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Pattern of the global syndrome for multiprocessor system of H^4 type for the MM model

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A problem of identification of faulty processors of a multiprocessor system is investigated. A method to reduce a pattern of a global syndrome for multiprocessor system which has a 4-cube topology, for the MM model, is presented. Results of the method for some topologies are also presented.

I. Introduction

IN the paper a problem of identification of faulty processors of multiprocessor system is investigated. Identification of faulty processors is a problem which is analysed in many publications ([2]-[8],[12]-[15]). The process of identifying faulty processors in a system by analysing the outcomes of available inter-processor tests is the system level diagnosis. Faulty processors, beside fault-free processors, are involved in testing process. The basis of the system level diagnosis and original diagnostic model, namely the PMC model, were proposed by Preperata, Metze and Chien in [9]. An example of another diagnostic model is the MM model (comparison-base diagnosis model) [3], [2], [8]. PMC model and MM model assume that communication links between the processors are reliable (useable). In PMC model all tests are performed between two adjacent processors, and it was assumed that a test result is reliable (respectively, unreliable) if the processor that initiates the test is fault-free (respectively, faulty). In the MM model, the same job is assigned to a pair of processors of the network and their outputs are compared by a central observer. This central observer performs diagnosis using the outcomes of these comparisons. The comparison-based diagnosis model was extended [2] to allow comparisons carried out by processors themselves. In [7] authors proposed that comparisons have no central observer involved. The diagnosability of hypercube under the comparison-based diagnosis model were presented (i.a.) in [9].

A multiprocessor network (system), presented in the paper, belongs to class of the fault-tolerant system ([1]) and is mounted onto objects which are difficult to access. The network belongs to the class of self-diagnosable systems [7] as well. The multiprocessor system, in general, has a regular logical structure (e.g.: torus, hypercube) and is homogenous. A faulty processor in the system is not interchanged nor repaired. The faulty processor is removed from the logical structure of the network and access to it is blocked. If certain conditions are met the system with degraded structure continues to operate, and tasks of faulted processors are taken over by fault-free processors of the system (network) with degraded structure. The diagnosability of the network (system) is defined as the maximum number t such that the network is self-diagnosable as long as the number of the faulty processors is not greater than t.

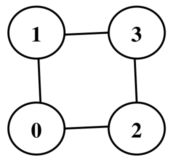
This paper focuses on selected issues connected with identifying of faulty processors of the 4-dimensional hypercube network and its node induced subgraphs (H^4 class for short) based on MM model and comparison diagnosis for such t-diagnosable system, where $1 \leq t \leq 2$. The rest of this paper is organised as follow. Section 2 gives some preliminaries; Section 3 focuses on a method of reduction of number of comparative trials. Section 4 concludes the paper.

II. Preliminaries

The processors network topology is represented by an undirected graph $G = \langle V, E\rangle$, where each node $u \in V$ denotes a processor and each edge $(u, v) \in E$ denotes a two-way link between nodes u and v. In the paper graph G is a 4-dimensional hypercube (4-cube for short) or nodes-included subgraph of 4-cube. A 4-cube (H^4) is such undirected graph $G = \langle V, E\rangle$, $|V| = 2^4$, $|E| = 4 \cdot 2^{4-1}$. Each node $u \in V$ is assigned an unique 4-bit binary vector (a coordinate) and each edge $(u, v) \in E$ links only those nodes whose coordinates differ in exactly one bit position (the Hamming distance between coordinates of linked nodes is equal 1).

Comparison diagnosis is based on inference of a network state on the basis of a set of results of comparative trials. Three processors are involved in the comparative trial: comparator $c \in V$, and comparative pair: $\{p_1, p_2\} \in V$: $\{p_1, p_2\} \subset V(c)$ ($V(c)$ is a set of nodes adjacent to c). A comparator c instructs adjacent nodes p_1, p_2 to perform the same task and then checks to see if test results are the same. A $\psi = (c; p_1, p_2)$ is named a comparative trial.

a)



b)

V^1 \ ψ	$d((0;1,2),V^1)$	$d((1;0,3), V^1)$	$d((2;0,3), V^1)$	$d((3;1,2), V^1)$
{∅}	0	0	0	0
{0}	x	1	1	0
{1}	1	x	0	x
{2}	1	0	x	1
{3}	0	1	1	x

Fig. 1 A pattern of the global syndrome (b) for H^2 network (a)

Let $\Psi(H^4)$, $E(\psi)$, $K(\psi)$ and $P(\psi)$ denote respectively: a set of all possible comparative trials of H^4, a set of processors participating in the comparative trail ψ, a comparator of ψ, and a comparative pair of ψ.

