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Perception of vector and triangle representations of fuzzy number most possible value changes

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Abstract— **The aim of the study is to investigate and evaluate user preferences regarding two visual representations of uncertainty estimates for decision-making purposes. The research is concerned with the perception of fuzzy numbers, which are depicted either as triangles or as specifically constructed vectors. The study involves a series of pairwise comparisons in which participants must determine which representation reflects the change in the most possible value in a more salient way. The results are then analyzed and formally verified statistically. The study shows that there are specific circumstances where vector representations are more desirable than their triangle-based counterparts. The findings also suggest that there may be some differences in assessing these representations depending on gender. This examination expands our understanding of how subjects perceive different graphical methods for presenting change in a selected parameter uncertainty feature. From a practical standpoint, the findings offer suggestions for designing graphical user interfaces that present fuzzy data to users.**

***Index Terms*—Fuzzy number visualization, Fuzzy number vector representation, Visual processing, Project uncertainty, Usability.**

I. Introduction

UNCERTAINTY is a pervasive issue that must be addressed in numerous fields both those typically associated with precision such as physics or engineering, as well as areas where rather soft computing is prevalent, e.g., management, economics, and other social sciences. In particular, uncertainty needs to be handled while analyzing risk, building various models and making decisions [1]–[3], and not taking into account uncertainty can lead to often negative or even catastrophic results such as project failures [4], [5]. It is, thus, indispensable to control them and to analyse their behaviour on an ongoing basis.

The efficiency and effectiveness of managing uncertainty and solving related problems, for instance, in production, or project management can strongly depend how it is presented to persons analyzing the data. The importance of data visualization in the process of decision making has been stressed in many papers, for instance, showing its substantial positive influence on communication process [6]. There are also a number of various ways of graphically presenting uncertainty and they have been described and reviewed in numerous scientific publications (e.g., [7]–[10]).

In our previous papers [11], [12], we proposed the use of vectors to graphically visualize uncertainty defined by triangular fuzzy numbers. We have initially compared the usability and potential applications of triangle- and vectors-based representations of triangular fuzzy numbers. By performing experimental tasks, we found out that their capability to convey information may not be worse than that of triangles and is seen as even better by some of the users. This type of relatively simple fuzzy number requires to provide only three, point-type estimates and is commonly used in practice. For example, in project management, uncertainty of a time or cost estimate, encoded by a triangular fuzzy number, is fully defined by three parameters: the optimistic, the most possible, and the pessimistic value [13], [14]. This approach closely resembles the well-known PERT approach [15]–[17] that, albeit based on probability theory, also requires similar three parameters. In the PERT approach the beta distribution [18] is used with the corresponding *arithmetic* of random variables, while with triangular fuzzy numbers, the representation of the estimates and the mathematical operations are more straightforward and intuitional. Although in the literature one can find phrases "*fuzzy vector*", however it should be emphasized that these papers refer usually to fuzzy vectors that are different in nature from the current paper definition – compare, for example, the following works [19]–[23].

In the current study, we investigate the uncertainty visualizations as the traditional graphs of membership functions, and our own proposals of depicting them as vectors. Since the previous results have shown that the effectiveness and efficiency of both approaches are comparable, we decided to pursue this subject in more detail. Here, we focus particularly on the individuals' perception of the saliency of changes in the most possible value depicted both as triangles and vectors.

The focus on the most possible values results from the fact that in many cases the most possible values are taken as the basis for current decision making such as setting deadlines in

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project planning. Therefore, the changes in the most possible values should be carefully controlled and their most suitable graphical representation is of great importance here. Moreover, monitoring variability in various aspects seems to significantly increase the chances for achieving project management success [24]. To extend our understanding how individuals perceive these changes portrayed in a different visual form, we designed and performed an experiment. The obtained outcomes are analysed and discussed in this paper. Since previous results [11] suggest that there can be some differences between men and women while assessing triangle- and vector-based representations, the gender effect was also included into our study.

