

# Modelling Legal Interpretation in Structured Argumentation Framework

Tomasz Zurek

Institute of Computer Science,  
Maria Curie-Sklodowska University in Lublin  
Ul. Akademicka 9, 20-033 Lublin, Poland  
Email: zurek@kft.umcs.lublin.pl

Michał Araszkiwicz  
Department of Legal Theory, Jagiellonian University,  
Bracka Str. 12, 31-005 Cracow, Poland  
Email: michal.araszkiwicz@uj.edu.pl

**Abstract**—The paper discusses the problem of formal modeling of the interpretation of statutory legal norms. The authors propose a comprehensive framework that allows the representation of the interpretation process. The authors' proposal is illustrated by a real-life example.

## INTRODUCTION

LEGAL interpretation is one of the most important problems in legal theory and practice. The aim of our paper is to develop formalized descriptive model of statutory interpretation. We are interested in formal modelling of actual interpretive argumentation rather than in developing its idealized picture. The second aim of our work is to integrate our model of legal interpretation with one of the most popular formal model of argumentation (ASPIC+ [1]). The development of a fully-fledged descriptive model of legal interpretation is a complex research project, and perhaps rather a regulative idea rather than an operational goal. The realisation of such a goal requires dealing with certain problems that remain unsolved in the current state of the art. In this contribution we model an actual case involving statutory interpretation to represent different arguments developed by different agents for the sake of the realisation of goals important for these agents.

## I. INTERPRETIVE AGENTS

The authors of [2] notice that a significant role in the argumentation process is played not only by the interpretive argument itself, but also by the agent putting forward the argument, since in legal discourse every agent plays a particular role with his/her own preferences and goals. In [2] a semi-formal framework is presented which permits to model the agent's role in interpretive arguments. This framework will constitute the grounds for the model of agent in our argumentative system.

## II. ARGUMENTATION FRAMEWORK

ASPIC<sup>+</sup> is a well-developed framework for structured argumentation representation ([1], [3] and many others). As such, it does not specify any logical language to represent

arguments, but argumentation represented in this framework may be instantiated in different languages.

## III. THE MODEL FOR LEGAL INTERPRETATION

Let  $\mathcal{L}$  be any well-defined and closed under negation language. First, let us define certain postulated sets. Let  $S$  be a legal system in question, let  $C$  be a concrete or a hypothetical case in question, and let  $\mathcal{L}$  be the language under consideration.

*Definition 1 (Source Set):* Let  $K$  be a knowledge base in argumentation system  $AS$ . Then  $SRC(S, C)$  (Source Set under a legal system  $S$  in the context of a case  $C$ ) consists of:

- 1)  $ST(S)$  is the set of all explicit statutory norms under a system  $S$ .  $ST(S) \subset K_p$ . Each norm in  $ST(S)$  is represented by a predicate  $norm(\alpha, \beta, \gamma)$ , where  $\alpha \in \mathcal{L}$  is the name of the norm,  $\beta$  is a wff in  $\mathcal{L}$  which represents the conditional part of the norm, and  $\gamma$  is a wff in  $\mathcal{L}$  which represents the conclusion of the norm;
- 2)  $Cases(S)$  is the set of wff in  $\mathcal{L}$  which represents all accessible judicial opinions ruled under a system  $S$ .  $Cases(S) \subset K_p$ ;
- 3)  $Doctrine(S)$  is the set of wff in  $\mathcal{L}$  which represents all scholarly opinions concerning legal issues arising under a legal system  $S$ .  $Doctrine(S) \subset K_p$ ;
- 4)  $Materials(S)$  is the set of wff in  $\mathcal{L}$  which represents the remaining official materials that may be relevant for the sake of interpretation of statutory law under a system  $S$ , such as legislative opinions, soft law and the like.  $Materials(S) \subset K_p$ ;
- 5)  $CSK$  is the set of wff in  $\mathcal{L}$  which represents all available common sense knowledge propositions.  $CSK \subset K_p$ ;
- 6)  $SK$  is the set encompassing propositions which are referred to as Scientific Knowledge.  $SK \subset K_p$ ;
- 7)  $Facts(C)$  is the complete set of propositions describing the facts of a case  $C$  in question.  $Facts(C) \subset K_n$ ;
- 8)  $IT(\mathcal{L})$  is the set of all Interpretive Terms in a language  $\mathcal{L}$ , that is, terms that may be used for the sake of the interpretation of any term of  $ST(S)$ .  $IT(\mathcal{L}) \subset K_p$ ;

9)  $R_i \subset R_d$  is the set of all argumentation schemes (represented as defeasible inference rules) used to generate interpretive arguments from the knowledge contained in sets 1)-8) above, hereafter referred to as Source Sets.

