

Control design of an UAV-Q based on feedback linearization and optimum modulus methods

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Abstract—In the paper, we present the formulation of quadrotor control loops that are based on a decomposition into a cascade structure and the use of feedback linearization and optimum modulus methods to determine controller parameters. The dynamic model used in this paper considers the dynamics of the propeller rotor drive systems. The propeller rotor drive systems are considered as a linear actuated system. After the synthesizing of the controllers is completed, the system is simulated in MATLAB/Simulink. The results from this work can be useful for the development of autonomous algorithms for UAV - Q (Unmanned Aerial Vehicle - Quadrotor). The research results serve as the basis for control algorithms development for other similar systems.

Index Terms—GameUAV-Q, propeller rotor drive system, feedback linearization, modulus optimum.

I. INTRODUCTION

IN THE recent years, UAV-Q has been an active research topic because of their broad applications, especially in the field of military and media services. Among many types of UAV-Q have been widely used because of their advantages such as they have simple structure, compact size, etc. Despite of having simple and symmetric structures, the dynamic models of quadrotor are nonlinear ones. Therefore, the control of these types of UAV requires advanced techniques in order to get good control quality. There have been a lot of studies on the control system design of such UAVs. Some of them can be listed such as the use of PID controller based on linearized models of quadrotor [1, 5, 6], or a number of other approaches that use the sliding mode control, backstepping [11] or robust control H_∞ [5]. Additionally, most of the previous works have mainly considered the control inputs to quadrotor as the forces or moments, neglecting the dynamics of the rotors that driving the propellers. In fact, the dynamics of the propeller rotor drive systems should be taken into account. This, of course, will increase the complexity of the UAV-Q system, [6, 13, 14].

In this paper, we present the synthesis of position and attitude controllers by breaking down the system into a cascade structure and the use of the feedback linearization and optimum modulus methods to determine the controllers for the quadrotor control loops. The dynamic models of quadrotor take into account the dynamics of the propeller rotor drive system.

II. THE DYNAMIC MODEL OF UAV-Q

The dynamics of a quadrotor is presented in [5]. Earth inertial frame (E frame) and body frame (B frame) whose ori-

gin is chosen the as quadrotor center of mass are shown in Fig. 2.

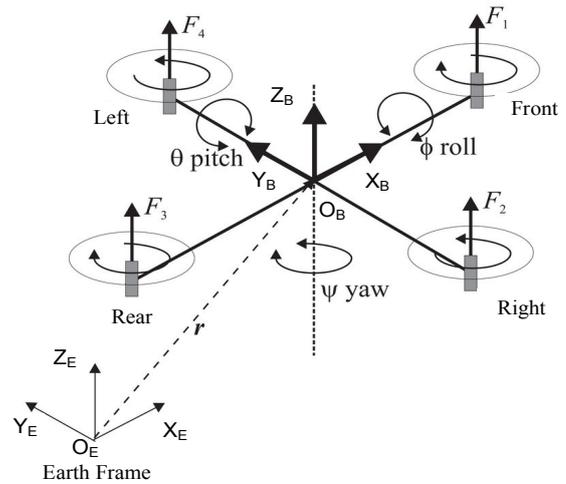


Fig. 1. The system coordinate of unmanned aerial vehicle quadrotor

The dynamics of a quadrotor [2, 5, 7, 8, 11] is described as follows:

$$\begin{bmatrix} mI_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & I \end{bmatrix} \begin{bmatrix} \dot{v}^B \\ \dot{\omega}^B \end{bmatrix} + \begin{bmatrix} \omega^B \times (mV^B) \\ \omega^B \times (I \omega^B) \end{bmatrix} = \begin{bmatrix} F^B \\ \tau^B \end{bmatrix} \quad (1)$$

where, $I_{3 \times 3}$ is the [3x3] identity matrix, $V^B [m.s^{-1}]$, $\dot{v}^B [m.s^{-2}]$ are the quadrotor linear velocity and acceleration vector expressed in the B-frame, while $\omega^B [rad.s^{-1}]$, $\dot{\omega}^B [rad.s^{-2}]$ is the quadrotor angular velocity and acceleration expressed in the B-frame, $F^B [N]$ is the quadrotor forces vector with respect to B-frame and $\tau^B [Nm]$ is the quadrotor moment vector expressed in the B-frame. A generalized force vector Λ is defined as:

$$\Lambda = [F^B \ \tau^B]^T = [F_x \ F_y \ F_z \ \tau_x \ \tau_y \ \tau_z]^T \quad (2)$$

