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Multi-Criteria Decision-Making by Approximation in the Domain of Linguistic Values

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***Abstract*—This paper presents a method of determining the set of alternatives, with respect to a subset of fuzzy criteria, that have the greatest degree of accordance with preferences given by a decision-maker. We apply two kinds of alternatives in the process of decision-making. The candidate solutions will be selected from a (large) universe of (real) alternatives. Their membership degrees in the linguistic values of all fuzzy criteria is assigned by an expert. A set of (imaginary) reference alternatives is generated to express the expectations of the decision-maker, who assigns membership degrees in the most preferred linguistic values of fuzzy criteria. We define the notion of approximation of a reference alternative by a real alternative in the domain of linguistic values of criteria, and introduce a measure of accordance of the real alternative with the reference alternative.**

I. Introduction

MODELLING and analysis of the decision-making activity preformed by humans strongly depends on the quality of data obtained from the real-world reasoning processes. The rough set theory [1] is a paradigm aimed at discovering uncertainty, inconsistency, and redundancy that can be often found in information systems. It could be successfully combined with different ways of knowledge representation in the form of hybrid approaches, such as fuzzy-rough decision models [2]–[6]. Both the rough set models and the fuzzy knowledge representation should be utilized in attempts to create systems that could be helpful in finding an optimal alternative in complex situations. It is especially important in the case of huge information systems, with a vast number of alternatives that are characterized by many criteria.

Even a skilled expert can hardly manage a difficult decision-making task, not only due to the size of the problem, but also because of natural contradictions between various criteria. Furthermore, when a group of experts is involved in finding an optimal decision, we can expect solutions that may be inconsistent. It is obvious that the human experts need to be supported in solving both the multi-criteria and the group decision-making tasks. The most popular algorithms applied to this end are SAW, TOPSIS, and AHP [7], [8].

In order to model actual decision-making processes, it is also necessary to admit subjectivity in goal functions and constrains. In a real-world situation, one cannot be restricted to objective (numerical) criteria only, such as cost or benefit, but has also to take into account subjective (vague) criteria. Therefore, we can observe that the standard (crisp-oriented) methods were adopted to work with fuzzy numbers and linguistic values in the multi-criteria [9]–[11], and in the group decision-making [12]–[15].

Motivation for our activity in the area of decision-making was not merely extending the standard approaches to deal with fuzzy instead of crisp values, but rather to propose directly a fuzzy-oriented knowledge representation, which is based on the notion of linguistic label [16]–[18]. This idea changes the way of comparing and classifying objects in a fuzzy information system. It is more convenient and computationally not demanding to simply discover classes of objects that share a common characteristic by having the same dominant linguistic values of attributes. We do not apply a standard fuzzy similarity relation for making a detailed analysis of similarity between particular objects of a fuzzy universe. This way we also avoid getting obscured results that depend on the used forms of fuzzy connectives and may be difficult to interpret.

In this paper, we introduce several new concepts into the label-based approach to multi-criteria decision-making. Firstly, we express the preferences of a decision-maker in the form of imaginary alternatives that constitute a reference for comparison and evaluation of real alternatives. Secondly, we define a notion of approximation of the reference alternatives by the real alternatives. This makes it possible to simplify the evaluation of accordance of the real alternatives with the preferences of the decision-maker.

II. Classification of alternatives using fuzzy linguistic labels

It is necessary to recall the basic notions that are used in the presented approach. The process of decision-making is performed by evaluating the alternatives that are characterized by fuzzy criteria. We denote by [16]:

U – a nonempty set (universe) of alternatives (elements),
A – a finite set of fuzzy criteria (attributes),
$\mathbb{V}_a$ – a set of linguistic values of every criterion $a \in A$,
$\mathbb{V}$ – a set of linguistic values of criteria, $\mathbb{V} = \bigcup_{a \in A} \mathbb{V}_a$,
f – an information function, $f : U \times \mathbb{V} \to [0, 1]$,
$f(u, V) \in [0, 1]$, for all $u \in U$, and $V \in \mathbb{V}$.