*Definition 2 (Extensional relations):* Extensional relation  $INC \subset \mathcal{L}$  is a set of binary relations encompassing inclusion relation  $\sqsubseteq$ , strict inclusion relation  $\sqsubset$ , and equivalence relation ( $\equiv$ ) defined on the set  $\mathcal{L}$ .

If  $X \sqsubseteq Y$  and  $X$  and  $Y$  are two expressions in  $\mathcal{L}$ , then we claim that  $X$  is within the scope (semantic extension) of  $Y$ .

*Definition 3 (Interpretation):*  $\bullet \in \mathcal{L}$  is a binary relation word denoting “legally counts as” or “is interpreted as”. We introduce this relation in order to grasp the phenomenon in which, in certain cases, an expression may legally count as an instance of another expression even though it is situated outside its semantic extension. It is worth to notice that the relation “is interpreted as” should be understood as a presumptive (defeasible) one.

Relation  $\bullet$  is reflexive (because  $\phi \bullet \phi$ ), not symmetric (because if  $\phi \bullet \psi$  then it is not necessary that  $\psi \bullet \phi$ ), and transitive (because if  $\phi \bullet \psi$  and  $\psi \bullet \delta$  then  $\phi \bullet \delta$ )

Each of the sources from a Source Set can be interpreted as a kind of a knowledge base which allows to infer and examine whether a given proposition is in the scope of meaning of a certain expression.

*Definition 4 (Interpretive Sentences):* All complex expressions of a language  $\mathcal{L}$  constructed by means of any of the elements from the set  $INC$  or by means of the relation word  $\bullet$  will be referred to as Interpretive Sentences.

The legal theory points out that Interpretive Sentences are justified by means of interpretive arguments or canons. In our work interpretive canons will be represented by interpretive inference rules:

*Definition 5 (Interpretive Inference rules):* All inference rules whose conclusions are interpretive sentences or undercutters of other interpretive inference rules will be referred to as interpretive inference rules. The set of all Interpretive Inference Rules will be denoted as  $R_i$  ( $R_i \subset R_d$ )

*Definition 6 (Interpetive Arguments):* If  $A$  is an argument constructed by means of a knowledge base  $K$  in AS and the last inference rule in  $A$  is built on the basis of an interpretive inference rule ( $TopRule(A) \in R_i$ ), then argument  $A$  is an interpretive argument.

Although we assumed that the sources of justifications do not have to be consistent, we assume that an interpretive argument should be internally consistent. By an internally consistent argument we understand an argument in which:

$$\bar{A}_{\phi, \psi} (\phi \in Prem(A) \wedge (\psi \in Prem(A) \vee Conc(A) = \psi) \wedge \psi \in \bar{\phi}).$$

#### A. Authorship of interpretive arguments

The authors of [2] point out that the notion of interpreting agent plays a crucial role in a descriptive model of legal interpretation. The notion of interpreting agents enables us to attribute certain statements and arguments to a given agent, which allows for a more fine-grained representation of argumentation in real-life cases.

*Definition 7 (Set of Agents):* Let  $IA \subseteq K_n$  be a collection of the agents’ names. Each  $ia \in IA$  will be the name of the agent present in a legal case  $c$ .

*Definition 8 (Authorship of an argument):* The relation of authorship is a subset of a Cartesian product:

$$\mathcal{R} \subseteq IA \times \mathcal{A}, \text{ i.e. a set of pairs: } (ia, A), \text{ where } ia \in IA \text{ and } A \in \mathcal{A}.$$

This relation shows who the author of a given argument is. The argument can have many authors; one agent may be the author of many arguments.

To consider the issue of argument authorship in structured argumentation framework, the SAF from [1] definition must be adjusted:

*Definition 9:* A structured authored argumentation framework (SAAF) is a tuple  $\langle \mathcal{A}, \mathcal{C}, \preceq, \mathcal{R} \rangle$  where:

- $\mathcal{A}$  is the smallest set of all finite arguments constructed from a knowledge base in AS;
- $\preceq$  is an ordering on  $\mathcal{A}$ ;
- $(X, Y) \in \mathcal{C}$  iff  $X$  attacks (is in conflict with)  $Y$ .
- $\mathcal{R}$  is an authorship relation on sets  $IA \subset K_n$  and  $\mathcal{A}$ .

