Abstract—This study is devoted to process optimization for commercial sharing of e-vehicles. The model describes a system with one-way trips and relocations of e-vehicles between sectors by service personnel according to a dynamically compiled list of service trips. The model includes an algorithm for increasing the expected income, depending on the dynamically selected e-vehicle transfer. The implementation of the MIP (Mixed-Integer Programming) type algorithm pays particular attention to its performance, as optimization should be performed dynamically within few hours' intervals. The developed model has been validated for its practical application in Riga, Latvia.

Keywords—Sharing of e-Vehicles, One-way Trips and Relocations, Mixed-Integer Programming.

I. INTRODUCTION

SUSTAINABLE development requirements in the EU provide guidelines for improving the transport system, first and foremost by reducing CO₂ emissions. One of the possible solutions to achieve the goal is to switch to the use of electric vehicles (e-vehicle). According to [1], e-vehicle sharing systems benefit both users and the society in general. The two main benefits for individual users include reduced personal transport costs and improved mobility. Research has shown that e-vehicle sharing reduces the average number of kilometers travelled by a vehicle and is likely to reduce traffic congestions [2] and CO₂ emissions [3].

Vehicle rental approaches tend to be classified in two large groups [4]: (1) traditional rental—when customers receive and transfer vehicles after use at specially arranged points of leasing firms and rental will take one or more days—and (2) vehicle sharing—when vehicles can be taken for use anywhere, even for a very short period of time, and may be left anywhere at the end of the trip. Vehicle sharing has quickly gained popularity. The growth rate of the service has increased during the COVID-19 pandemic, as it allows urban populations to avoid the need to travel to their destination via public transport.

The main challenge facing e-vehicle sharing rental systems is to achieve the optimal (the most profitable) deployment of vehicles in a city. This requires relocating e-vehicles quickly to the most profitable sectors of the city which in turn causes additional costs. The studies [5] show that technical relocation of vehicles take approximately 14% of the total distance carried out by the vehicles. Optimization algorithms are used to give recommendations to system holders on the need to relocate vehicles to achieve a “more cost-effective” deployment and, hence, higher returns.

This study offers an e-vehicle sharing model that considers the dynamics of relocating e-vehicles in a city. The proposed model is designed to fully meet the requirements of real systems and differs from all known solutions.

The paper is structured as follows: a theoretical background on vehicle sharing models (Section 2), an original vehicle sharing model proposed by the authors (Section 3), a short discussion on the research findings (Section 4), and conclusions (Section 5).

II. RELATED RESEARCH

A. Review of Sharing Models

Although scientific literature on e-vehicle sharing is broad, the authors of other works as well as the authors of this study conclude that the scientific literature currently available does not offer a model that, along with parameters such as the number, size and location of charging stations, the size of the car fleet, would also consider the dynamics of vehicle relocations and system balancing when reserving of e-vehicles is used. The existing models [6] and [7] either use station locations without considering vehicle relocations [7] or use station locations, assuming only a limited subset of stations corresponding to the current demand should be serviced [6]. If vehicle relocations are modeled vehicle movements and associated costs are only considered at the end of the operating period (usually daily) and, therefore, affect the size of the available fleet [1].

According to [8] which studies an example of a city in Southern California, even 3-6 vehicles can be sufficient to provide 100 trips daily and achieve optimal customer waiting times. Meanwhile, about 18–24 vehicles would be enough to reduce the number of vehicle relocations. The authors conclude that, in addition to the number of vehicles (per trip), the relocation algorithm and the charging approach used are key factors for the successful use of such a system.

Boyaci [1] highlights the importance of the service level, which, in his view, influences the access of potential users to
e-vehicle stations, i.e. (1) the distance between the location of the e-vehicle and the destination, respectively, from the point of start and arrival of the e-vehicle, and (2) the availability of e-vehicles at stations. On the other hand, the number and size of the stations and the size and availability of the e-vehicle fleet at “real time” at the “particular station” are affected by the costs of establishing and operating the e-vehicle sharing system.