Equation (1) is rewritten in a matrix form as:

$$M \dot{v} + C(v)v = \Lambda \quad (3)$$

where \dot{v} is generalized acceleration vector, M_B is inertia matrix, and $C_b(v)$ is Coriolis-centripetal matrix. The dynamic equations of quadrotor [3, 5] are given as follows:

$$\begin{cases} \ddot{X} = (\sin \psi \sin \phi + \cos \psi \sin \theta \cos \phi)U / m \\ \ddot{Y} = (\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi)U / m \\ \ddot{Z} = -g + \cos \phi \cos \theta)U / m \\ \ddot{\phi} = \frac{(I-I)}{I} \dot{\psi} \dot{\theta} - \frac{J}{I} \dot{\theta} \dot{\Omega} + \frac{lb}{I} (-\Omega + \Omega) \\ \ddot{\theta} = \frac{(I-I)}{I} \dot{\psi} \dot{\phi} + \frac{J}{I} \dot{\phi} \dot{\Omega} + \frac{lb}{I} (-\Omega + \Omega) \\ \ddot{\psi} = \frac{(I-I)}{I} \dot{\phi} \dot{\theta} + \frac{d}{I} (-\Omega + \Omega - \Omega + \Omega) \end{cases} \quad (4)$$

where the propellers' speed inputs U_1, U_2, U_3 and U_4 with respect to B-frame are given as:

$$\begin{cases} U_1 = b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \\ U_2 = lb(-\Omega_2^2 + \Omega_4^2) \\ U_3 = lb(-\Omega_1^2 + \Omega_3^2) \\ U_4 = d(-\Omega_1^2 + \Omega_2^2 - \Omega_3^2 + \Omega_4^2) \\ \Omega = -\Omega_1 + \Omega_2 - \Omega_3 + \Omega_4 \end{cases} \quad (5)$$

where, U_1 is responsible for the X, Y, Z coordinates of the quadrotor and their rates of change. U_2, U_3 and U_4 are responsible for the roll (ϕ), pitch (θ), and yaw (ψ) rotations and their rates of change. This model can be written in state space form $\dot{\mathbf{X}} = f(\mathbf{X}, \mathbf{U})$, where $\mathbf{U}^T = [U_1, U_2, U_3, U_4]$ is the input variables and $\mathbf{X}^T = (\dot{X}, \dot{Y}, \dot{Z}, \phi, \theta, \psi, \dot{\phi}, \dot{\theta}, \dot{\psi}, X, Y, Z)$ with:

$$\begin{array}{llll} x_1 = \dot{X} & x_4 = \phi & x_7 = \dot{\phi} & x_{10} = X \\ x_2 = \dot{Y} & x_5 = \theta & x_8 = \dot{\theta} & x_{11} = Y \\ x_3 = \dot{Z} & x_6 = \psi & x_9 = \dot{\psi} & x_{12} = Z \end{array}$$

Therefore, equations (4) are rewritten into (6). In the state space model (6), one can see the common model in the works on quadrotor control with control input U_1, U_2, U_3 and U_4 . Considering the dynamics of the propeller rotor drive systems, an additional system with nonlinear equations must be included which has the following general form:

$$\dot{\mathbf{X}}_{dc} = f(\mathbf{X}_{dc}, \mathbf{U}_{dc}) \quad (6)$$

$$\dot{\mathbf{X}} = \begin{bmatrix} (\sin X_6 \sin X_4 + \cos X_6 \sin X_5 \cos X_4)U_1/m \\ (\cos X_4 \sin X_5 \sin X_6 - \sin X_4 \cos X_6)U_1/m \\ -g + (\cos X_4 \cos X_5)U_1/m \\ X_7 \\ X_8 \\ X_9 \\ \frac{(I_{YY} - I_{ZZ})}{I_{XX}} X_9 X_8 - \frac{J_{TP}}{I_{XX}} X_8 \Omega + \frac{U_2}{I_{XX}} \\ \frac{(I_{ZZ} - I_{XX})}{I_{YY}} X_9 X_7 + \frac{J_{TP}}{I_{YY}} X_7 \Omega + \frac{U_3}{I_{YY}} \\ \frac{(I_{XX} - I_{YY})}{I_{ZZ}} X_7 X_8 + \frac{U_4}{I_{ZZ}} \\ X_{10} \\ X_{11} \\ X_{12} \end{bmatrix} \quad (7)$$