#### B. Model of Interpreting Agent

*Definition 10 (Agent):* Basing on the model from [2], it is assumed that an agent  $ia$  in a structured authored argumentation framework SAAF will be a tuple:

$$(KB(ia), preferences(ia), authority(ia))$$

Where:

- $KB(ia) \subseteq SRC(S, C)$  The knowledge base of an agent  $IA$  is a subset of the Source Set
- $preferences(ia) \subseteq K_n$  and  $preferences(ia) = NormPref(ia) \cup SubPref(ia)$ 
  - $NormPref(ia) = (\prec_{NP(ia)}, \mathcal{L})$   
 $NormPref(ia)_i$  is a partial order on set wff in  $\mathcal{L}$
  - $SubPref(ia) = (\prec_{SP(ia)}, \mathcal{L})$   
 $SubPref(ia)$  is a partial order on set wff in  $\mathcal{L}$
- $authority(ia) \subseteq K_n$  The relation of Authority is a subset of a Cartesian product  $authority(ia) \subseteq S(ia) \times IA$ , where  $S(ia)$  is the set of all sentences stated by an Interpretive Agent in the case  $c$  (formally:  $S(ia) = \{Conc(A_n) : (Conc(A_n), ia) \in \mathcal{R}\}$ , i.e. a set of pairs of statements given by the agent in a case  $c$  and agents formally bound by these sentences).

On the basis of the relations  $preferences(ia)$ ,  $authority(ia)$ , and the relevant inference rules (to appear in future work), it will be possible to establish a relation of order between conflicting arguments in a structured argumentation framework (relation  $\preceq$ ).

## IV. EXAMPLE

This section presents a modelling of interpretation in an actual case also discussed in [2]. However, here, in addition to presentation of the knowledge bases of the relevant agents, we also reconstruct the arguments developed and used by them. The legal issue at stake was as follows. Generally, according to the Personal Income Tax Act (PITA), the taxpayer’s total

revenue is taken into account in the calculation of taxable income, unless this revenue is exempted. Pursuant to the provision of 21.1.47c of the PITA, revenues raised by a natural person from a governmental or an executive agency, where the agency is financed from the state budget, are exempted from tax. The protagonist of the case obtained a housing benefit from the Military Housing Agency and claimed that this revenue was exempted from tax. However, the tax authorities disagreed, pointing out that the legislative materials suggested that the exemption is question was intended to apply to entrepreneurs, while the protagonist of the case was not one. The assessment of the Court is that the opinions presented by the tax office are not sufficiently justified both in relation with the provisions of tax law and the factual circumstances of the case. Such a conclusion results primarily from the outcomes of the linguistic interpretation of the Act. It follows from this regulation that in order to apply the exemption in question, two conditions must be fulfilled: first, the taxpayer is to receive a specific amount from a government or executive agency; second, this agency is to receive funds for this purpose from the state budget. In the view of the Court, both conditions which determine the exemption are fulfilled in this case. It should be highlighted that in the interpretation of tax law provisions, the linguistic interpretation of the text of the Act has the primary and dominant weight. Under no circumstances is it permitted in the tax law system to apply teleological, systemic, or historical interpretation of a provision of tax law to the factual circumstances should its result (even if obtained correctly) be inconsistent with the result of the linguistic interpretation.

### Basics:

First, the alphabet of the language is defined:

- Propositional atoms:  $\{housing\_benefit, natural\_person, person, enterprise, revenue\_from\_agency, revenue, agency\_financed\_from\_the\_state\_budget, tax\_law, tax, d_{ling}, d_{hist}, d_{mod}, r1, r2\}$
- symbols:  $\{\neg, \wedge, \vee, \supset, \sqsubseteq, \sqsubset, \equiv, \bullet, norm(, ,)\}$
- Interpreting Agents:  
 $IA = \{ia_{person}, ia_{taxOffice}, ia_{judge}\}$

### Knowledge Base

The authors of [4] (section 4.0) distinguish two ways of utilization of the  $ASPIC^+$  framework: domain-specific vs. general inference rules. In order to model our example, we will use the second one.