The outline of the paper is as follows: In Section II, we present basic information about the triangular and vector representations of triangular fuzzy numbers. Section III includes the all the details about the experimental design and study subjects' characteristics which is followed by the results analysis section. The paper ends with a discussion and conclusions.

II. Membership functions versus vectors – two uncertainty visualization approaches

Let us suppose that a project cost or time item has been estimated by experts in the form of three values $\underline{r}, \hat{r}, \overline{r}$ – the optimistic, most possible and pessimistic values, respectively. The respective triangular fuzzy number will be denoted as $\tilde{R} = (\underline{r}, \hat{r}, \overline{r})$. Its membership function $\mu_R(x)$ is defined on the set $\mathfrak{R}$ of real numbers and represents the possibility degrees of the respective real numbers. An example of $\tilde{R}$ (2,3,5) is shown in Fig.1. The alternative representation, put forward and examined in our previous study [11], [25], is based on vectors (Fig. 1). The vector $\vec{R}(\underline{r}, \hat{r}, \overline{r}) = \{m_R, s_R, \gamma_R\}$ will be defined by its starting point $(m_R, 0)$, where $m_R = (\overline{r} + \underline{r})/2$, its length $s_R = \overline{r} - \underline{r}$, the angle in relation to the line $x = m_R$ computed as $\gamma_R = arc\,tan(\hat{r} - m_R)$; positive angles denote the inclination to the right and negative ones – to the left.

It is important to notice that the angle γ_R will be zero only if the most possible value $\hat{r}$ is equal to m_R, the arithmetic mean of the pessimistic and optimistic values $\overline{r}$ and $\underline{r}$. These two representations described above are investigated in an experiment described in following sections.

A. Study Subjects

In total, 88 individuals participated in the survey. However, five persons (four females and one male) were excluded from further analysis due to incomplete data.

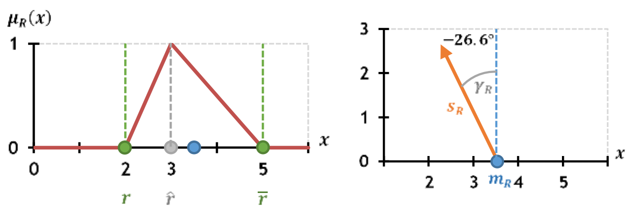

Fig. 1. Membership function based visualisation of parameter *R* determined by numbers 2, 3, 5 and its vector visualization.

III. Method

Thus, all the presented in this study results refer to 83 participants. They were primarily volunteer students aged between 18 and 46 years old from Wroclaw University of Science and Technology in Poland. The mean age was 23.42 years, with a standard deviation of 3.45. The group was highly homogenous in this regard, as the 25th age centile amounted to 23 and 75th – 24 years. Out of the participants, the majority were women, specifically 61 individuals, making up 73.5% of the total. All of them provided their informed consent to participate in the study.

A. Experimental Design and Task Description

The experimental design aimed to gain a deeper understanding of how individuals perceive changes in fuzzy number most possible value in both triangular and vector visualizations. Participants' task was to give subjective assessment of different variants of these representations in terms of their saliency of fuzzy number feature change. In particular, the effects described in the next subsection were examined.

Examined factors

The present study investigated two graphical representations of triangular fuzzy numbers: traditional triangle and vector-based visualizations. They were the first factor examined in the study, and their mathematical properties were concisely explained in Section II. The study specifically focused on how participants perceived changes in a single property related to fuzzy numbers: the information about the most possible value of the imprecise parameter in question. Graphically, this feature is associated with either the position of the maximum of the triangular fuzzy number membership function or the inclination angle of the vector. The study explored three distinct levels of change in the most possible value, which included (i) a change of two units from zero to two, (ii) a change of four units from zero to four, and (iii) a change of two units from two to four. The factors investigated and their corresponding levels are depicted in Fig. 2.