Where $\mathbf{X}_{dc} \in R^n$ is the vector of the state variables of rotors; $\mathbf{U}_{dc} \in R^m$ is the vector of the input variables to control rotors. The output variables of (6) are the rotor velocities, that are the components $\Omega_1, \Omega_2, \Omega_3, \Omega_4$ in expression (5). The decomposition technique is used to transform the state space equations (5), (7) and (6) into the below subsystems: The first subsystem S_1 includes the different equations, which describes the quadrotor's angular rates.

$$\begin{bmatrix} \dot{X}_7 \\ \dot{X}_8 \\ \dot{X}_9 \end{bmatrix} = \begin{bmatrix} \frac{(I_{YY} - I_{ZZ})}{I_{XX}} X_9 X_8 - \frac{J_{TP}}{I_{XX}} X_8 \Omega + \frac{U_2}{I_{XX}} \\ \frac{(I_{ZZ} - I_{XX})}{I_{YY}} X_9 X_7 + \frac{J_{TP}}{I_{YY}} X_7 \Omega + \frac{U_3}{I_{YY}} \\ \frac{(I_{XX} - I_{YY})}{I_{ZZ}} X_7 X_8 + \frac{U_4}{I_{ZZ}} \end{bmatrix} \quad (8)$$

The UAV - Quadrotor's Euler angles can be calculated by simply integrating \dot{X}_7, \dot{X}_8 and \dot{X}_9 . The second subsystem includes the different equations, which describe the velocities of UAV-Q inputs of the subsystem are Euler angles and variable U_1 .

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{bmatrix} = \begin{bmatrix} (\sin X_6 \sin X_4 + \cos X_6 \sin X_5 \cos X_4) \frac{U_1}{m} \\ (\cos X_4 \sin X_5 \sin X_6 - \sin X_4 \cos X_6) \frac{U_1}{m} \\ -g + (\cos X_4 \cos X_5) \frac{U_1}{m} \end{bmatrix} \quad (9)$$

The third subsystem is described by system of nonlinear different equations (7), that describes the propeller rotor drive systems of quadrotor.

III. THE CONTROLLER DESIGN

According to the paper's approach, structure of quadrotor control system includes the propeller rotor speed control loop, the attitude loop and the position control loop. In this control architecture, the inner attitude control loop faster dynamics compared to the outer loop responses. In this paper a linear model of the propeller rotor drive systems depicted as:

$$W_m = \frac{\Omega_m}{U_m} = \frac{K_m}{s(T_1s+1)(T_2s+1)} \quad (10)$$

where Ω_m is the rotor speed, U_m is the voltage input. The controller synthesis is based on the optimum modulus method [8, 15, 16] and select factor $a=2$, a desired transfer function of open loop is defined by below expression:

$$W_{ol} = \frac{1}{2T_1s(T_1s+1)} \text{ then} \\ W_{cm} = \frac{W_{ol}}{W_m} = \frac{1}{2T_1s(T_1s+1)} \cdot \frac{s(T_1s+1)(T_2s+1)}{K_m} = \frac{T_2s+1}{2K_mT_1} \quad (11)$$

According to [3], [10], the transfer function of the controller (C0) in this case is determined by equation (11). This is a Proportional-Derivative (PD) controller.

A. Attitude control design

It is assumed that the gyroscopic terms are ignored. The attitude loop stabilizes the Euler angles following a desired vector $(\phi_d, \theta_d, \psi_d)$ or (X_{4d}, X_{5d}, X_{6d}) . The subsystem is described in (7) is of nonlinear system. Applying feedback linearization [1, 2] and [11] the following linear system can be obtained

$$\begin{cases} U_2 = f_2(X_7, X_8, X_9) + U_2^* \\ U_3 = f_3(X_7, X_8, X_9) + U_3^* \\ U_4 = f_4(X_7, X_8, X_9) + U_4^* \end{cases} \quad (12)$$

Where U_2^*, U_3^*, U_4^* are new control variables. Substituting (12) into the equation (8) and neglecting the gyroscopic terms, we are received the equation (13).

