfied, while $S_{k,l}^\nu$ is the number of cases in which $R(a_{C_k, O_i}, a_{C_k, O_j})$ and $\bar{R}(a_{C_l, O_i}, a_{C_l, O_j})$ are simultaneously satisfied, it is obvious, that

$$S_{k,l}^\mu + S_{k,l}^\nu \leq \frac{n(n-1)}{2}.$$

Further, for every k, l , satisfying $1 \leq k < l \leq m$, and for $n \geq 2$,

$$\mu_{C_k, C_l} = 2 \frac{S_{k,l}^\mu}{n(n-1)}, \quad \nu_{C_k, C_l} = 2 \frac{S_{k,l}^\nu}{n(n-1)} \quad (9)$$

are defined. Therefore, $\langle \mu_{C_k, C_l}, \nu_{C_k, C_l} \rangle$ is an IFP. Next, the following IM is constructed:

| | C_1 | \dots | C_m |
|----------|--|----------|--|
| C_1 | $\langle \mu_{C_1, C_1}, \nu_{C_1, C_1} \rangle$ | \dots | $\langle \mu_{C_1, C_m}, \nu_{C_1, C_m} \rangle$ |
| \vdots | \vdots | \ddots | \vdots |
| C_m | $\langle \mu_{C_m, C_1}, \nu_{C_m, C_1} \rangle$ | \dots | $\langle \mu_{C_m, C_m}, \nu_{C_m, C_m} \rangle$ |

that determines the degrees of correspondence between criteria C_1, \dots, C_m .

In the most of the obtained pairs $\langle \mu_{C_k, C_l}, \nu_{C_k, C_l} \rangle$, the sum $\mu_{C_k, C_l} + \nu_{C_k, C_l}$ is equal to 1. However, there may be some pairs, for which this sum is less than 1. The difference

$$\pi_{C_k, C_l} = 1 - \mu_{C_k, C_l} - \nu_{C_k, C_l} \quad (10)$$

is considered as a degree of “uncertainty”.

IV. NUMERICAL RESULTS AND DISCUSSION

In order to obtain reliable results for convergence time, optimization criterion and model parameters estimations, thirty independent runs of SGA have been performed for each value of crossover and mutation rates for both examined here Case studies. Obtained results have been averaged and two IMs are constructed for each Case study, involving values for crossover or mutation rates, respectively. In other words, altogether four IMs are constructed: IMs $A_{1(xovr)}$ (Eq. (11)) and $A_{1(mutr)}$ (Eq. (12)) for the Case study 1 and IMs $A_{2(xovr)}$ (Eq. (13)) and $A_{2(mutr)}$ (Eq. (14)) for the Case study 2.

IM $A_{1(xovr)}$ presents average estimates of the model parameters $\mu_{2S}, \mu_{2E}, k_S, k_E$, and $Y_{S/X}$, as well as the resulting convergence time T and objective function value J , respectively for $xovr = \{0.65; 0.75; 0.85; 0.95\}$, denoted as $GA_{1,1}^{xovr} \div GA_{1,4}^{xovr}$. In the same way, IM $A_{1(mutr)}$ presents

the results for $\mu_{2S}, \mu_{2E}, k_S, k_E$, and $Y_{S/X}, T, J$ and $mutr$, respectively for $mutr = \{0.02; 0.04; 0.06; 0.08; 0.1\}$, denoted as $GA_{1,1}^{mutr} \div GA_{1,5}^{mutr}$.

IMs $A_{2(xovr)}$ and $A_{2(mutr)}$ for the Case study 2 have been created by analogy with the Case study 1.

Based on Eq. (9), ICA algorithm calculates the IFP $\langle \mu, \nu \rangle$ for every two pairs of considered criteria based on the obtained IMs $A_{1(xovr)}, A_{1(mutr)}, A_{2(xovr)}$ and $A_{2(mutr)}$. Values of π (Eq. (10)) are calculated too. Obtained results are grouped in Table 1 for both Case studies, considering dependences between crossover and mutation rates, optimization criterion, convergence time and model parameters themselves.

Applied here non-linear models for two Case studies (respectively Eqs. (1)-(3) and Eqs. (4)-(6)) are a prerequisite some closer relations between observed criteria to be expected after ICA application. On the other hand, some differences in the parameters relations might appear caused by the different specific growth rates in *S. cerevisiae* and *E. coli* FP.