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The corresponding family of linguistic values of a fuzzy criterion $a_i \in A$, where $i = 1, \ldots, n$, is expressed as $\mathbb{A}_i = \{A_{i1}, \ldots, A_{in_i}\}$. The membership degrees of any alternative $u \in U$ in the linguistic values of all fuzzy criteria should be assigned under the following conditions [18]:

$$\exists A_{ik} \in \mathbb{A}_i \quad (\mu_{A_{ik}}(u) \geq 0.5, \\ \mu_{A_{ik-1}}(u) = 1 - \mu_{A_{ik}}(u) \ \vee \\ \mu_{A_{ik+1}}(u) = 1 - \mu_{A_{ik}}(u)), \tag{1}$$

$$\text{power}\,(\mathbb{A}_i(u)) = \sum_{k=1}^{n_i} \mu_{A_{ik}}(u) = 1. \tag{2}$$

Because of the requirements (1) and (2), every alternative $u \in U$ must have a dominant linguistic value for each fuzzy criterion. Furthermore, due to the requirement (1), every alternative $u \in U$ can take a nonzero membership degree in at most two neighbouring linguistic values.

Introducing the requirement (1) was inspired by practical applications of fuzzy inference systems. It can be observed in a real-world case that an expert usually assigns a nonzero membership degree in only two neighbouring linguistic values of a criterion. This is in accordance with our intuition. We can describe, e.g., an object to be "high" and "middle" to some degree, but not "low" at the same time. Hence, the parameters of membership functions that are applied in fuzzy control systems should be set in such a way that only two neighbouring membership functions can be activated by any crisp input value [19], [20].

The requirement (2) can be perceived as a generalization of the property that can be observed in crisp information systems that constitute a special case of fuzzy information systems. Every element of a crisp universe can have a nonzero membership, which equal to 1, in only one single value of a selected crisp attribute. All the membership degrees in the remaining values of the crisp attribute must be equal to zero. This property of possessing exactly one attribute value can be also expressed by applying the law of excluded middle, which is a crucial principle of the standard bi-valued logic. In a fuzzy information system, an element of the universe can possess a membership degree, which can be any real number in the interval $[0, 1]$, in more than one attribute value. The process of assigning the degree of membership in fuzzy sets is a fundamental issue in applications of the fuzzy set theory. Some researches do not impose strict constraints in this regard. This can be observed especially in the area of neuro-fuzzy systems where the membership degrees are often tuned freely during a training process. Unfortunately, we lose the correspondence to logic in such an arithmetic-oriented approach. Therefore, a well-defined fuzzy information system should satisfy a requirement that can be seen as a counterpart of the law of excluded middle in fuzzy logic.

The requirement (2) is important in real-world applications, e.g., in fuzzy control [19], because it helps to construct a consistent system of fuzzy decision rules that can be easily interpreted. Another example is the fuzzy flow graph approach introduced in [21]. It that case, the requirement (2) has to be be used if we want to retain the flow conservation equations that describe the flow distribution in a fuzzy flow graph.

Both the requirements (1) and (2) allow to significantly simplify the process of constructing a fuzzy information system. Obviously, the formulae (1) and (2) are fully compatible with the standard bi-valued logic in a special case of crisp information systems.

We also require a certain level of membership degree to the dominant linguistic values, which can further restrict the subset of alternatives taken into consideration. Such (positive) alternatives can be easily discovered by finding the unique membership degrees, for all criteria $a \in A$ that exceed a threshold of similarity, denoted by β, that satisfies the inequality: $0.5 < \beta \leq 1$.

Depending on the value of membership degree, and the threshold β, three kinds of linguistic values [17] can be distinguished. For every alternative $u \in U$, and any attribute $a \in A$, we define the set $\widehat{\mathbb{V}}_a(u) \subseteq \mathbb{V}_a$ of positive linguistic values

$$\widehat{\mathbb{V}}_a(u) = \{V \in \mathbb{V}_a : f(u, V) \geq \beta\}, \tag{3}$$

the set $\overline{\overline{\mathbb{V}}}_a(u) \subseteq \mathbb{V}_a$ of boundary linguistic values

$$\overline{\overline{\mathbb{V}}}_a(u) = \{V \in \mathbb{V}_a : 0.5 \leq f(u, V) < \beta\}, \tag{4}$$

and the set $\check{\mathbb{V}}_a(u) \subseteq \mathbb{V}_a$ of negative linguistic values

$$\check{\mathbb{V}}_a(u) = \{V \in \mathbb{V}_a : 0 \leq f(u, V) < 0.5\}. \tag{5}$$

