

An investigation of the complexities of a malignant tumor's fractional-order mathematical model

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Abstract—In this paper, model of a malignant tumor & associated problems are examined using fractional-order method. We consider a case where the malignant tumor cells' net death rate is solely time-dependent. Fractional homotopy decomposition method (HDM) has been applied to determine model's series solution. The answer to the HDM is given using the Maclaurin expansion. This method's use of the Mathematica software package allows for fast and simple computation of series solutions, which is one of its benefits.

Index Terms—Homotopy decomposition method, Series solution, Caputo fractional derivative, Fractional malignant tumor model.

I. INTRODUCTION

Fractional calculus and its applications are currently experiencing rapid development, with more and more compelling real-world applications. Fractional calculus is an attractive area of applied analysis that is utilized to simulate biological problems in science. Fractional calculus has been very beneficial in the modeling of many diseases. To understand its assessment, presence, stability, and control, research into mathematical models of infectious diseases is crucial. Fractional differential equations have been used in recent years to provide mathematical representations of biological processes [1–6]. This work aims to apply the homotopy decomposition technique HDM [7], a new method, to solve the Time-Fractional Malignant Tumor Growth Model [5]. Malignant tumor is another term for a cancerous tumor. When a tumor is called "malignant," it means it is cancerous and has a high chance of spreading beyond its original location. Although they can infect other organs, the cancer cells that spread to other body regions are identical to the original ones. For instance, if lung cancer spreads to the liver, the cancer cells there are still lung cancer cells [2-4].

Korpinar Z et al. [4] explain how RPS method is used to analyze the most recent series solutions of several fractional cancer tumor models. Saadeh R et al. [3] described a new approach LRPS method to find numerical solution of some model of cancer tumor. Iyiola OS et al. [5] released their paper in which they said that 3 distinct scenarios of net death rate are taken into consideration, even in situations when the net death rate of cancerous cells depends on cluster of cells. With help of q-HAM, find answer of the time-fractional partial differential equation. Gandhi H et al. [2] says that the situation where net death rate & tumor growth are considered & therapy is time-dependent. To find model's solution, the fractional RDT method was used.

Our study is moving forward with a time-fractional malignant tumor model under a beginning condition to account for the fact that the model is provided via a time-fractional differential equation and that the "net death rate of tumor cells" is solely dependent on time.

$$\frac{\partial^\alpha \wp(s, w)}{\partial w^\alpha} = \frac{\partial^2 \wp(s, w)}{\partial s^2} - w^2 \wp(s, w)$$

where $w \geq 0, 0 \leq s \leq 1, 0 < \alpha \leq 1$, with initial condition as

$$\wp(s, 0) = e^{rs}$$

where $\wp(s, w)$ is tumor cell density at position s at time w & r is dependent on therapy death rate at time w . In this work, Caputo fractional derivative is used.

II. PRELIMINARIES

Definition 2.1. Riemann-Liouville (RL) fractional integral operator for order $\alpha \geq 0$, of a function $\phi \in L^1(a, b)$ is given as in [12,13],

$$I^\alpha \phi(w) = \frac{1}{\Gamma(\alpha)} \int_0^w (w-\tau)^{\alpha-1} \phi(\tau) d\tau, w > 0, (2.1)$$

where Γ indicates the Gamma function and $I^\alpha \phi(w) = \phi(w)$.

Definition 2.2. In Caputo's definition, a fractional derivative is

$$D^\alpha \phi(w) = I^{i-\alpha} \phi(w) = \frac{1}{\Gamma(i-\alpha)} \int_0^w (w-\tau)^{i-\alpha-1} \phi^{(i)}(\tau) d\tau, (2.2)$$

where $i - 1 < \alpha \leq i, i \in \mathbb{N}, w > 0$, assuming the right side is present.

Lemma 2.1. Let $w \in (a, b]$. Then

$$[I^\alpha (w-a)^\beta](w) = \frac{\Gamma(\beta+1)}{\Gamma(\beta+\alpha+1)} (w-a)^{\beta+\alpha}, (2.3)$$

where $\alpha \geq 0, \beta > 0$.