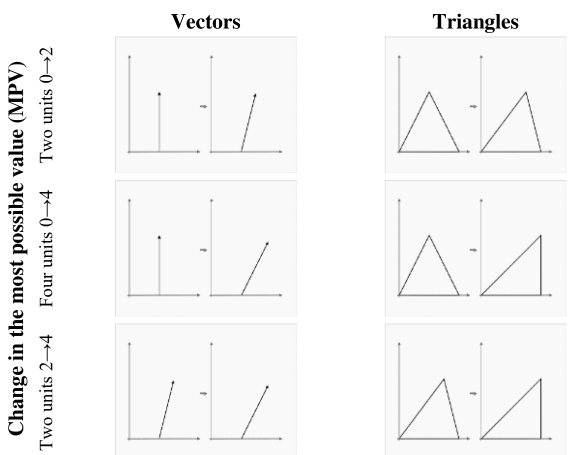

Fig. 2. Factors and their levels examined in the current study: visual representation (vectors, triangles), the most possible value change (MPV: Two units 0→2, Four units 0→4, Two units 2→4).

Dependent measures

We utilized two dependent measures to assess the subjective opinions of the respondents. To assess preferences, we obtained relative weights through pairwise comparisons of all experimental conditions. Pairwise comparisons have been demonstrated to enhance the accuracy of evaluations [26], [27] and have been successfully implemented in numerous studies for establishing hierarchies of preferences see, e.g., [28]–[32]). In this study, we utilized this approach within the Analytic Hierarchy Process (AHP) framework [33] to determine stimulus subjective perceptions and calculate consistency ratios for each participant.

To determine which figure showed a more noticeable increase in the most possible value of the fuzzy number, participants were asked to provide responses on a 5-point, two-directional linguistic scale recommended in the AHP approach.

By combining different levels of the two factors examined, we were able to identify six distinct experimental conditions. These conditions were generated by varying two types of graphical representations of triangular fuzzy numbers and three levels of indeterminacy changes (as shown in Fig. 2). To test all six experimental conditions, we applied a within-subject design where each subject participated in every condition.

B. Experimental Procedure

The data collection process for the study was conducted entirely over the internet. Participants were provided with general information about the research and a hyperlink to a slideshow containing a detailed audio explanation of the research. The final slide contained a hyperlink to the experimental application based on React.js, which opened in their default web browser upon clicking. After opening the software, the first page presented participants with the informed consent form, which they were required to read and accept before starting the examination. After providing their gender and age, the software displayed all necessary pairwise comparisons of the experimental conditions one by one, in a random order. During this process, the data were collected locally in the web browser's internal variables and were subsequently sent to a remote server after the completion of the entire procedure.

IV. Results

A. Descriptive statistics

The experimental data that was gathered was brought together and transferred to TIBCO Statistica version 13.3 software. The analysis included both consistency ratios and relative weights and was carried out taking advantage of typical descriptive statistics and analysis of variance. The outcomes of this examination are exhibited in the following sub-sections.

Consistency ratios

The consistency ratios of all the examined individuals were equaled, on average, 0.381 with a standard deviation amounting to 0.296. The median value of 0.263 was much lower than the mean, which suggest that the distribution was positively skewed. The consistency ratio ranged from a minimum value of 0.0421 to a maximum of 1.42.

Relative weights for studied stimuli

Table I displays the key descriptive statistics for the relative weights computed for all the stimuli that were investigated.

The greater the relative weight values, the stronger the perception of saliency of the change in the fuzzy number most possible value in a particular experimental condition. Pictures illustrating the change by four units exhibit the highest mean and median values, signifying the most salient perception of the changes. This observation was consistent for both triangle and vector representations.

In all cases, the median value was slightly smaller than the mean value, which indicates a somewhat positively skewed distribution. Furthermore, these experimental conditions had the highest variability, as evidenced by the larger standard deviations and mean standard errors.

B. Analysis of variance

The analysis of variance technique was employed to formally verify if the observed differences in average values were statistically significant. We have conducted this method to both consistency rations and relative weights. The results of these two analyses of variance are presented in the following subsections.