We can identify alternatives $u \in U$ that have nonempty sets $\widehat{\mathbb{V}}_a(u)$ for all criteria $a \in A$, according to (3). Those alternatives are marked by distinct labels, which are combinations of their positive linguistic values of criteria.

The set of linguistic labels $\widehat{\mathbb{L}}(u)$ is expressed as the Cartesian product of the sets of positive linguistic values $\widehat{\mathbb{V}}_a(u)$, for all $a \in A$:

$$\widehat{\mathbb{L}}(u) = \prod_{a \in A} \widehat{\mathbb{V}}_a(u). \tag{6}$$

By inspecting the membership degrees of every element u of the universe U in all linguistic values of criteria, we obtain classes (granules) of similar alternatives that share the same linguistic label.

We denote by U_L the subset of those elements u of the universe U that correspond to a linguistic label $L \in \mathbb{L}$, for all fuzzy attributes $a \in A$:

$$U_L = \{u \in U : L(u) = L\}. \tag{7}$$

The subset U_L is called the set of characteristic elements of the linguistic label L.

The linguistic label $L \in \mathbb{L}$ can be expressed as an ordered tuple of positive linguistic values, for all attributes $a \in A$:

$$L = \left(\hat{V}_{a_1}^L, \ldots, \hat{V}_{a_n}^L\right). \tag{8}$$

In the proposed novel approach presented in this paper, we denote by X the universe of real alternatives having the membership degrees assigned by an expert. The alternatives $x \in X$

will be evaluated and ranked with respect to the subjective preferences provided by a decision-maker. In contrast to our previous work, we propose a different way of specifying the preferences of the decision-maker. This is done by introducing ideal (imaginary) reference alternatives that should reflect the expectations of the decision-maker, who must assign their membership degrees in the preferred linguistic values of each fuzzy criterion. The reference alternatives will be denoted by y, and their universe by Y.

We would usually expect the universe Y of imaginary alternatives to contain only one or a small number of elements, because the decision-maker ought to strictly specify his or her requirements. However, in a real-world case, the criteria should not be treated as totaly independent characteristics of alternatives. Rather, we must assume that the decision-maker considers several (possible) versions of ideal alternatives having different combinations of preferred linguistic values of criteria. Moreover, such imaginary alternatives can be treated as solution variants that are not equally desirable.

III. Approximation of alternatives in the domain of linguistic values

In the first step, we determine the set of linguistic labels $\mathbb{L}^E$ of the real alternatives $x \in X$ described by the expert, and the set $\mathbb{L}^D$ of linguistic labels of the reference alternatives $y \in Y$ provided by the decision-maker.

The preferences of the decision-maker can be satisfied only, if we can find linguistic labels that are shared by the expert and the decision-maker, i.e., $\mathbb{L}^E \cap \mathbb{L}^D \neq \emptyset$. Only the real alternatives belonging to the characteristic sets of the common linguistic labels constitute variants of acceptable solutions. All the other real alternatives are not in accordance with the preferences of the decision-maker. In other words, they do not support linguistic labels of the decision-maker, hence, they will be discarded from further consideration.

In the next step, we need to calculate the degree of accordance of the real alternatives with the corresponding reference alternative. We refer to the idea of approximation of sets, which is a fundamental concept of the rough set theory. The notions of the lower and upper crisp set approximations, proposed by Pawlak [1], were extended by many researchers, who developed various generalizations for the case of fuzzy information systems, e.g., [2]–[6].

In approximation of crisp sets a unique indiscernibility relation is used, whereas fuzzy sets can be approximated with the help of a similarity relation. Contrary to the crisp indiscernibility relation, there is no unique way how a fuzzy similarity relation is defined. However, it could be shown [4] that the fuzzy approximations based on the (residual) R-implicators satisfy the largest number of basic properties of rough sets. This fact inspired us to consider R-implicators as a suitable tool for approximation of alternatives in the domain of linguistic values.