According to the classification of [4], the e-vehicle sharing system analyzed in this paper is:

1. a commercial solution as the aim is to generate maximum income,
2. station-based – e-vehicles are deployed in any available parking place and the city is divided into areas - stations,
3. one-directional as the customer is allowed to not return the e-vehicle to the start point of the trip,
4. with relocations as the service staff moves e-vehicles to potentially more favourable places in the city,
5. with dynamic booked trips as an e-vehicle may be rented by the customer anywhere, at any time without prior e-vehicle reservation.

Increased profits for commercial e-vehicle sharing can be achieved by supplying vehicles to the places in a city where customers will need them with the highest probability as well as increasing the relocation efficiency. The e-vehicle sharing models proposed by other authors differ significantly from those proposed in this work.

B. Optimization Algorithms

An optimization algorithm that was matching to the task of this study is the Mixed-Integer Programming (MIP) model, which maximizes profits on the assumption that next trips, the availability of fixed stations and availability of re-location staff are known.

The model described in [9] has a deficiency from model of this study. It is designed on the assumption that future trips are known, e.g. customers order the vehicles indicating starting and ending stations and the duration of the trip. Originally, the authors examined the possibility of predicting customer trips based on the data history of many previous trips. However, the experiments failed to obtain a sufficiently reliable forecast for future trips, so this idea was rejected. On the other hand, if the trips forecast is not sufficiently precise, the model defined by [9] does not provide a credible relocation plan, i.e., the vehicles will possibly be moved to places where customers will not need them.

Consequently, the [9] algorithm is not used directly in the study, and the authors have developed an original algorithm.

III. Vehicle Deployment Model

The model consists of several successive steps:

- dividing the city into sectors and estimating the costs of moving e-vehicles between sectors,
- identification of repeated trips,
- determining the value of an e-vehicle in a specific sector and time,
- forecasting of booked trips,
- an estimate of the total profitability of the e-vehicle at a given moment before relocation,
- compiling a list of profitable relocations,
- optimization of the relocation execution plan,
- estimation of the total profitability of the e-vehicle after relocation,
- creating of relocations plans.

In this chapter, the model will be discussed informal fleetly, leaving a description of the formalized model to other paper.

A. Station-based Algorithm

According to [9], the continuous division of the transport sharing service area into sectors (other studies referred to as stations) does not significantly affect optimization. Cluster analyzing the history of trips and knowing the specific characteristics of the area, Riga city was divided into sectors as can be seen in Fig. 1.

![Division of Riga, Latvia in sectors](www.openstreetmap.org/copyright)

However, division of territory into sectors must be carried out under several conditions. First, the sectors need to be relatively small to place a vehicle in the area for the client to reach it within “reasonable” time (the accumulated real data set shows that customers are ready to spend up to five minutes for reaching a vehicle). Secondly, the driving time between two adjacent sectors must be comparable. Thirdly, within one sector, customers’ behavior must be comparable, i.e., customers make trips from the respective sector uniformly frequently.

B. Forecasting User Trips

While sharing e-vehicle users in Riga city do not make all requests a day in advance, certain user trips can be scheduled with high possibility, using historical trip data. Using cluster analysis, “routine arcs” can be found: regular trips...
that consistently start and end the day from day to day in the same sectors, at approximately the same time.

C. Estimation of Sectors’ Income

The purpose of relocation is to place e-vehicle in areas where they are in demand and profits are expected accordingly. In sharing systems with booking in advance, a full estimate of demand and expected profit is known prior to the planning of relocation operations. But in our case, requests are made in real time. Therefore, to take tactical decisions on relocation operations, it is necessary to be able to carry out an alternative assessment to which stations to move the e-vehicles.

One potential solution that this study looks at is the modelling of expected income using historical data. The model of expected incomes describes the average expected benefits over a specific time period from an e-vehicle parked in a particular sector that can be rented by users. The expected income most probably will vary from station to station, as well as it will change over a day: In “peak hours” the expected income will be higher, in “quiet hours” less.

Modelling the expected incomes may not only provide tactical support in the planning of resettlement operations, but also give general impression on the behavior of sharing e-vehicle users. Comparing the expected income at different times, different weekdays, and different stations, it will be possible to draw conclusions that can also help you to make strategic, long-term decisions, such as handling different sectors or deploying charging stations.

D. Estimation of Expected Income Using Historical Data

From the history of e-vehicle rentals in different sectors, you can do an assessment of expected income in each sector for the next day. This data is taken from historical information about e-vehicle rentals.