III. BASIC CONCEPT OF THE HDM

In order to show how this technique's basic structure works as in [16,18], focus on a common nonlinear non-

homogeneous fractional PDE using the following structural beginning conditions:

$$\frac{\partial^\alpha \varpi(s, w)}{\partial w^\alpha} = L(\varpi(s, w)) + N(\varpi(s, w)) + \wp(s, w), \tag{3.1}$$

where $\alpha > 0$.

subject to initial conditions

$$D_0^{\alpha-l} \varpi(u, 0) = \wp_l(s), \quad l = (0, \dots, n-1),$$

$$D_0^{\alpha-n} \varpi(s, 0) = 0, \quad n = [\alpha],$$

$$D_0^l \varpi(s, 0) = \check{h}_l(s), \quad l = (0, \dots, n-1),$$

$$D_0^n \varpi(s, 0) = 0, \quad n = [\alpha],$$

where, $\frac{\partial^\alpha}{\partial w^\alpha}$ indicate Caputo or Riemann-Liouville fractional derivative, \wp is commonly used function, N is common nonlinear fractional differential operator, & L represent linear fractional differential operator. HDM prime pace is used to switch fractional PDE to fractional PIE via using inverse operator I_v^α is utilized on both sides of "3.1". Riemann-Liouville fractional derivative case,

$$\varpi(s, w) = \sum_{j=1}^{n-1} \frac{\wp_j(s)}{\Gamma(\alpha-j+1)} W^{\alpha-j} + \frac{1}{\Gamma(\alpha)} \int_0^w (w-\alpha)^{\alpha-1} \tag{3.2}$$

$$[L(\varpi(s, \tau)) + N(\varpi(s, \tau)) + \wp(s, \tau)] d\tau;$$

Caputo fractional derivative case,

$$\varpi(s, w) = \sum_{j=1}^{n-1} \frac{\check{h}_j(s)}{\Gamma(\alpha-j+1)} W^j + \frac{1}{\Gamma(\alpha)} \int_0^w (w-\alpha)^{\alpha-1} \tag{3.3}$$

$$[L(\varpi(s, \tau)) + N(\varpi(s, \tau)) + \wp(s, \tau)] d\tau.$$

or by placing generally

$$\sum_{j=1}^{n-1} \frac{\wp_j(s)}{\Gamma(\alpha-j+1)} W^{\alpha-j} = G(s, w) \quad \text{or}$$

$$G(s, w) = \sum_{j=1}^{n-1} \frac{\check{h}_j(s)}{\Gamma(\alpha-j+1)} W^j,$$

we find

$$\varpi(s, w) = G(s, w) + \frac{1}{\Gamma(\alpha)} \int_0^w (w-\alpha)^{\alpha-1} [L(\varpi(s, \tau)) + N(\varpi(s, \tau)) + \wp(s, \tau)] d\tau \tag{3.4}$$

The answer may be expressed as a power series in p , according to a Key statement of HPM technique:

$$\varpi(s, w) = \sum_{n=0}^{\infty} p^n \varpi_n(s, w), \tag{3.5a}$$

$$\varpi(s, w) = \lim_{n \rightarrow \infty} \varpi(s, w, p) \tag{3.5b}$$

Decomposing a nonlinear term yields

$$N\varpi(s, w) = \sum_{n=0}^{\infty} p^n \mathcal{H}_n(\varpi) \tag{3.6}$$

where parameter for embedding is $p \in (0, 1]$.

$\mathcal{H}_n(\varpi)$ is He's polynomial & is created through

$$\mathcal{H}_n(\varpi_0, \dots, \varpi_n) = \frac{1}{n!} \frac{\partial^n}{\partial p^n} \left[N \left(\sum_{j=0}^{\infty} p^j \varpi_j(s, w) \right) \right], \tag{3.7}$$

$n = 0, 1, 2, \dots$

HDM is achieved via an elegant mixture of homotopy method & Abel integral.

$$\sum_{n=0}^{\infty} p^n \varpi_n(s, w) - G(s, w) = \frac{p}{\Gamma(\alpha)} \int_0^w (w-\tau)^{\alpha-1} \left[\wp(s, w) + L \left(\sum_{n=0}^{\infty} p^n \varpi_n(s, w) \right) + N \left(\sum_{n=0}^{\infty} p^n \varpi_n(s, w) \right) \right] d\tau. \tag{3.8}$$