A set $\Psi' \subseteq \Psi(H^4)$ is a comparative trial cover of set of processors if $P(\Psi') = V$. In other words a set $\Psi' \subseteq \Psi\ (H^4)$ is a comparative trial cover of set of processors if $\forall(e \in V)\ \exists(\psi \in \Psi')\ e \in P(\psi)$.

A diagnostic structure of H^4 based on comparative trails is such an ordered pair $\langle H^4, \Psi'\rangle$ ($\Psi' \subseteq \Psi(H^4)$), that a set Ψ' is a comparative trials cover of set of processors of H^4.

Let V^1, V^0 and $d(\psi, V^1)$ denote respectively the set of faulty processors of the network, the set of fault-free processors of the network, and the result of comparative trial ψ for set V^1, wherein $d(\psi, V^1) = 0$ denotes that test results of comparative trail for both processors are identical, and $d(\psi, V^1) = 1$ denotes that test results of comparative trail for both processors are different

The following rule of inference based on comparative trials is valid [2], [3].

$$[(K(\psi) \in V^0) \wedge (P(\psi) \cap V^1 = \emptyset)] \Rightarrow [d(\psi,V^1) = 0];$$
$$[(K(\psi) \in V^0) \wedge (P(\psi) \cap V^1 \neq \emptyset)] \Rightarrow [d(\psi,V^1) = 1]; \quad (1)$$
$$[(K(\psi) \in V^1)] \Rightarrow [d(\psi,V^1) = x;\ x \in \{0,1\}].$$

From formula 1 and Fig. 1 follows that if the network has faulty processors, then there exist several syndromes which faulty processors could produce. Let $\sigma(V^1)$ represents the set of syndromes which could be produced. Two distinct sets V', $V'' \subset V$ are said to be indistinguishable if and only if $\sigma(V') \cap \sigma(V'') \neq \emptyset$; otherwise, V', V'' are said to be distinguishable. Let σ^* denotes a set of patterns of syndromes which faulty processors could produce, then from Fig.1 follows that i.e. $\sigma^*(\{1\}) \cap \sigma^*(\{2\}) \neq \emptyset$ which implies that there is no possibility to point out a faulty processor (for $t \geq 1$).

A processors network is one-step t-diagnosable by a set of comparative trials $\Psi' \subseteq \Psi(H^4)$ if each pair such sets V', V'' ($|V'| \leq t$, $|V''| \leq t$) of faulty nodes is distinguishable by at least one comparative trail $\psi \in \Psi'$.

Theorem 1[7]: For any V', V'' where $V', V'' \subset V$ and $V' \neq V''$ is a distinguishable pair if and only if at least one of following conditions is satisfied:

1) $(\exists(i, k \in V \setminus \{V' \cup V''\}) \wedge \exists\ (j \in \{(V' \setminus V'') \cup (V'' \setminus V')\})) \Rightarrow (\psi = (k; i, j) \wedge \psi \in \Psi')$,
2) $(\exists(i, j \in V' \setminus V'') \wedge \exists(k \in V \setminus \{V' \cup V''\})) \Rightarrow (\psi = (k\ ; i, j) \wedge \psi \in \Psi')$, (2)
3) $(\exists(i, j \in V'' \setminus V') \wedge \exists(k \in V \setminus \{V' \cup V''\})) \Rightarrow (\psi = (k\ ; i, j) \wedge \psi \in \Psi')$.

Theorem 2[7]: A system, is t-diagnosable if and only if each node has order of at least t and for each distinct pair of sets V', $V'' \subset V$ such that $|V'| = |V''| = t$, at least one of the conditions of theorem 1 is satisfied.

Let us note, for example, that, for sets $V' = \{0\}$, $V'' = \{3\}$ (graph Fig. 1a) and diagnosability $t = 1$ none of conditions of theorem 1 is satisfied.

For a processors network given by H^4 and $\langle H^4, \Psi'\rangle$ and for a set of nodes $X \subset V$, $T(X)$ denotes the set of those nodes in $V \setminus X$ which are compared to some nodes of X by some nodes of X:

$$T(X) = \{j \in V \setminus X : \exists(\psi \in \Psi')\ \ \psi = (k\ ; i, j) \wedge i, k \in X\}.$$

Theorem 3[7]: A system with n nodes is t-diagnosable if and only if

1) $n \geq 2t + 1$,
2) each node has order of at least t,
3) $\forall(0 \leq p \leq t - 1)\ \forall(X \subset V : |X| = |V| - 2t + p) : |T(X)| > p$.