$$\begin{cases} \dot{X}_7 = I_1 X_9 X_8 + (f_2(X_7, X_8, X_9) + U_2^*) / I_{xx} \\ \dot{X}_8 = I_2 X_9 X_7 + (f_3(X_7, X_8, X_9) + U_3^*) / I_{yy} \\ \dot{X}_9 = I_3 X_7 X_8 + (f_4(X_7, X_8, X_9) + U_4^*) / I_{zz} \end{cases} \quad (13)$$

where $I_1 = (I_{yy} - I_{zz}) / I_{xx}$, $I_2 = (I_{zz} - I_{xx}) / I_{yy}$ and $I_3 = (I_{xx} - I_{yy}) / I_{zz}$. In order to obtain a linear system, the

new control variables U_2^*, U_3^*, U_4^* are selected in the right side of the equation system (13), which becomes a linear system. Toward this end, the following conditions must be fulfilled:

$$\begin{cases} I_1 X_9 X_8 + (f_2(X_7, X_8, X_9) + U_2^*) / I_{xx} = K_2 X_7 \\ I_2 X_9 X_7 + (f_3(X_7, X_8, X_9) + U_3^*) / I_{yy} = K_3 X_8 \\ I_3 X_7 X_8 + (f_4(X_7, X_8, X_9) + U_4^*) / I_{zz} = K_4 X_9 \end{cases} \quad (14)$$

with unknown parameters K_2, K_3, K_4 . From above expression, it is shown that

$$\begin{cases} f_2(X_7, X_8, X_9) = I_{xx} (K_2 X_7 - I_1 X_9 X_8) \\ f_3(X_7, X_8, X_9) = I_{yy} (K_3 X_8 - I_2 X_9 X_7) \\ f_4(X_7, X_8, X_9) = I_{zz} (K_4 X_9 - I_3 X_7 X_8) \end{cases} \quad (15)$$

From equation (13) and (15) one can derive a linear equation system (16) as the following:

$$\begin{cases} \dot{X}_7 = K_2 X_7 + U_2^* / I_{xx} \\ \dot{X}_8 = K_3 X_8 + U_3^* / I_{yy} \\ \dot{X}_9 = K_4 X_9 + U_4^* / I_{zz} \end{cases} \quad (16)$$

$$\text{and} \quad \begin{cases} U_2 = K_2 X_7 - I_1 X_9 X_8 + U_2^* \\ U_3 = K_3 X_8 - I_2 X_9 X_7 + U_3^* \\ U_4 = K_4 X_9 - I_3 X_7 X_8 + U_4^* \end{cases} \quad (16)$$

To determine the coefficients K_2, K_3, K_4 , we use the following Lyapunov candidate function

$$V(X_7, X_8, X_9) = (X_7^2 + X_8^2 + X_9^2) / 2 \quad (17)$$

Taking the first time derivative of V results in:

$$\dot{V} = X_7 \dot{X}_7 + X_8 \dot{X}_8 + X_9 \dot{X}_9 \quad (18)$$

From equation (16), equation (18) can be rewritten as:

$$\dot{V} = X_7 \dot{X}_7 + X_8 \dot{X}_8 + X_9 \dot{X}_9 = K_2 X_7^2 + K_3 X_8^2 + K_4 X_9^2 \quad (20)$$

where $K_2, K_3, K_4 < 0$. After performing the linearized transformation of attitude control systems of quadrotor we obtain the following equations:

$$\begin{cases} \ddot{X}_4 = K_2 \dot{X}_4 + U_2^* / I_{xx} \\ \ddot{X}_5 = K_3 \dot{X}_5 + U_3^* / I_{yy} \\ \ddot{X}_6 = K_4 \dot{X}_6 + U_4^* / I_{zz} \end{cases} \quad (21)$$

From (22) we can get the transfer function of each channel. In this case we implement for roll motion:

$$W_\phi = \frac{\phi}{U_{c\phi}} = \frac{1/(-I_{xx}K_2)}{s((-1/K_2)s+1)} \quad (22)$$

The control design for this channel uses optimum modulus method [7, 8, 15, 16]. The the roll angle controller is determined as:

$$W_{c\phi} = K_\phi = 0.5I_{xx}K_2^2 \quad (23)$$

The other controllers of pitch and yaw channel can be done in the same way, and they are presented in the following expressions: $W_{c\theta} = K_\theta = 0.5I_{yy}K_3^2$; $W_{c\psi} = K_\psi = 0.5I_{zz}K_4^2$.