Consistency ratios

There were differences in the CR mean values for men and women with males being on average more consistent (0.356) than females (0.390). However due to the considerable standard deviations, the discrepancy occurred to not be meaningful, which was supported by performing one-way analysis of variance. Its results for gender differences in consistency ratios showed statistical insignificance at the level of $p = 0.65$ [$F(1, 81) = 0.21$].

Relative weights for studied stimuli

To determine the statistical significance and extent of the differences in the mean relative weight values for the studied

TABLE I.
KEY DESCRIPTIVE STATISTICS OF RELATIVE WEIGHTS FOR ALL EXPERIMENTAL CONDITIONS

Graphic Representation	Change Type	Mean	Median	Min	Max	Std Dev	Mean Std Error
Triangle	CT_0→2	0.101	0.082	0.024	0.523	0.072	0.0080
	CT_0→4	0.327	0.323	0.032	0.593	0.149	0.0163
	CT_2→4	0.128	0.105	0.019	0.348	0.090	0.0099
Vector	CT_0→2	0.122	0.098	0.021	0.423	0.095	0.0105
	CT_0→4	0.247	0.234	0.051	0.543	0.125	0.0138
	CT_2→4	0.075	0.058	0.013	0.283	0.058	0.0064
All		0.167	0.115	0.013	0.593	0.137	0.0061

effects, we performed a three-way analysis of variance, specifically analyzing the *Change Type* and *Graphical Representation* factors. We have also included the *Gender* effect, since our previous study [11] suggests that this may differentiate the results regarding the investigated stimuli. The results show that two of the three factors investigated were statistically significant. Specifically, the factors of *Change Type* and *Graphical Representation* had significant effects with: $F(2, 486) = 154.7$, $p < 0.0001$, and $F(1, 486) = 5.24$, $p = 0.0225$, respectively. The gender effect alone was statistically irrelevant, but its interactions with both *Graphical Representation* and *Change Type* were meaningful. The former one (*Graphical Representation* × *Gender*) at the level of $\alpha < 0.05$ [$F(2, 486) = 7.78$, $p = 0.0055$], whereas the latter *Change Type* × *Gender* at the level of $\alpha < 0.1$ [$F(2, 486) = 2.87$, $p = 0.0578$]. Additionally, the interaction between *Change Type* and *Graphical Representation* was statistically significant at the level of $\alpha < 0.05$ [$F(2, 486) = 7.13$, $p = 0.0009$].

Fig. 3 presents a visual representation of the mean relative weight values for the *Change Type* effect. The figure indicates that the study subjects perceived the most salient change in the most possible value for four-unit changes (CT_0→4). On the other hand, the difference between one-unit changes (CT_0→2 and CT_2→4) appears to be less clear-cut.

In order to explore the distinctions between the levels of the *Change Type* effect, a set of pairwise LSD post-hoc statistical tests were conducted. The results of these calculations indicate that the sole discrepancy that is not statistically meaningful pertains to two levels that entail changes of two units in the fuzzy number most possible value (CT_0→2 vs. CT_2→4 with $p = 0.368$). In other cases, differences were significant at $\alpha < 0.0001$.

Fig. 3 also displays the average relative weights for the two levels of the *Graphical Representation* effect. These findings corroborate the initial analysis graphically shown in the key descriptive statistics section. Specifically, the study subjects evidently recognized that changes in the fuzzy number most possible values visualized as triangles were more noticeable than those presented as vectors.

It seems that the most interesting results are associated with the interaction between *Change Type* and *Graphical Representation* [$F(2, 486) = 7.13$, $p = 0.0009$]. Fig. 4 graphically presents the differences in mean relative weights for this effect.

These data suggest that triangles were better suited for visualizing the change in the most possible value for CT_0→4 and CT_2→4 change types. However, the situation was reversed for the CT_0→2 level. In this case, vector representations were better rated than its triangular counterparts.