### Facts of the case:

$Facts(C) = \{housing\_benefit, natural\_person, revenue\_from\_agency, agency\_financed\_from\_the\_state\_budget, tax\_law\}$

### Commonsense knowledge:

$CSK(S) = \{revenue\_from\_agency \sqsubseteq revenue\}$

### Applicable law:

$ST(S) = \{norm(r1, housing\_benefit \wedge revenue\_from\_agency \wedge agency\_financed\_from\_the\_state\_budget, \neg tax),$

$norm(r2, revenue, tax)\}$

There are two legal rules: it follows from the first one that revenues raised from a governmental or an executive agency, where the agency is financed from the state budget, are exempted from tax, whereas the other rule states that all kinds of revenue are taxable.

### Historical materials:

$Materials(S) = \{norm(r1, \alpha, \beta) \wedge natural\_person \supset \neg(natural\_person \wedge \alpha \bullet \alpha)\}$

According to historical materials, the legal rule  $r1$  is not intended for natural persons, but for companies: a natural person does not fulfill the conditions of rule  $r1$ .

### Doctrine:

$Doctrine(S) = \{(n(A) = d_{hist} \wedge tax\_law) \supset \neg A)\}$

The use of historical interpretation is forbidden in tax law.

### Inference Rules:

Interpretive inference rules  $R_i =$

- linguistic interpretation  $d_{ling} : \alpha, \alpha \checkmark \beta \Rightarrow \alpha \bullet \beta$ , where  $\checkmark$  is one of the extensional relations:  $\sqsubseteq, \sqsubset, \equiv$ .
- historical interpretation  
 $d_{hist} : \alpha, (\alpha \bullet \beta) \in Materials(S) \Rightarrow \alpha \bullet \beta$
- interpretation  $d_{int} : \alpha, \alpha \bullet \beta \Rightarrow \beta$

Defeasible inference rules  $R_d =$

- defeasible modus ponens  $d_{mod} : \alpha, (\alpha \supset \beta) \Rightarrow \beta$
- legal rule application:  $d_{legal} : \alpha, norm(r, \alpha, \beta) \Rightarrow \beta$

Where  $\alpha, \beta$  are formulae in  $\mathcal{L}$ ,  $r \in \mathcal{L}$  is a legal rule name.

### Interpreting Agents

The models of interpreting agents are adapted from [2]: Since none of the agents from our case built arguments on the basis of the sets  $Cases$ ,  $CSK$ , and  $SK$ , we assume that they are empty for all agents.

**Agent:**  $ia_{judge} KB(ia_{judge}) :$

- $ST(ia_{judge}) = \{norm(r1, housing\_benefit \wedge revenue\_from\_agency \wedge agency\_financed\_from\_the\_state\_budget, \neg tax), norm(r2, revenue, tax)\}$
- $Doctrine(ia_{judge}) = \{(InfRule(A) = d_{hist} \wedge tax\_law) \supset \neg A)\}$
- $Materials(ia_{judge}) = \emptyset$
- $Facts(ia_{judge}) = Facts(C)$
- $R_d(ia_{judge}) = R_d$

**Preferences:** In the analyzed case, the agent does not use his/her preferences

**Authority:** Interpretive statements made by the judge are binding on the tax office and the person:

If  $\alpha \in S(ia_{judge})$  then  $(\alpha, ia_{person}) \in authority(ia_{judge})$  and  $(\alpha, ia_{taxOffice}) \in authority(ia_{judge})$

**Agent:**  $ia_{taxOffice}$

$KB(ia_{taxOffice}) :$

- $ST(ia_{taxOffice}) = \{norm(r1, housing\_benefit \wedge revenue\_from\_agency \wedge agency\_financed\_from\_the\_state\_budget, \neg tax), norm(r2, revenue, tax)\}$
- $Doctrine(ia_{taxOffice}) = \emptyset$

- $Materials(ia_{taxOffice}) : \{norm(r1, \alpha, \beta) \wedge natural\_person \supset \neg(natural\_person \wedge \alpha \bullet \alpha)\}$
- $Facts(ia_{taxOffice}) = Facts(C)$
- $R_d(ia_{taxOffice}) = R_d$

**Preferences:**  $NormPref(ia_{taxOffice} = SubPref(ia_{taxOffice}))$ .

The agent prefers rules which increases the collected tax:

$$NormPref(ia_{taxOffice}) = \{r1 <_{NP(ia_{taxOffice})} r2\}$$

In the analyzed case, the agent does not use his/her preferences.