As it could be seen from Table 1, there is a strong relation between $T \leftrightarrow xovr/mutr$ for the Case study 1, while in the Case study 2 a weak relation is observed. The similar discrepancy is identified in the correlation between $Y_{S/X} \leftrightarrow xovr/mutr$: in the Case study 2 there is a strong relation for GA parameter $xovr$, while in the Case study 1 – a weak. These discrepancies might be explained by the stochastic nature of GA. Crossover rate strongly influences evaluation of model parameter μ_{2E} in Case study 1. In the Case study 2, there is a significant indication for high correlation between $J \leftrightarrow mutr$. For the rest of model parameters the observed correlations are weak – there are no significant dependencies between T and these parameters.

Going further in investigation of relations between algorithm accuracy J and model parameters, higher μ -values is observed between $Y_{S/X} \leftrightarrow J$ in Case study 1 and for GA parameter $mutr$. Less stronger correlations are identified in the Case study 1 for GA parameter $mutr$ between $\mu_{2S} \leftrightarrow J$, as well as in Case study 2 for GA parameter $mutr$ between $Y_{S/X} \leftrightarrow J$. These similarities are caused by the physical meaning of considered model parameters. For the rest of parameters the observed correlations are weak – there are no significant dependencies between these parameters and J .

When considering the influence of convergence time T over the model parameters, higher μ is observed in pairs $\mu_{2E} \leftrightarrow T$ in Case study 1 for GA parameter $xovr$. In the Case study 2, higher μ -values are observed between $\mu_{max} \leftrightarrow T$ and $k_S \leftrightarrow T$ for $mutr$ GA parameter. Observed μ -values for the rest of pairs of model parameters and T show that there are no significant correlations between them.

The last group of examined correlations is between model parameters themselves in both considered Case studies. Different model structures in both FP complicate the extraction of some common correlations. Although that fact, there are some coincidences for both Case studies. In the Case study 1 for GA parameter $xovr$, the strongest correlations are found respectively for $\mu_{2S} \leftrightarrow k_S$ and $\mu_{2S} \leftrightarrow Y_{S/X}$, while less stronger correlations are identified for the pairs $k_S \leftrightarrow k_E$,

$$A = \begin{matrix} & O_1 & \dots & O_k & \dots & O_l & \dots & O_n \\ \begin{matrix} C_1 \\ \vdots \\ C_i \\ \vdots \\ C_j \\ \vdots \\ C_m \end{matrix} & \begin{matrix} a_{C_1,O_1} & \dots & a_{C_1,O_k} & \dots & a_{C_1,O_l} & \dots & a_{C_1,O_n} \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ a_{C_i,O_1} & \dots & a_{C_i,O_k} & \dots & a_{C_i,O_l} & \dots & a_{C_i,O_n} \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ a_{C_j,O_1} & \dots & a_{C_j,O_k} & \dots & a_{C_j,O_l} & \dots & a_{C_j,O_n} \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ a_{C_m,O_1} & \dots & a_{C_m,O_k} & \dots & a_{C_m,O_l} & \dots & a_{C_m,O_n} \end{matrix} \end{matrix}, \quad (8)$$

Case study 1, IM $A_{1(xovr)}$:

$$A_{1(xovr)} = \begin{matrix} & GA_{1,1}^{xovr} & GA_{1,2}^{xovr} & GA_{1,3}^{xovr} & GA_{1,4}^{xovr} \\ \begin{matrix} J \\ T \\ xovr \\ \mu_{2S} \\ \mu_{2E} \\ k_S \\ k_E \\ Y_{S/X} \end{matrix} & \begin{matrix} 0.0222 & 0.0222 & 0.0222 & 0.0221 \\ 69.140600 & 70.212400 & 69.475000 & 71.359200 \\ 0.65 & 0.75 & 0.85 & 0.95 \\ 0.962120 & 0.949840 & 0.974790 & 0.923920 \\ 0.103840 & 0.107940 & 0.115320 & 0.129580 \\ 0.124640 & 0.119580 & 0.128700 & 0.119780 \\ 0.799020 & 0.798700 & 0.798860 & 0.798960 \\ 0.417885 & 0.413705 & 0.413850 & 0.409500 \end{matrix} \end{matrix} \quad (11)$$