By comparing the components of similar degrees of p , solutions in various orders with the initial term can be obtained.

$$\varpi_0(s, w) = G(s, w). \tag{3.9}$$

Theorem ([7]) Assume that $F: U \rightarrow V$ is a contraction of nonlinear mapping and that U and V are Banach spaces. If the sequence produced by the HDM technique is considered

$$\varpi_n(s, w) = F(\varpi_{n-1}(s, w)) = \sum_{i=0}^{n-1} \varpi_i(s, w)$$

$n = 1, 2, 3, \dots$, If so, the following is true:

- (1) $\|\varpi_n(s, w) - \varpi(s, w)\| \leq \varphi^n \|G(s, w) - \varpi(s, w)\|;$
- (2) $\varpi_n(s, w)$ is forever in neighborhood of $\varpi(s, w)$ meaning $\varpi_n(s, w) \in B(\varpi(s, w), r) = \{\varpi^*(s, w) / \|\varpi^*(s, w) - \varpi(s, w)\|\};$
- (3) $\lim_{n \rightarrow \infty} \varpi_n(s, w) = \varpi(s, w).$

Proof: (1) By induction on n , $\|\varpi_1 - \varpi\| = \|H(\varpi_0) - \varpi\|$, & based on Banach fixed point theorem, F has a fixed point ϖ indicating $F(\varpi) = \varpi$; for that reason,

$$\|\varpi_1 - \varpi\| = \|H(\varpi_0) - \varpi\| = \|H(\varpi_0) - H(\varpi)\| \leq \varphi \|\varpi_0 - \varpi\| = \varphi \|G(s, w) - \varpi\|$$

because F is contraction mapping.

Suppose that $\|\varpi_{n-1} - \varpi\| \leq \varphi^{n-1} \|G(s, 0) - \varpi(s, w)\|$ is an induction hypothesis, then

$$\|\varpi_n - \varpi\| = \|H(\varpi_{n-1}) - H(\varpi)\| \leq \phi \|\varpi_{n-1} - \varpi\| \leq \phi \phi^{n-1} \|G(s, w) - \varpi\|.$$

(2) First demonstrate that $G(s, w) \in B(\varpi(s, w), r)$, and this is find by induction on m . So, for $m = 1$,

$$\|G(s, w) - \varpi(s, w)\| = \|\varpi(s, 0) - \varpi(s, w)\| \leq r \text{ with } \varpi(s, 0) \text{ initial condition.}$$

Suppose that $\|G(s, w) - \varpi(s, w)\| \leq r$ for $m-2$ is induction hypothesis, then

$$\begin{aligned} \|G(s, w) - \varpi(s, w)\| &= \left\| G_{m-2}(s, w) - \frac{\wp_m(s)}{\Gamma(\alpha - m - 1)} w^{\alpha - m} \right\| \\ &\leq \|G_{m-1}(s, w) - \varpi(s, w)\| + \left\| \frac{\wp_m(s)}{\Gamma(\alpha, m, 1)} w^{m-1} \right\| = r. \end{aligned}$$

Now, utilizing (1), we have for all $n \geq 1$

$$\|\varpi_n - \varpi\| \leq \phi^n \|G(s, w) - \varpi\| \leq \phi^n r \leq r.$$

(3) By (2) & the fact $\lim_{n \rightarrow \infty} \phi^n = 0$ produces that $\lim_{n \rightarrow \infty} \|\varpi_n - \varpi\| = 0$; therefore,

$$\lim_{n \rightarrow \infty} \varpi_n = \varpi.$$

IV. TIME-FRACTIONAL MALIGNANT TUMOR GROWTH MATHEMATICAL MODEL

Malignant cells' net death rate is solely time-dependent as in [5] & model comes by time-fractional differential equation

$$\frac{\partial^\alpha \wp(s, w)}{\partial w^\alpha} = \frac{\partial^2 \wp(s, w)}{\partial s^2} - w^2 f(s, w) \tag{4.1}$$

where $w \geq 0, 0 \leq s \leq 1, 0 < \alpha \leq 1$, with initial condition as

$$\wp(s, 0) = e^{rs} \tag{4.2}$$

where $\wp(s, w)$ is tumor cell density at position s at time w & r is dependent on therapy death rate at time w .