Let us define the accordance relation of a real alternative with a reference alternative, with respect to particular linguistic values of criteria. To this end, we represent any alternative x as the fuzzy set $\widetilde{X}$, and any alternative y as the fuzzy set $\widetilde{Y}$ on the domain of linguistic values of criteria. Now, we are able to determine the covering of the set $\widetilde{Y}$ by the set $\widetilde{X}$. The covering will be expressed as a fuzzy set in the domain of linguistic values of criteria, and denoted by $\mathrm{COV}(\widetilde{X}, \widetilde{Y})$.

For a linguistic value A_{ik} of a criterion a_i, we require that the membership degree in A_{ik} of the real alternative x must be at least equal to the membership degree in A_{ik} of the reference alternative y, as a condition to assign the highest possible covering degree (with respect to A_{ik}) of $\widetilde{Y}$ by $\widetilde{X}$:

$$\mu_{\mathrm{COV}(\widetilde{X},\widetilde{Y})}(A_{ik}) = 1 \iff \mu_{\widetilde{Y}}(A_{ik}) \leq \mu_{\widetilde{X}}(A_{ik}). \qquad (9)$$

This requirement can be satisfied, when the covering of fuzzy sets is defined by applying residual implicators, which are generally expressed with the help of a t-norm operator T, for $a, b \in [0, 1]$:

$$\mathrm{I}(a, b) = \sup\{\lambda \in [0, 1] : \ \mathrm{T}(a, \lambda) \leq b\}. \qquad (10)$$

The R-implicator of Gaines turned out to be the most suitable in our approach, because it takes into account the ratio between its arguments. It has the following form:

$$\mathrm{I}(a, b) = \begin{cases} 1, & \text{if } a \leq b, \\ b/a, & \text{otherwise.} \end{cases} \qquad (11)$$

By applying the selected R-implicator, we define the covering set $\mathrm{COV}(\widetilde{X}, \widetilde{Y})$ of a nonempty set $\widetilde{Y}$ by the set $\widetilde{X}$ as follows:

$$\mu_{\mathrm{COV}(\widetilde{X},\widetilde{Y})}(A_{ik}) = \begin{cases} \mathrm{I}(\mu_{\widetilde{Y}}(A_{ik}), \mu_{\widetilde{X}}(A_{ik})), \\ \text{if } \mu_{\widetilde{X}}(A_{ik}) > 0, \\ 0, \ \text{otherwise.} \end{cases} \qquad (12)$$

On the other hand, we should take also into account the preferences for neighbouring linguistic values of criteria that are specified by the decision-maker. Even in the case of a full covering with respect to a dominant linguistic value, the decision-maker and the expert can assign a nonzero membership of alternatives in different neighbouring linguistic values of a criterion. Therefore, we apply the membership degrees of the reference alternative in the linguistic values of criteria as weighting factors to precisely determine the accordance between the reference and the real alternative. Summarizing, the accordance degree $acc_{A_{ik}}(x, y)$ of a real alternative $x \in X$ with a reference alternative $y \in Y$, with respect to a linguistic value A_{ik} of a criterion $a_i \in A$, is defined as

$$acc_{A_{ik}}(x, y) = \mu_{A_{ik}}(y) \times \mu_{\mathrm{COV}(\widetilde{X},\widetilde{Y})}(A_{ik}). \qquad (13)$$

By adding up the accordance degrees $acc_{A_{ik}}(x, y)$ calculated for all linguistic values of a criterion $a_i \in A$, we get the accordance degree $acc_{a_i}(x, y)$ of the alternative x with the alternative y, with respect to a_i:

$$acc_{a_i}(x, y) = \sum_{k=1}^{n_i} acc_{A_{ik}}(x, y). \qquad (14)$$

The approximation $\mathrm{ACC}_x(y)$ of the reference alternative $y \in Y$ by the real alternative $x \in X$ is a fuzzy set in the domain of linguistic values of criteria:

$$\mathrm{ACC}_x(y) = \{\, acc_{A_{11}}(x,y)/A_{11}, \ldots, acc_{A_{ik}}(x,y)/A_{ik}, \ldots, acc_{A_{nn_n}}(x,y)/A_{nn_n}\}. \tag{15}$$