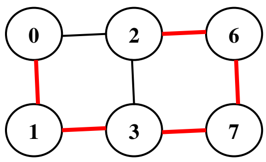
Let us note, for example, that for the graph on Fig. 1a for set $X = \{0,3\}$ and $t = 1$ theorem 3 is not satisfied.

On the basis of the above definitions and notation the problem of measurement of the multiprocessors system integrity is addressed in the next section.

III. Reducing the Number of Comparative Trails

The system diagnosability of H^4 class depends on orders of nodes and the number of nodes. It is known (*Theorem 3*) that diagnosability is not greater than the minimum order of the network node. Note that, for a processors networks having n nodes, maximum number of comparative trials is $\sum_{i=0}^{n-1} \binom{\mu(i) : \mu(i) > 1}{2}$, where i is a node label and $\mu(i)$ is an order of i node.

a)



b)

Ψ′ \ V^1	(0;1,2)	(1;0,3)	(2;0,3)	(2;0,6)	(2;3,6)	(3;1,2)	(3;1,7)	(3;2,7)	(6;2,7)	(7;3,6)
{∅}	0	0	0	0	0	0	0	0	0	0
{0}	x	1	1	1	0	0	0	0	0	0
{1}	1	x	0	0	0	1	1	0	0	0
{2}	1	0	x	x	x	1	0	1	1	0
{3}	0	1	1	0	1	x	x	x	0	1
{6}	0	0	0	1	1	0	0	0	x	1
{7}	0	0	0	0	0	0	1	1	1	x

Fig. 2. A pattern of global syndrome (b) computed for multiprocessor system (a)

Corollary 1: Path graph[11] of graph $G = \langle V,E\rangle$ which has at least 6 nodes ($|V| \geq 6$) describes an 1-diagnosable system under MM model.

Proof: We must show that $\forall(X \subset V : |X| = |V|-2) : |T(X)| > 0$ (*Theorem 3* point 3). If $(|V| \geq 6$ then $|X| \geq 4$, $|V \setminus X| = 2$ and $\exists(k; i, j) : k, i \in X \wedge j \in V \setminus X \wedge (k, i) \in E \wedge (k, j) \in E$.

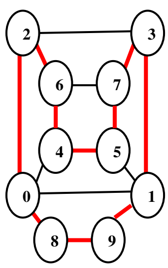
If a processors network of H^4 class is described by graph $G = \langle V, E\rangle$: $|V| \geq 6$ which has a Hamiltonian path[10] then the number of comparative trails for diagnosability of 1 is $|V| - 2$.

Example 1: Given cube $G' = \langle V',E'\rangle$: $V' = \{0,1,2,3,6,7\}$, we want to find a pattern for global syndrome under MM model for diagnosability of 1. On Fig.2 is presented the entire pattern of global syndrome. Red edges (Fig. 2a.) and cells of table filled with red colour (Fig. 2b) presents the global pattern for Hamiltonian path of graph *G'* (after reduction).

Corollary 2: Hamiltonian graph[11] of graph $G = \langle V,E\rangle$ which has at least 10 nodes ($|V| \geq 10$) describes 2-diagnosable system under MM model.

Proof: We must show that that $(\forall(X \subset V : |X| = |V| - 4) : |T(X)| > 0) \wedge (\forall(X \subset V : |X| = |V| - 3) : |T(X)| > 1)$ (*Theorem 3* point 3). If $(p = 0$ and $|V| \geq 10$ then $|X| \geq 6$, $|V \setminus X| = 4$ and $\exists(k; i, j) : k, i \in X \wedge j \in V \setminus X \wedge (k, i) \in E \wedge (k, j) \in E$. If $p = 1$ and $|V| \geq 10$ then $|X| \geq 7$, $|V \setminus X| = 3$ and $\exists\Psi'':|\Psi''| > 1$

a)

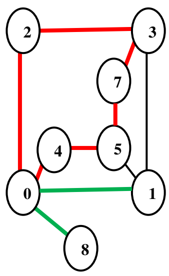


b)