B. Velocity control design

If the attitude control loop is sufficiently fast, it is assumed that the desired roll, pitch, and yaw angles are achieved very fast. Therefore, without loss of generality, the attitude loop can be regarded as a unity gain. According to (8), the position subsystem can be depicted follows

$$\begin{cases} \dot{X}_1 = (\sin X_{6d} \sin X_{4d} + \cos X_{6d} \sin X_{5d} \cos X_{4d}) \frac{U_1}{m} \\ \dot{X}_2 = (\sin X_{4d} \cos X_{6d} - \cos X_{4d} \sin X_{5d} \sin X_{6d}) \frac{U_1}{m} \\ \dot{X}_3 = -g + (\cos X_{4d} \cos X_{5d}) \frac{U_1}{m} \end{cases} \quad (24)$$

where X_{4d}, X_{5d}, X_{6d} and U_1 are the input variables. These are the nonlinear equations and can be rewritten as follows:

$$\begin{cases} \dot{X}_1 = \tilde{U}_1 = f_1(X_{4d}, X_{5d}, X_{6d}, U_1) \\ \dot{X}_2 = \tilde{U}_2 = f_2(X_{4d}, X_{5d}, X_{6d}, U_1) \\ \dot{X}_3 = \tilde{U}_3 = f_3(X_{4d}, X_{5d}, X_{6d}, U_1) \end{cases} \quad (26)$$

$$\begin{cases} \tilde{U}_1 = n_1(X_{1d} - X_1) \\ \text{and } \tilde{U}_2 = n_2(X_{2d} - X_2) \\ \tilde{U}_3 = n_3(X_{3d} - X_3) \end{cases} \quad (25)$$

with newly defined input variables $\tilde{U}_1, \tilde{U}_2, \tilde{U}_3$ are selected for a proportional controller in the following form (27). Here, the parameters of the controllers n_1, n_2, n_3 could be selected such that the outer loop is sufficiently fast compared to attitude control loop. These transformed input variables is used to calculate the real input variables X_{4d}, X_{5d}, X_{6d} and U_1 by evaluating (25). It is noted that any desired velocity vector can be reached regardless any yaw rotation, so the system of equations (25) can be simplified as follows:

$$\begin{cases} \tilde{U}_1 = \sin X_{5d} \cos X_{4d} \frac{U_1}{m} \\ \tilde{U}_2 = \sin X_{4d} \frac{U_1}{m} \\ \tilde{U}_3 = -g + \cos X_{4d} \cos X_{5d} \frac{U_1}{m} \end{cases} \quad (26)$$

These above equations can be solved analytically by applying the considering relation:

$$\begin{cases} \lambda = \sin X_{4d} \Rightarrow \cos X_{4d} = \pm\sqrt{1-\lambda^2} \\ \gamma = \sin X_{5d} \Rightarrow \cos X_{5d} = \pm\sqrt{1-\gamma^2} \end{cases} \quad (27)$$

Substituting λ, γ into equation (27), we obtain the following relations:

$$\begin{cases} \tilde{U}_1 = \gamma \frac{U_1}{m} \pm \sqrt{1-\lambda^2} \\ \tilde{U}_2 = \lambda \frac{U_1}{m} \\ \tilde{U}_3 = -g + \left(\pm\sqrt{1-\lambda^2} \pm \sqrt{1-\gamma^2} \cdot \frac{U_1}{m} \right) \end{cases} \quad (30)$$

$$\Rightarrow \begin{cases} \gamma = \pm \frac{1}{\sqrt{\left[\frac{(g + \tilde{U}_3)}{\tilde{U}_1} \right]^2 + 1}} \\ U_1 = \pm m \sqrt{\tilde{U}_1^2 / \gamma^2 + \tilde{U}_2^2} \\ \lambda = \tilde{U}_2 \frac{m}{U_1} \end{cases} \quad (28)$$

If $\tilde{U}_1 \neq 0$, we solve the equation system (26) and obtain the following solution λ, γ and U_1 . U_1 is always positive, so from (27) for U_1 we obtain the unique solution:

$$U_1 = m \sqrt{(\tilde{U}_1^2 / \lambda^2) + \tilde{U}_2^2} \quad (29)$$

λ is unique, hence $X_{4d} = \arcsin \lambda$ is uniquely obtained in $[\pm\pi/2]$. In the similar fashion, it can be show that $X_{5d} = \arcsin \gamma$, but γ and X_{5d} could be positive or negative. This is explained in the following:

$$\tilde{U}_1 = -\cos X_{4d} \sin X_{5d} U_1 / m \quad (30)$$

In (30), the first term is positive in the interval $[\pm\pi/2]$ and the last term (U_1/m) is also positive. Therefore, X_{5d} is negative if \tilde{U}_1 is positive and vice versa.