V. ANALYTICAL SOLUTION

The following problem in "(4.1)" can be found using the HDM technique.

$$\begin{aligned} \sum_{n=0}^{\infty} P^n \wp_n(s, w) - \wp(s, 0) &= \frac{P}{\Gamma(\alpha)} \int_0^w (w - \tau)^{\alpha - 1} \\ &\left(\frac{\partial^2}{\partial s^2} \left(\sum_{n=0}^{\infty} P^n \wp_n(s, \tau) \right) - w^2 \sum_{n=0}^{\infty} P^n \wp_n(s, \tau) \right) d\tau \end{aligned} \tag{5.1}$$

At this time, compare the coefficient of the similar order of P , integral equations receive:

$$P^0 : \wp_0(s, w) = \wp(s, 0) = e^{rs}, \quad \wp_0(s, 0) = \wp(s, 0) \tag{5.2}$$

$$\begin{aligned} P^1 : \wp_1(s, w) &= \frac{1}{\Gamma(\alpha)} \int_0^w (w - \tau)^{\alpha - 1} \left(\frac{\partial^2 \wp_0}{\partial s^2} - w^2 \wp_0 \right) d\tau, \\ \wp_1(u, 0) &= 0, \end{aligned} \tag{5.3}$$

$$\begin{aligned} P^2 : \wp_2(u, v) &= \frac{1}{\Gamma(\alpha)} \int_0^w (w - \tau)^{\alpha - 1} \left(\frac{\partial^2 \wp_1}{\partial s^2} - w^2 \wp_1 \right) d\tau, \\ \wp_2(u, 0) &= 0, \end{aligned} \tag{5.4}$$

$$\begin{aligned} P^3 : \wp_3(s, w) &= \frac{1}{\Gamma(\alpha)} \int_0^w (w - \tau)^{\alpha - 1} \left(\frac{\partial^3 \wp_2}{\partial s^3} - w^2 \wp_2 \right) d\tau, \\ \wp_3(u, 0) &= 0, \end{aligned} \tag{5.5}$$

$$\begin{aligned} P^4 : \wp_4(s, w) &= \frac{1}{\Gamma(\alpha)} \int_0^w (w - \tau)^{\alpha - 1} \left(\frac{\partial^4 \wp_3}{\partial s^4} - w^2 \wp_3 \right) d\tau, \\ \wp_4(s, 0) &= 0, \end{aligned} \tag{5.6}$$

$$\vdots$$

$$\begin{aligned} P^n : \wp_n(u, v) &= \frac{1}{\Gamma(\alpha)} \int_0^w (w - \tau)^{\alpha - 1} \left(\frac{\partial^2 \wp_{n-1}}{\partial s^2} - w^2 \wp_{n-1} \right) d\tau, \\ \wp_n(s, 0) &= 0. \end{aligned} \tag{5.7}$$

The following solution are obtained:

$$\wp_0(s, w) = e^{rs}, \tag{5.8}$$

$$\wp_1(s, w) = \frac{e^{rs}(r^2 - w^2)}{\Gamma(1 + \alpha)} w^\alpha, \tag{5.9}$$

$$\wp_2(s, w) = \frac{e^{rs}(r^2 - w^2)^2}{\Gamma(1 + 2\alpha)} w^{2\alpha}, \tag{5.10}$$

$$\wp_3(s, w) = \frac{e^{rs}(r^2 - w^2)^3}{\Gamma(1 + 3\alpha)} w^{3\alpha}, \tag{5.11}$$

$$\wp_4(s, w) = \frac{e^{rs}(r^2 - w^2)^4}{\Gamma(1 + 4\alpha)} w^{4\alpha}, \tag{5.12}$$

$$\vdots$$

$$\wp_n(s, w) = \frac{e^{rs}(r^2 - w^2)^n}{\Gamma(1 + n\alpha)} w^{n\alpha}. \tag{5.13}$$