**Authority:** Interpreting statements made by the tax office are binding on the person:

If  $\alpha \in S(ia_{taxOffice})$  then  $(\alpha, ia_{person}) \in authority(ia_{taxOffice})$

**Agent:**  $ia_{person}$

$KB(ia_{person}) :$

- $ST(ia_{person}) = \{norm(r1, housing\_benefit \wedge revenue\_from\_agency \wedge agency\_financed\_from\_the\_state\_budget, \neg tax), norm(r2, revenue, tax)\}$
- $Doctrine(ia_{person}) = \emptyset$
- $Materials(ia_{person}) = \emptyset$
- $Facts(ia_{person}) = Facts(C)$
- $R_d(ia_{person}) = R_d$

**Preferences:** In the analyzed case, the agent does not use his/her preferences

**Authority:** Interpreting statements made by a person are not binding on anyone:

$$authority(ia_{person}) = \emptyset$$

### Arguments:

First of all, the arguments of the agent *person* are presented:

$A_1 : natural\_person$

$A_2 : housing\_benefit$

$A_3 : revenue\_from\_agency$

$A_4 : agency\_financed\_from\_the\_state\_budget$

$A_5 : norm(r1, \alpha, \beta)$

where:  $\alpha = housing\_benefit$

$\wedge revenue\_from\_agency \wedge$

$agency\_financed\_from\_the\_state\_budget,$

$\beta = \neg tax,$

$A_6 : A_1, A_2, A_3, A_4 \sqsubseteq \alpha$

$A_7 : A_6 \Rightarrow (A_1 \wedge A_2 \wedge A_3 \wedge A_4) \bullet \alpha$  (Inference rule:  $d_{ling}$ )

$A_8 : A_7 \Rightarrow \alpha$  (Inference rule:  $d_{int}$ )

$A_9 : A_8, A_5 \Rightarrow \neg tax$  (Inference rule:  $d_{legal}$ )

It follows from the above arguments that since the conditions of the legal rule  $r1$  are fulfilled, the revenue should not be taxable.

Next, the arguments of agent *taxOffice* are presented:

$B_1 : Materials(ia_{taxOffice}) : norm(r1, \alpha, \beta) \wedge$

$natural\_person \supset \neg(natural\_person \wedge \alpha \bullet \alpha)$

$B_2 : B_1, A_1, A_2, A_3, A_4, A_5 \Rightarrow \neg((A_1 \wedge A_2 \wedge A_3 \wedge A_4) \bullet \alpha)$  (Inference rule:  $d_{hist}$ )

The arguments of tax authorities are based on historical materials from which it can be concluded that the legal rule  $r1$  does not apply to natural persons, and therefore the natural

person (even if theoretically the conditions of  $r1$  are fulfilled) cannot be interpreted as fulfilling the conditions of  $r1$ .

Arguments  $A_7$  and  $B_2$  are in conflict ( $B_2$  rebuts  $A_7$ ), and hence the case is decided by the judge:

The arguments of the agent *judge*:

$C_1 : Doctrine(ia_{judge}) = \{(InfRule(A) = d_{hist} \wedge tax\_law) \supset \neg A\}$

$C_2 : InfRule(B_2) = d_{hist}$

$C_3 : tax\_law$

$C_4 : C_1, C_2, C_3 \Rightarrow \neg B_2$  (Inference rule:  $d_{mod}$ ).

Arguments  $A_7$  i  $B_2$  are, on the basis on definition 11, contradictory.

According to the doctrine (arg  $C_1$ ), it is forbidden to use historical interpretation in tax law, and hence argument  $C_4$  attacks (undercuts) argument  $B_2$ .

Since argument  $B_2$  is undercut by argument  $C_4$  and:

- $(ia_{judge}, C_4) \in \mathcal{R}$
- $(Conc(C_4), ia_{person}) \in authority(ia_{judge}),$
- $(Conc(C_4), ia_{taxOffice}) \in authority(ia_{judge})$

then  $C_4$  defeats  $B_2$ .

Since  $C_4$  defeats  $B_2$ , then  $C_4$  defends  $A_7$ .

## V. CONCLUSIONS

The main aim of our work was to develop a formal descriptive model of statutory interpretation which can be integrated with one of the most popular argumentation frameworks (ASPIC+ [1]). Our proposal was illustrated by a model of a real life example of a legal case. Compared to the models presented in [5], [6], [7], our model is more comprehensive and abstract. We focused on the problem of integrating interpretation with the entire argumentation process, on the roles played by agents, disregarding the discussion of the structure of interpretive arguments.

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