C. The Synthesis of position control system

The design of position controller is implemented after the inner-loop controllers are synthesized. The way to

design for controller X, Y, Z is the same. We assume that the velocity loop is of second order. In this section we synthesize the controller for altitude channel Z . Therefore, the transfer function of Z channel has a form:

$$W_{pz} = \frac{Z}{U_{cz}} = \frac{K_z}{s(T_{z1}s + 1)(T_{z2}s + 1)} \quad (31)$$

According to the optimum modulus method [3], [10], we can obtain the transfer function of Z channel controller in the following form:

$$W_{cz} = \frac{T_{z2}s + 1}{2K_z T_{z1}} \quad (32)$$

IV. NUMERICAL SIMULATIONS

From equations describe the quadrotor dynamics (4) and controllers for control loop are synthesized above with quadrotor parameters simulation, we implement the numerical simulation via Matlab. The parameters of controller in loops are chosen as: $K_p=3, K_D=0.5, T_1 = 0.01, T_2 = 0.1, K_1=K_2=K_3=-80$; Coefficients of velocity controller: $n_1=n_2=n_3=1, K_p=0.25, K_D=0.1$. The thrust factor $b = 2,92 \cdot 10^{-6} \text{ kg.m}$, total rotational moment of inertia around the propeller axis $J_{TP} = 3,36 \cdot 10^{-5} \text{ kg.m}^2$, the aerodynamic drag factor $d = 1,1 \cdot 10^{-7} \text{ kg.m}^2$, mass of quadrotor $m = 0.5 \text{ kg}$, the body moment of inertia $I_{XX}=I_{YY} = 4,85 \cdot 10^{-3} \text{ kg.m}^2$ and $I_{ZZ} = 8,81 \cdot 10^{-3} \text{ kg.m}^2$, the center-to-center distance between the quadrotor and the propeller $l = 0,24 \text{ m}$.

Case 1: The desired inputs $\psi_d = 0 \text{ rad}, X_d = 0 \text{ [m]}, Y_d = 0 \text{ m}, Z_d = 15 \text{ m}$: as shown in the picture Fig 5 and 6 presents the results of X, Y, Z and Euler angles responses.

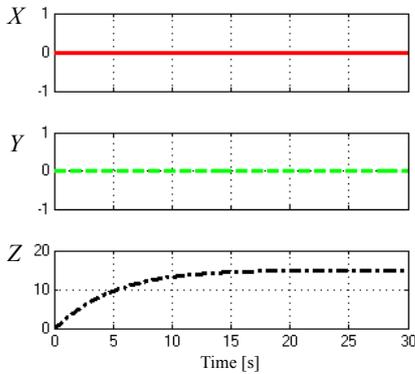


Fig. 2.

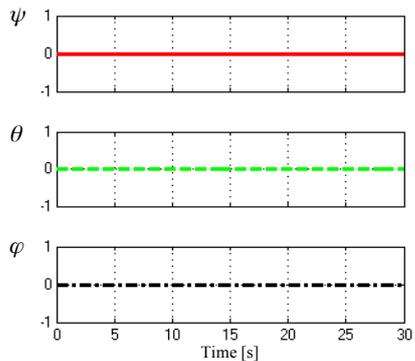


Fig. 3.

Case 2: The desired inputs $\psi_d = 0 \text{ [rad]}, X_d = 10 \text{ [m]}, Y_d = 0 \text{ [m]}, Z_d = 15 \text{ [m]}$: Fig 7 and 8 present the results of value X, Y, Z and Euler angles responses.

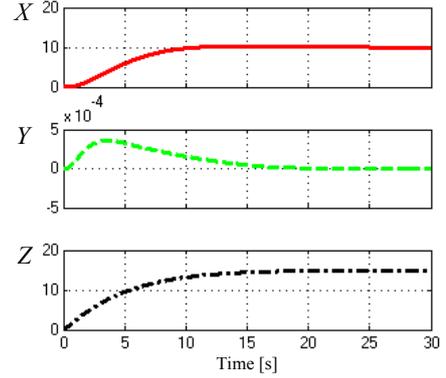


Fig. 4.