One has to remember that the criteria are not equally important in a typical multi-criteria optimization task. This restriction can be formally expressed with the help of weights $w_1, \ldots, w_n$ that can take values from the interval $[0,1]$, and must add up to 1: $\sum_{i=1}^{n} w_i = 1$. The values of the weights for particular criteria can be freely changed, depending on the choice of the decision-maker.

Finally, we define the measure of acceptance of a real alternative x as follows:

$$acc(x,y) = \sum_{i=1}^{n} w_i \times acc_{a_i}(x,y). \tag{16}$$

The final ranking of alternatives can obtained basing on the value of the acceptance measure, but it should be performed separately for each reference alternative of the decision-maker.

IV. Example

In order to illustrate the presented approach, let us assume a fuzzy information system that contains 10 real alternatives characterized by three fuzzy criteria a_1, a_2, and a_3. All criteria can take three linguistic values and have the same importance, i.e., the weights w_1, w_2, and w_3 are set to $\frac{1}{3}$. The similarity threshold β is equal to 0.6.

For the reference alternatives y_1, y_2, and y_3 provided by the decision-maker (Table II), we get the linguistic labels L_1, L_2, and L_3, respectively. By inspecting the universe X containing the real alternatives, we can find the same linguistic labels and their corresponding sets of characteristic elements X_{L_1}, X_{L_2}, X_{L_3}, respectively:

$$\begin{aligned} L_1 &= (A_{13}A_{22}A_{32}): \quad X_{L_1} = \{x_3, x_{10}\}, \\ L_2 &= (A_{12}A_{23}A_{32}): \quad X_{L_2} = \{x_2, x_6, x_9\}, \\ L_3 &= (A_{13}A_{23}A_{33}): \quad X_{L_3} = \{x_5, x_8\}. \end{aligned}$$

The alternative x_4 has no linguistic label, because all its membership degrees in the linguistic values of the criterion a_2 are below the similarity threshold β. The alternatives x_1, and x_7 do not support the reference labels of the decision-maker. Hence, the alternatives x_1, x_4, and x_7 will be discarded from the solution space.

To demonstrate the details of determining the approximation of the reference alternatives, we select the fuzzy sets $\widetilde{Y}_1(a_1)$, and $\widetilde{X}_3(a_1)$ that represent the alternatives y_1, and x_3, respectively, in the domain of linguistic values of the criterion a_1:

$$\begin{aligned} \widetilde{Y}_1(a_1) &= \{0.00/A_{11}, 0.30/A_{12}, 0.70/A_{13}\}, \\ \widetilde{X}_3(a_1) &= \{0.00/A_{11}, 0.20/A_{12}, 0.80/A_{13}\}. \end{aligned}$$

By applying the formulae 12, 13, and 14, we get the results given in Table III, including the covering set $\mathrm{COV}(\widetilde{X_3}, \widetilde{Y_1})$, the approximation of the reference alternative y_1 by the real alternative x_3, and the accordance degrees with respect to the criteria a_1, a_2, and a_3.

Tables IV, V, VI contain the accordance degrees of the supporting real alternatives with the reference alternatives y_1, y_2, and y_3, respectively. As we can see, the acceptance degrees of the supporting real alternatives are relatively high. However, those three groups of alternatives should be seen as distinct solution variants with separate rankings, and may have different meaning or importance for the decision-maker.

V. Conclusions

Finding an optimal solution by evaluating and ranking the alternatives that are characterized by subjective criteria can be done by utilizing the notion of fuzzy linguistic label. In this paper, we propose to express the subjective preferences of a decision-maker as ideal reference alternatives. They can be represented by respective linguistic labels, in the same way like the real alternatives that constitute the solutions space. By unifying the description in the multi-criteria decision-making process, we are able to define a notion of approximation in the domain of linguistic values of criteria, and to use it for introducing an accordance relation for comparing both kinds of alternatives. The presented method can be a starting point to develop novel hybrid approaches to solving complex multi-criteria decision-making tasks, with respect to mixed subjective and objective criteria.