To further explore which of these differences were statistically significant a series of pairwise LSD post-hoc tests were carried out. The outcomes indicate that the suitability of triangle-based representation for the fuzzy number change in the most possible value is statistically significantly higher for CT_0→4 and CT_2→4 *Change Type* levels ($p < 0.001$ in

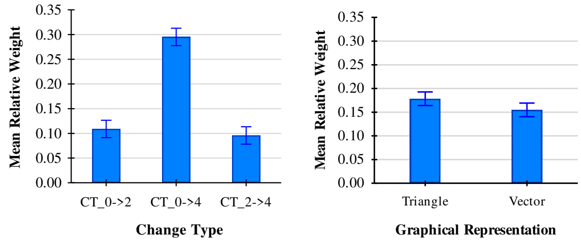

Fig. 3. Mean relative weights for *Change Type* [$F(2, 486) = 154.7$, $p < 0.0001$] and *Graphical Representation* [$F(1, 486) = 5.24$, $p = 0.0225$]. Bars denote 0.95 confidence intervals.

both cases). Although, according to participants, the mean relative weights for vector representations were bigger for CT_0→2, the difference was statistically inconclusive ($p = 0.176$).

We also further examined the *Change Type* × *Gender* interaction effect [$F(2, 486) = 2.87$, $p = 0.058$], which is illustrated in Fig. 5. This graph suggest that women were more prone to perceive the changes in the most possible value as more salient than men if the changes were smaller, that is, amounted to two units. This phenomenon was inverted for the much bigger change involving four units.

Additional pairwise tests were employed to check which of the differences were statistically meaningful. The results of the LSD post-hoc tests, revealed that gender differences for smaller changes in most possible factors were irrelevant ($p > 0.15$). However, the difference between female and male study subjects for the bigger change was statistically significant at $p = 0.057$.

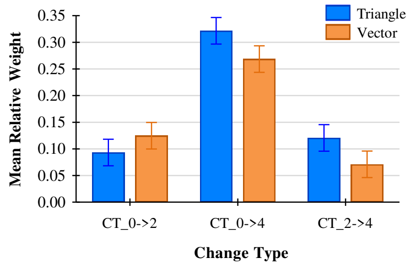

Fig. 4. Mean relative weights for *Change Type* × *Graphical Representation* interaction. Bars denote 0.95 confidence intervals [$F(2, 486) = 7.13$, $p = 0.0009$].

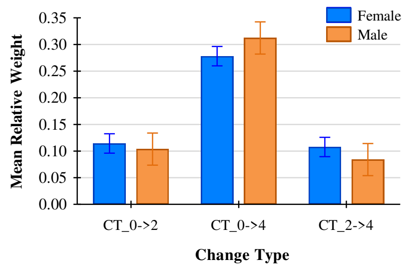

Fig. 5. Mean relative weights for *Change Type* × *Gender* interaction. Bars denote 0.95 confidence intervals [$F(2, 486) = 2.87$, $p = 0.058$].

The last significant effect from the performed analysis of variance, namely the *Graphical Representation* × *Gender* interaction, is visually demonstrated in Fig. 6. The graph shows that women considered triangular representations as better fitted to exhibit changes in most possible values than men did. On the other hand, males rated higher vector visualizations than women.

Again, we used pairwise LSD post-hoc analysis to verify the significance of the observed differences in mean relative weights. The findings, put together in Table VI, indicate that the observed gender discrepancies both for triangle and vector representations are statistically considerable ($p = 0.049$ and $p < 0.001$, respectively). Moreover, females significantly better perceived the change saliency if the fuzzy number change was presented as triangles than vectors ($p < 0.001$). For males, such an outcome was not detected ($p = 0.770$).