![Expected profit for various sectors on Monday](image)

Fig. 2. Expected income for the sectors on Monday

Estimated forecasted incomes on Monday for the four sectors are summarized in Fig. 2. Data shows that there are significant differences between revenues for different sectors, as well as expected income changes by day. Moreover, some of the sectors can be very profitable only in specific time intervals and unprofitable in all others. For example, in the morning it is more convenient to move a car to sector D than to sector C, but after 1 PM a car in Sector C will be more profitable than a car parked in Sector D.

E. Weekly variations

There is a difference in estimated income for a particular sector between weekdays. From simple assumptions about the behavior of e-vehicle sharing users, there can be expected that demand for shared cars, so expected income, could vary significantly between business days and holidays. Indeed, such a phenomenon can be observed in the estimated expected income for the sector A, as shown in Fig. 3. Although there is a variation in the expected income between business days, there is a very significant difference in the expected income on holiday.

![Expected profit in “Sector A”](image)

Fig. 3. Expected income in “Sector A” in different days of week

As there is a significant but hard-to-predict difference between weekdays, it is necessary to calculate the expected earnings for each day of the week separately.

F. Variation in Historical Data

A related subject matter related to the modelling of forecasted income from historical data is how old historical data is effectively used. Since the number of shared cars is limited, a full history may be considered for usage to reduce the “noise” in the data gathered. However, as a counter argument for the use of historical data, there can be mentioned that user demand for shared cars is not static but is changing in the result of seasonal or other long-term processes. It is also concluded that, to keep the calculation of the intended income used up to date, the expected income should be recalculated on a regular basis.

G. Limitations of the Proposed Method

While the method of estimating the expected income provides a valuable numerical estimate of the cost of parking sharing e-vehicle in specific sectors, effective use of the described method must be aware of its shortcomings and limitations. For example, the method may provide inaccurate results if too many e-vehicles are placed in a particular sector, or the area of the sector is too small. Similarly, it is necessary to have a reliable history of bookings to estimate the expected income, and it may not be available when starting a sharing e-vehicle operation in a new region.
The following chapters describe two main shortcomings and limitations for estimating of the expected income.

**H. Linearity Assumption**

In the definition and calculation process described above, there is an assumption that the estimated income for a particular e-vehicle parked in a sector does not depend on the total number of parked vehicles in that sector. At a large number of e-vehicle parked in the same sector, you can see that this assumption is flawed; if the number of e-vehicle parked in the sector significantly exceeds the demand for shared e-vehicle in this sector, the average per e-vehicle income will be low.

Information collected from historical data (see Fig. 4) allows you to analyze the veracity of the statement described. The graph shows the average number of rented cars in the sector F over two hours, depending on the number of parked cars in this sector.

![Proportion of rented vehicles in "Sector F"](image)

The picture shows that the percentage of cars rented in this sector is almost constant at a small number of cars (55% of the cars located in the sector will be rented within two hours). Hence, the possibility of renting a particular car does not depend on the total number of cars in the sector at low number of available cars. And therefore, the expected per car income does not depend on the total number of parked cars in the sector.

The breakpoint in the sector F, when the described revenue model is ineffective, is around 14 cars. However, the typical number of cars in the sector F is lower, so the described model for calculating the expected income is an acceptable approximation. Similar data analysis and finding a breakpoint can be used to find a flexible demand limit for rapid responding to changes in demand for shared e-vehicle, or to price policies. The described expected income estimation algorithm will work most accurately if variations in the shared e-vehicle system are minimal.

Small sectors may lack data to adequately calculate expected income due to data noise. This phenomenon limits the lower size of sectors, thereby affecting the constructing of sectors.

**J. Car-sharing Income Optimization by E-vehicle Relocation**

Increasing income is possible by moving cars from low-income sectors to higher-potential sectors, but of course considering the costs of relocation. The location of e-vehicles at a given time determines the total value of all e-vehicles. This can be increased by moving the e-vehicle between sectors and finding the optimal location with the highest possible value.