Case study 1, IM $A_{1(mutr)}$:

$$A_{1(mutr)} = \begin{matrix} & GA_{1,1}^{mutr} & GA_{1,2}^{mutr} & GA_{1,3}^{mutr} & GA_{1,4}^{mutr} & GA_{1,5}^{mutr} \\ \begin{matrix} J \\ T \\ mutr \\ \mu_{2S} \\ \mu_{2E} \\ k_S \\ k_E \\ Y_{S/X} \end{matrix} & \begin{matrix} 0.022200 & 0.022167 & 0.022133 & 0.022300 & 0.022100 \\ 71.677000 & 76.104333 & 90.479000 & 101.400667 & 98.161667 \\ 0.02 & 0.04 & 0.06 & 0.08 & 0.1 \\ 0.963433 & 0.987333 & 0.943333 & 0.960033 & 0.914933 \\ 0.113100 & 0.111900 & 0.129733 & 0.094967 & 0.146100 \\ 0.124000 & 0.123333 & 0.128167 & 0.117033 & 0.121300 \\ 0.799867 & 0.799500 & 0.799600 & 0.792433 & 0.797833 \\ 0.410841 & 0.411348 & 0.407914 & 0.421965 & 0.398290 \end{matrix} \end{matrix} \quad (12)$$

Case study 2, IM $A_{2(xovr)}$:

$$A_{2(xovr)} = \begin{matrix} & GA_{2,1}^{xovr} & GA_{2,2}^{xovr} & GA_{2,3}^{xovr} & GA_{2,4}^{xovr} & GA_{2,5}^{xovr} & GA_{2,6}^{xovr} \\ \begin{matrix} J \\ T \\ xovr \\ \mu_{max} \\ k_S \\ Y_{S/X} \end{matrix} & \begin{matrix} 0.010700 & 0.000310 & 0.000320 & 0.000170 & 0.000450 & 0.000310 \\ 143.156 & 77.782 & 218.234 & 104.719 & 158.078 & 86.953 \\ 0.5 & 0.6 & 0.7 & 0.8 & 0.9 & 1 \\ 0.553000 & 0.549000 & 0.550000 & 0.551000 & 0.549000 & 0.548000 \\ 0.011700 & 0.009800 & 0.010100 & 0.010000 & 0.009800 & 0.009900 \\ 0.500275 & 0.499975 & 0.499950 & 0.500000 & 0.500250 & 0.500500 \end{matrix} \end{matrix} \quad (13)$$

$Y_{S/X} \leftrightarrow k_S$ and $Y_{S/X} \leftrightarrow k_E$. Considering GA parameter $mutr$, the strongest correlations are between $k_S \leftrightarrow k_E$ and $\mu_{2S} \leftrightarrow Y_{S/X}$. Comparing to Case study 2 and taking into account the simpler specific growth rate model structure, the similar result for the pair $\mu_{max} \leftrightarrow k_S$ is observed. The highest correlation is observed for both GA parameters $xovr$ and $mutr$. These strong parameter dependencies are again caused by the physical meaning of FP models parameters. For the

rest correlations between model parameters themselves, the μ -values are low – there are no significant dependencies.

It is also interesting to be noted that during the investigation of $xovr$ influence, there are some pairs of considered criteria with reported degree of uncertainty π . For the Case study 1, all observed appearances of degrees of uncertainty are in pairs with optimization criterion value, while in Case study 2 – in pairs of optimization criterion value or specific growth rate. All

Case study 2, IM $A_{2(mutr)}$:

| | $GA_{2,1}^{mutr}$ | $GA_{2,2}^{mutr}$ | $GA_{2,3}^{mutr}$ | $GA_{2,4}^{mutr}$ | $GA_{2,5}^{mutr}$ |
|----------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| J | 0.019000 | 0.000360 | 0.007300 | 4.130700 | 25.622800 |
| T | 53.250000 | 116.594000 | 193.641000 | 70.937000 | 39.234000 |
| $A_{2(mutr)} = mutr$ | 0.001 | 0.01 | 0.1 | 0.5 | 1 |
| μ_{max} | 0.546000 | 0.550000 | 0.554000 | 0.599000 | 0.432000 |
| k_S | 0.007800 | 0.010200 | 0.011000 | 0.044400 | 0.002300 |
| $Y_{S/X}$ | 0.499500 | 0.500250 | 0.500501 | 0.500325 | 0.518403 |

(14)

these facts have an obvious explanation – as it can be seen from IM $A_{1(xovr)}$ for Case study 1, there are equal values for optimization criterion value. In analogy, as seen from IM $A_{2(xovr)}$, there are equal evaluations of optimization criterion value and specific growth rate in Case study 2. Observed equal values logically cause an uncertainty and makes difficult the process of decision making.