Ψ′ \ V^1	(8;9,0) (9;8,1) (1;3,9) (3;1,7) (7;5,3) (5;4,6) (4;5,6) (6;4,2) (2;0,6) (0;2,8)	Ψ′ \ V^1	(8;9,0) (9;8,1) (1;3,9) (3;1,7) (7;5,3) (5;4,6) (4;5,6) (6;4,2) (2;0,6) (0;2,8)	Ψ′ \ V^1	(8;9,0) (9;8,1) (1;3,9) (3;1,7) (7;5,3) (5;4,6) (4;5,6) (6;4,2) (2;0,6) (0;2,8)
{∅}	0000000000	{1,2}	001x11x100	{4,5}	x1000001x1
{0}	100000001x	{1,3}	10001x101x	{4,6}	x11x100001
{1}	01x1000000	{1,4}	01x11x1000	{4,7}	x10001x101
{2}	00000001x1	{1,5}	00001x11x1	{4,8}	x1001x1001
{3}	001x100000	{1,6}	001x1x1000	{4,9}	x100001x11
{4}	000001x100	{1,7}	00001xx100	{5,6}	x101x10001
{5}	00001x1000	{1,8}	1000001x1x	{5,7}	1x1000001x
{6}	0000001x10	{1,9}	01x1001x10	{5,8}	1xx1000000
{7}	0001x10000	{2,3}	0000001xx1	{5,9}	1x100001x1
{8}	x100000001	{2,4}	001x101x10	{6,7}	1x1x100000
{9}	1x10000000	{2,5}	000001xx10	{6,8}	1x1001x100
{0,1}	11x100001x	{2,6}	00001x1x10	{6,9}	1x101x1000
{0,2}	10000001xx	{2,7}	1001x1001x	{7,8}	1x10001x10
{0,3}	01x10001x1	{2,8}	01x1x10000	{7,9}	1x11x10000
{0,4}	101x10001x	{2,9}	0001x101x1	{8,9}	xx10000001
{0,5}	01xx100000	{3,4}	001xx10000		
{0,6}	001x1001x1	{3,5}	0001x1x100		
{0,7}	100001x11x	{3,6}	0001xx1000		
{0,8}	01x101x100	{3,7}	0001x11x10		
{0,9}	000001x1x1	{3,8}	x10000001x		
		{3,9}	x1x1000001		

Fig. 3 A reduced pattern of global syndrome (b) computed for multiprocessor system (a)

a)



b)

V^1 \ Ψ′	(2;3,0) (3;2,7) (7;5,3) (5;4,7) (4;5,0) (0;1,4) (0;2,8)
{∅}	0000000
{0}	10001xx
{1}	0000010
{2}	x100001
{3}	1x10000
{4}	0001x10
{5}	001x100
{7}	01x1000
{8}	0000001

Fig. 4 A reduced pattern of global syndrome (b) computed for multiprocessor system (a) which has not Hamiltonian cycle or a Hamiltonian path

$\Rightarrow(\forall(k;\, i, j)\in\Psi'' : k, i \in X \wedge j \in V \setminus X \wedge (k, i) \in E \wedge\ (k, j) \in E)$.

If a processors network of H4 class is described by graph $G = \langle V, E\rangle$: $|V| \geq 10$ is a Hamiltonian graph then number of comparative trails for diagnosability of 2 is $|V|$.

Example 2: Given cube $G' = \langle V', E'\rangle$: $V' = \{0,1,2,3,6,7,8,9\}$, we want to find a reduced pattern for global syndrome under MM model for diagnosability of 2. Fig.3b presentes the reduced pattern of global syndrome. Red edges (Fig. 3a.) present the Hamiltonian cycle of graph G'.

If a multiprocessor system G' has no Hamiltonian path or is not a Hamiltonian graph then you should find longest path or longest graph cycle and add the missing nodes.

Example 2: Given cube $G' = \langle V', E'\rangle$: $V' = \{0,1,2,3,5,7,8\}$, we want to find a reduced pattern for global syndrome under MM model for diagnosability of 1. On Fig.4b is presented the reduced pattern of global syndrome. Red edges (Fig. 4a.) presents the longest path of graph G' and green edges present links to missing nodes.

IV. Conclusion

The problem of reduction of number of comparative trials for MM model is complex. The paper only addresses the problem of generating a pattern of global syndrome for multiprocessor system of H^4 class under MM model and reduce the global pattern. Corollaries and examples presented in the paper shown the benefits of reducing a pattern of global syndrome. Other issues that should be considered are: the development of diagnostic procedures and the development of a test for a single processor system.

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