The work of optimization is primarily inspired by the Mixed-Integer Programming (MIP) model proposed by C. Gambella, E. Malaguti, F. Masini, and D. Vigo for optimizing relocation operations in electric car-sharing [9]. The parameters discussed in the previous sections are passed to the optimization algorithm and it finds a new e-vehicle location by sectors that give the highest potential revenue, as well as a relocation plan to obtain this location.

In the case of many sectors and e-vehicles, the work of the algorithm may require significant computational resources/time. Therefore, it was assumed that the optimization algorithm was given a time limit during which the best of the considered variants is found, without guaranteeing to find the optimal solution.

As a result of the optimization algorithm execution, three reports are generated:
- a plan for relocation of e-vehicles between sectors,
- work plan for e-vehicle relocators,
- potential income change report.

**IV. RESEARCH FINDINGS**

The proposed solution uses historical data of shared e-vehicles — the intensity of trips across different sectors of the city, depending on season, day of week and clock time. This data can be obtained by recording events that the service provider information system can manage. The division of the city into sectors is also carried out using historical data, which in turn affects the permissible set of relocations and their costs. The model described is therefore applicable after the introduction of a shared transport service and the accumulation of historical data.

It should be acknowledged that e-vehicle rental services can be organized in many ways. The company offering services in Riga city, Latvia, provides a dynamic optimization of e-vehicle relocation approach. Obviously, the service capabilities determine the complexity of the model and its effectiveness. The rapid development of IoT will offer ever-new service capabilities that will require ever-new solutions.
This calls into question the need for a single, universal solution.

To estimate the potential income growth that can be achieved by moving e-vehicles between sectors, the relocation for a different number of e-vehicles and sectors was simulated (see Table 1). In the result of the simulation, there was concluded that e-vehicle relocation increases the total income by more than 15%.

Table 1.

<table>
<thead>
<tr>
<th>Number of vehicles</th>
<th>Number of relocators</th>
<th>Number of sectors</th>
<th>Number of relocation</th>
<th>Potential income Before</th>
<th>Potential income After</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>2</td>
<td>30</td>
<td>9</td>
<td>82.44</td>
<td>167.03</td>
</tr>
<tr>
<td>40</td>
<td>5</td>
<td>30</td>
<td>9</td>
<td>502.20</td>
<td>571.85</td>
</tr>
<tr>
<td>56</td>
<td>7</td>
<td>30</td>
<td>23</td>
<td>552.72</td>
<td>583.50</td>
</tr>
<tr>
<td>60</td>
<td>5</td>
<td>57</td>
<td>22</td>
<td>781.56</td>
<td>952.10</td>
</tr>
</tbody>
</table>

The potential income calculated in the table is an estimate of all cars in the sectors combined. The potential income is, of course, indicative and cannot be determined as an absolute value but its changes show the relative effect of relocations on the potential income.

V. CONCLUSIONS

The study offers a model for the use of shared e-vehicles, described as a commercial system with one-way trips and dynamic relocations of e-vehicles between city sectors, without pre-booked trips. The model consists of the following set of parameters: breakdown of the city in sectors, maximal number of available cars, number of cars per sector, set of possible relocations, parameters characterizing e-vehicle relocations, number of available relocators.

The parameters used in the model allow to describe the operation of a real system:

- The strategic level determines the number of e-vehicle available for sharing and the maximum number of e-vehicle to be placed in each urban sector.
- At the tactical level, historical shared vehicle data allows calculating the profitability of e-vehicle depending on the season, day of the week, usage time and city sector in which e-vehicle is placed.
- At the operational level, the total daily income is estimated as a sum of the expected average income over a specific period of time from all e-vehicle placed in a specific sector that can be rented by users.

The study provides an algorithm that optimizes expected income based on the set of selected relocations using the values of the above parameters. When implementing an algorithm, special attention should be paid to its performance as optimization must be performed dynamically, within few hours’ interval.

The vehicle sharing model proposed in the study is only one step towards an optimal solution. The model only partly describes real-life processes, such as e-vehicle battery capacity and technical parameters and prices. Similarly, the model does not consider cases where it is beneficial for several customers to use the same e-vehicle when a route is agreed. In addition, the work does not analyze the risks posed by concurrent usage of shared e-vehicle. Such a study has been conducted for e-commerce [10] and e-scooters [11]. These issues may be the content of further studies.

REFERENCES