As a summary of ICA implementation, the following main results might be outlined:

- Considered GA parameters $xovr$ and $mutr$ show a high correlation with T in both Case studies. In Case study 2, parameter $mutr$ is in a high correlation with J and model parameter $Y_{S/X}$. The values of $xovr$ and $mutr$ reflect on T because of the more complex model used in Case study 1 [1], [12], [15]. In opposite, the more simple model structure in Case study 2 allows the relations between $mutr$ and J and one of the most sensitive model parameter $Y_{S/X}$ [17] to be outlined.
- When looking at T and J relations, strong connections are observed for $J \leftrightarrow Y_{S/X}$, especially in Case study 1; between specific growth rates (respectively μ_{2E} and μ_{max}) and T in both Case studies, as well as for $k_S \leftrightarrow T$ in Case study 2. The stochastic nature of GA is a preposition of a relatively small number of observed strong relations [10], [12], [15].
- In the last group of examined correlations between model parameters themselves, higher dependencies are obtained between specific growth rates μ_{2S} and μ_{max} and model parameter k_S in both Case studies, especially in Case study 2 at GA parameter $mutr$. Considering Case study 1, strong correlations are observed for $k_S \leftrightarrow k_E$ and for $Y_{S/X} \leftrightarrow \mu_{2S}$. The ascertained results are caused by the physical meaning of FP models parameters, as well as by the strong non-linearity of FP model structures [9], [11], [14], [16].

V. CONCLUSION

In this paper the recently proposed InterCriteria Analysis is applied to establish the relations and dependencies between two GAs parameters – crossover and mutation rates, on one hand, and convergence time, model accuracy and FP model parameters, on the other hand. Simple GA with different values of crossover and mutation rates is used for parameter identification of two FP models – of yeast *S. cerevisiae* and bacteria *E. coli*.

The obtained results from ICA show some existing relations and dependencies that result from the physical meaning of the model parameters, on one hand, and from stochastic nature of the considered meta-heuristic, on the other hand. Moreover, derived additional knowledge for ascertained correlations will be useful in further identification procedures of FP models and, in general, for more accurate SGA application.

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TABLE I
RESULTS FROM THE ICA OF *S. cerevisiae* AND *E. coli* FED-BATCH FERMENTATION PROCESSES