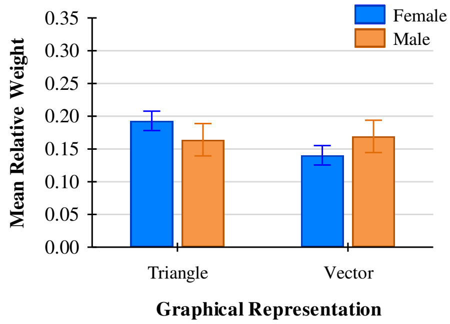

Fig. 6. Mean relative weights for *Graphical Representation* × *Gender* interaction. Bars denote 0.95 confidence intervals [$F(1, 486) = 7.78$, $p = 0.0055$].

V. Discussion of the Results and Conclusion

Our experimental study presented in this paper aimed to expand our understanding of how users perceive changes in triangular fuzzy numbers that are commonly used for expressing uncertainty. Specifically, we investigated two different visual representations of these fuzzy numbers, namely vector and classic, triangle-based ones along with various conditions concerned with their most possible values. The changes were depicted graphically through variations either in the vector angle or location of the maximum value of the membership function, and depended on the form of representation used. To gather study subjects' preferences towards the perceived saliencies regarding the change in the most possible value, we utilized pairwise comparisons within the AHP framework. Such an approach provided us with relative weights for all examined experimental conditions and allowed for comprehensive detailed formal statistical analysis. Gender differences in consistency ratios, computed according to this methodology, occurred to be statistically irrelevant. Furthermore, we identified statistically significant differences in the average relative scores for the two out of three factors studied, and three two-way interactions. The triangular-based fuzzy number representation, in general, were assessed as more appropriate than vectors for presenting changes in the most possible value. This factor considerably interacted with the *Change Type* effect. Triangles were better perceived in this experimental setup than vectors for large changes, and for changes more distant from the symmetrical case, that is vertical vectors and isosceles triangle. However, for smaller changes starting from that symmetrical situation, the reverse tendency was noticed suggesting that vectors could be better suited for detecting changes in such a case. Although this phenomenon was not statistically significant alone, the significance of the whole interaction certainly indicates that the application of vector representations should be studied in more detail in future research.

The general picture of the results obtained is further complicated by two additional gender interactions with *Graphical Representation* and *Change Type*. Females rated triangles considerably better than vectors, whereas for males the difference between these representations was unnoticeable. Moreover, triangles were relatively worse than vectors in terms of change saliency for men than women, but vectors were better perceived by male than female participants. As to the interaction with *Change Type* effect, men perceived big changes as more salient than women did. On the other hand, smaller changes of the most possible values were subjectively more noticeable for females than males. This suggest that females may be more sensitive in detecting smaller changes and less sensitive in identifying larger changes than males. This hypothesis, naturally, requires further empirical evidence. The discussed findings indisputably show that prospective research regarding graphical representations of uncertainty must involve gender-related analysis. This should be paid attention to already while designing and conducting the experiment, for instance, by ensuring similar number of man and women taking part in the study.

There are several possibilities of extending the presented study. Here, we confined only to the changes in one uncertainty feature of fuzzy numbers, that is, the most possible value. It is not clear, what would be the study subjects' perception of the saliency of changes if also the indeterminacy would be involved in the experimental setup. Thus, it should be subject to examination in future works as well. Since this study results showed that subjects' opinions depend considerably on the interaction between *Graphical Representation* and *Change Type* factors, another extension could include more levels of the *Change Type* effect to obtain a more comprehensive view of this outcome.

The results of this study contribute to the existing knowledge on how people perceive graphical representations of triangular fuzzy numbers. With the increasing use of artificial intelligence methods for handling inexact or ambiguous data, it has become crucial to develop suitable recommendations for user interfaces in computer programs that assist in solving problems with uncertainties.

The current investigation outcomes extend our understanding of individuals' opinions on the suitability of fuzzy number graphical visualizations in demonstrating uncertainty changes.

This can translate to provide appropriate recommendations for developing better graphical user interfaces with applications in such areas as production or project management. Given the presented results such interfaces should be carefully tailor-made individually for men and women.

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