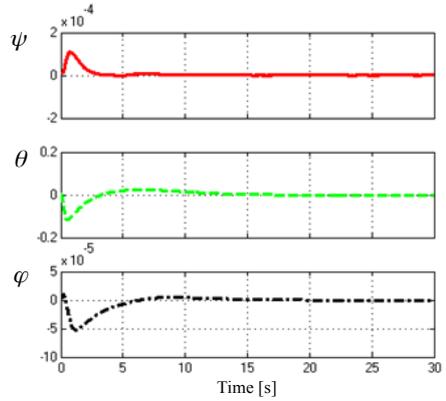


Fig. 5.

From the simulation results, we see that all the desired state reached along channel Z, X, Y with fast response. These results prove that the controllers work well with good tracking performance. Compared with previous studies, as shown in [9, 13], the research results of the paper are much better than previous studies.

CONCLUSIONS

The paper has presented the synthesis results for quadrotor control loops (position control loop, attitude control loop, propeller rotor control loop), which used the feedback linearization and optimum modulus methods. The dynamic model of the quadrotor is derived and numerically implemented. Through the simulation, the nonlinear vehicle control system is verified and its efficiency is demonstrated. The results, which obtained from this paper, contribute to the development of algorithms for autonomous UAV-Q.

REFERENCES

- [1] A. J. Fossard and D. Normand-Cyrot (Eds.). "Nonlinear Systems", Vol. 3: Control, Springer, (1996).
- [2] A. Isidori, "Nonlinear Control Systems", 3rd Edition, Springer, (1995).
- [3] S.-K. Kim, K.-G. Lee, and K.-B. Lee, "Singularity-free adaptive speed tracking control for uncertain permanent magnet synchronous motor", IEEE Transactions on Power Electronics, vol. 31, no. 2, pp. 1692-1701, (2016).
- [4] J. J. Slotine and W. Li, "Applied Nonlinear Control". Englewood Cliffs, NJ: Prentice-Hall, (1991).

- [5] J. Ghandour, S. Aberkane, J-C. Ponsart, "*Feedback Linearization approach for Standard and Fault Tolerant control: Application to a Quadrotor UAV Testbed*", Journal of Physics: Conference Series 570, (2014).
- [6] Utkin V., Guldner J., Shi J., *Sliding Mode Control in Electromechanical Systems*, CRC Press LLC, (1999).
- [7] Pedro Castillo, Rogelio Lozano and Alejandro E. Dzul, "*Modelling and Control of Mini-Flying Machines*", Springer, Compiègne, France, (2004).
- [8] Ali Emadi, "*Advanced Electric Drive Vehicles*", CRC Press is an imprint of Taylor & Francis Group, an Informa business, Springer International Publishing; USA, (2016).
- [9] S. Bouabdallah, P. Murrieri, and R. Siegwart, "*Design and control of an indoor micro quadrotor*". In Robotics and Automation, Proceedings. ICRA'04. IEEE (2004).
- [10] Hyeonbeom Lee and H. Jin Kim, "*Trajectory Tracking Control of Multirotors from Modelling to Experiments: A Survey*", International Journal of Control, Automation and Systems, pp. 1-12, (2017).
- [11] Tommaso Bresciani, "*Modelling, Identification and Control of a Quadrotor Helicopter*", master thesis, October (2008).
- [12] László Keviczky, Ruth Bars, Jenő Hetthéssy, "*Csilla Bányász, Control Engineering: MATLAB Exercises*", Springer Nature Singapore Pte Ltd, USA ISSN 1439-2232, (2019).
- [13] M. Navabi, H. Mirazei, "*Robust optimal adaptive trajectory tracking control of quadroto helicopter*", Lat. Am. J. solids struct. vol 14 No 6 Rio de Janeiro June (2017),
- [14] Luo, Shaohua, Gao, Ruizhen, "*Chaos control of the permanent magnet synchronous motor with time-varying delay by using adaptive sliding mode control based on DSC*", Volume 355, Issue 10 Pages 4147-4163, July (2018).
- [15] Ключев В. И, Теория электропривода Учебник для вузов Изд. Энергоатомиздат, 3-е (2001)
- [16] Б. К. Чемоданов - Следящие приводы Т1, 2.- М.: Изд. МГТУ им Баумана (1999).