| Correlation | <i>S. cerevisiae</i> fed-batch fermentation process | | | | <i>E. coli</i> fed-batch fermentation process | | | |
|---------------------------------------|---|-------|----------------------------|-------|---|-------|----------------------------|-------|
| | <i>xovr</i> | | <i>mutr</i> | | <i>xovr</i> | | <i>mutr</i> | |
| | $\langle \mu, \nu \rangle$ | π | $\langle \mu, \nu \rangle$ | π | $\langle \mu, \nu \rangle$ | π | $\langle \mu, \nu \rangle$ | π |
| $T \leftrightarrow xovr/mutr$ | 0.8, 0.2 | 0 | 0.9, 0.1 | 0 | 0.5, 0.5 | 0 | 0.4, 0.6 | 0 |
| $J \leftrightarrow xovr/mutr$ | 0, 0.5 | 0.5 | 0.3, 0.7 | 0 | 0.3, 0.6 | 0.1 | 0.8, 0.2 | 0 |
| $\mu_{2S} \leftrightarrow xovr/mutr$ | 0.3, 0.7 | 0 | 0.2, 0.8 | 0 | | | | |
| $\mu_{2E} \leftrightarrow xovr/mutr$ | 1, 0 | 0 | 0.6, 0.4 | 0 | | | | |
| $\mu_{max} \leftrightarrow xovr/mutr$ | | | | | 0.2, 0.7 | 0.1 | 0.6, 0.4 | 0 |
| $Y_{S/X} \leftrightarrow xovr/mutr$ | 0.2, 0.8 | 0 | 0.4, 0.6 | 0 | 0.7, 0.3 | 0 | 0.9, 0.1 | 0 |
| $k_S \leftrightarrow xovr/mutr$ | 0.5, 0.5 | 0 | 0.3, 0.7 | 0 | 0.3, 0.7 | 0 | 0.6, 0.4 | 0 |
| $k_E \leftrightarrow xovr/mutr$ | 0.5, 0.5 | 0 | 0.2, 0.8 | 0 | | | | |
| $T \leftrightarrow J$ | 0, 0.5 | 0.5 | 0.4, 0.6 | 0 | 0.6, 0.3 | 0.1 | 0.2, 0.8 | 0 |
| $\mu_{2S} \leftrightarrow J$ | 0.5, 0 | 0.5 | 0.7, 0.3 | 0 | | | | |
| $\mu_{2E} \leftrightarrow J$ | 0, 0.5 | 0.5 | 0.1, 0.9 | 0 | | | | |
| $\mu_{max} \leftrightarrow J$ | | | | | 0.5, 0.3 | 0.2 | 0.4, 0.6 | 0 |
| $Y_{S/X} \leftrightarrow J$ | 0.5, 0 | 0 | 0.9, 0.1 | 0 | 0.5, 0.4 | 0.1 | 0.7, 0.3 | 0 |
| $k_S \leftrightarrow J$ | 0.3, 0.2 | 0.5 | 0.4, 0.6 | 0 | 0.5, 0.3 | 0.2 | 0.4, 0.6 | 0 |
| $k_E \leftrightarrow J$ | 0.2, 0.3 | 0.5 | 0.5, 0.5 | 0 | | | | |
| $\mu_{2S} \leftrightarrow T$ | 0.2, 0.8 | 0 | 0.3, 0.7 | 0 | | | | |
| $\mu_{2E} \leftrightarrow T$ | 0.8, 0.2 | 0 | 0.5, 0.5 | 0 | | | | |
| $\mu_{max} \leftrightarrow T$ | | | | | 0.6, 0.3 | 0.1 | 0.8, 0.2 | 0 |
| $Y_{S/X} \leftrightarrow T$ | 0, 1 | 0 | 0.5, 0.5 | 0 | 0.4, 0.6 | 0 | 0.5, 0.5 | 0 |
| $k_S \leftrightarrow T$ | 0.3, 0.7 | 0 | 0.2, 0.8 | 0 | 0.7, 0.3 | 0 | 0.8, 0.2 | 0 |
| $k_E \leftrightarrow T$ | 0.3, 0.7 | 0 | 0.1, 0.9 | 0 | | | | |
| $\mu_{2S} \leftrightarrow \mu_{2E}$ | 0.3, 0.7 | 0 | 0.2, 0.8 | 0 | | | | |
| $\mu_{2S} \leftrightarrow k_S$ | 0.8, 0.2 | 0 | 0.5, 0.5 | 0 | | | | |
| $\mu_{2E} \leftrightarrow k_S$ | 0.5, 0.5 | 0 | 0.7, 0.3 | 0 | | | | |
| $\mu_{max} \leftrightarrow k_S$ | | | | | 0.8, 0.2 | 0 | 1, 0 | 0 |
| $\mu_{2S} \leftrightarrow k_E$ | 0.5, 0.5 | 0 | 0.6, 0.4 | 0 | | | | |
| $\mu_{2E} \leftrightarrow k_E$ | 0.5, 0.5 | 0 | 0.6, 0.4 | 0 | | | | |
| $k_S \leftrightarrow k_E$ | 0.7, 0.3 | 0 | 0.9, 0.1 | 0 | | | | |
| $Y_{S/X} \leftrightarrow \mu_{2S}$ | 0.8, 0.2 | 0 | 0.8, 0.2 | 0 | | | | |
| $Y_{S/X} \leftrightarrow \mu_{2E}$ | 0.2, 0.8 | 0 | 0, 1 | 0 | | | | |
| $Y_{S/X} \leftrightarrow \mu_{max}$ | | | | | 0.4, 0.5 | 0.1 | 0.5, 0.5 | 0 |
| $Y_{S/X} \leftrightarrow k_S$ | 0.7, 0.3 | 0 | 0.3, 0.7 | 0 | 0.5, 0.5 | 0 | 0.5, 0.5 | 0 |
| $Y_{S/X} \leftrightarrow k_E$ | 0.7, 0.3 | 0 | 0.4, 0.6 | 0 | | | | |

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