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TABLE I
REAL ALTERNATIVES WITH MEMBERSHIP DEGREES ASSIGNED BY THE EXPERT

	a_1			a_2			a_3		
	A_{11}	A_{12}	A_{13}	A_{21}	A_{22}	A_{23}	A_{31}	A_{32}	A_{33}
x_1	0.00	**0.75**	0.25	0.00	0.20	**0.80**	**0.70**	0.30	0.00
x_2	0.00	**0.65**	0.35	0.00	0.30	**0.70**	0.00	**0.75**	0.25
x_3	0.00	0.20	**0.80**	0.00	**0.80**	0.20	0.00	**0.80**	0.20
x_4	**0.80**	0.20	0.00	**0.50**	**0.50**	0.00	0.00	0.30	**0.70**
x_5	0.00	0.25	**0.75**	0.00	0.35	**0.65**	0.00	0.40	**0.60**
x_6	0.15	**0.85**	0.00	0.00	0.25	**0.75**	0.25	**0.75**	0.00
x_7	**0.55**	0.45	0.00	**0.70**	0.30	0.00	**0.60**	0.40	0.00
x_8	0.00	0.35	**0.65**	0.00	0.25	**0.75**	0.00	0.10	**0.90**
x_9	0.00	**0.80**	0.20	0.00	0.15	**0.85**	0.00	**0.65**	0.35
x_{10}	0.00	0.25	**0.75**	0.30	**0.70**	0.00	0.00	**0.75**	0.25

TABLE II
REFERENCE ALTERNATIVES OF THE DECISION-MAKER

	a_1			a_2			a_3		
	A_{11}	A_{12}	A_{13}	A_{21}	A_{22}	A_{23}	A_{31}	A_{32}	A_{33}
y_1	0.00	0.30	**0.70**	0.00	**0.75**	0.25	0.00	**0.85**	0.15
y_2	0.00	**0.80**	0.20	0.00	0.30	**0.70**	0.00	**0.70**	0.30
y_3	0.00	0.30	**0.70**	0.00	0.25	**0.75**	0.00	0.20	**0.80**

TABLE III
ACCORDANCE OF x_3 WITH y_1

	a_1			a_2			a_3		
	A_{11}	A_{12}	A_{13}	A_{21}	A_{22}	A_{23}	A_{31}	A_{32}	A_{33}
x_3	0.00	0.20	0.80	0.00	0.80	0.20	0.00	0.80	0.20
y_1	0.00	0.30	0.70	0.00	0.75	0.25	0.00	0.85	0.15
$\mathrm{COV}(\widetilde{X_3}, \widetilde{Y_1})$	0.00	0.67	1.00	0.00	1.00	0.80	0.00	0.94	1.00
$\mathrm{ACC}_{x_3}(y)$	0.00	0.20	0.70	0.00	0.75	0.20	0.00	0.80	0.15
$acc_{a_i}(x_3, y_1)$		0.90			0.95			0.95	

TABLE IV
ACCEPTANCE DEGREE OF THE ALTERNATIVES OF X_{L_1} WITH y_1

	a_1	a_2	a_3	$acc(x_1, y_1)$	Position
x_3	0.90	0.95	0.95	0.933	1
x_{10}	0.95	0.70	0.90	0.851	2

TABLE V
ACCEPTANCE DEGREE OF THE ALTERNATIVES OF X_{L_2} WITH y_2

	a_1	a_2	a_3	$acc(x_2, y_2)$	Position
x_2	0.85	1.00	0.95	0.9325	2
x_6	0.80	0.95	0.70	0.8165	3
x_9	1.00	0.85	0.95	0.9340	1

TABLE VI
ACCEPTANCE DEGREE OF THE ALTERNATIVES OF X_{L_3} WITH y_3

	a_1	a_2	a_3	$acc(x_3, y_3)$	Position
x_5	0.95	0.90	0.80	0.884	2
x_8	0.95	1.00	0.90	0.950	1

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