Using the software mathematica, then find all the terms of the above integral equations. Several components of the series solutions are taken into consideration in this case, & approximate result is provided-

$$\wp(s, w) = \wp_0(s, w) + \wp_1(s, w) + \wp_2(s, w) + \wp_3(s, w) + \wp_4(s, w) + \dots \tag{5.14}$$

$$\wp(s, w) = e^{rs} + \frac{e^{rs}(r^2 - w^2)}{\Gamma(1 + \alpha)} w^\alpha + \frac{e^{rs}(r^2 - w^2)^2}{\Gamma(1 + 2\alpha)} w^{2\alpha} +$$

$$\frac{e^{rs}(r^2 - w^2)^3}{\Gamma(1 + 3\alpha)} w^{3\alpha} + \frac{e^{rs}(r^2 - w^2)^4}{\Gamma(1 + 4\alpha)} w^{4\alpha} + \dots \tag{5.15}$$

$$\begin{aligned} \wp(s, w) &= e^{rs} \left(1 + \frac{(r^2 - w^2)}{\Gamma(1 + \alpha)} w^\alpha + \frac{(r^2 - w^2)^2}{\Gamma(1 + 2\alpha)} w^{2\alpha} + \right. \\ &\quad \left. \frac{(r^2 - w^2)^3}{\Gamma(1 + 3\alpha)} w^{3\alpha} + \frac{(r^2 - w^2)^4}{\Gamma(1 + 4\alpha)} w^{4\alpha} + \dots \right) \\ &= e^{rs} \sum_{n=0}^{\infty} \frac{(r^2 - w^2)^n}{\Gamma(1 + n\alpha)} w^{n\alpha}. \end{aligned} \tag{5.16}$$

Using "(5.16)" in

$$\wp(s, w) = \sum_{n=0}^{\infty} \wp_n(s, w) \tag{5.17}$$

In particular as $\alpha \rightarrow 1$

$$\begin{aligned} \wp(s, w) &= e^{rs} \left(1 + \frac{(r^2 - w^2)}{1!} w + \frac{(r^2 - w^2)^2}{2!} w^2 + \right. \\ &\quad \left. \frac{(r^2 - w^2)^3}{3!} w^3 + \frac{(r^2 - w^2)^4}{4!} w^4 + \dots \right) \\ &= e^{rs} e^{(r^2 - w^2)w}, \end{aligned} \tag{5.18}$$

$$\wp(s, w) = e^{-w^3 + wr^2 + rs} \tag{5.19}$$

Researchers found an accurate solution to the problem by using the homotopy decomposition method, whereas Iyiola and Zaman et al. [5] used the homotopy analysis method and found an approximate analytical solution to the problem. The HDM is a useful method for dealing with non-linear PDEs, as we have seen.

VI. TIME-FRACTIONAL MALIGNANT TUMOR GROWTH MODEL GRAPHICAL SOLUTION

Finally, using the analytical solution "(5.16)", some graphical results were produced for various value of the fractional parameter α , that lies between 0 and 1 with parameters $r = -1, 0 \leq u \leq 2, 0 \leq v \leq 1.5$.

Outcome demonstrates that under the initial conditions and the chosen parameters, the death rate r caused the tumor cells quantity to drop over time & eventually approach zero. When we modified value for α , it affected the convergence resolution using Figs 1, 2, 3, 4, 5, 6 changed when we changed the value $\alpha = 0.5, 0.8, 0.9, 0.95, 0.98, 1$.

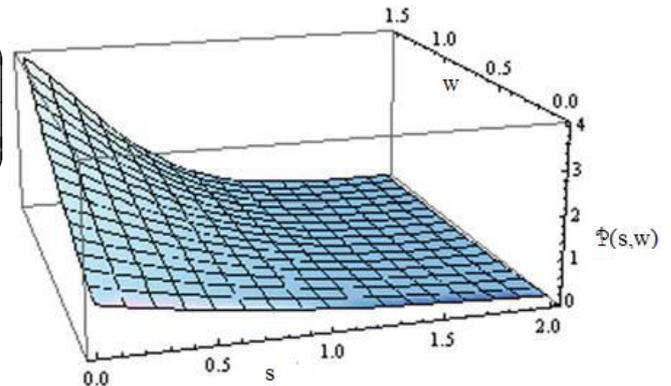


Fig.2 $r = -1, \alpha = 0.8, 0 \leq s \leq 2, 0 \leq w \leq 1.5$

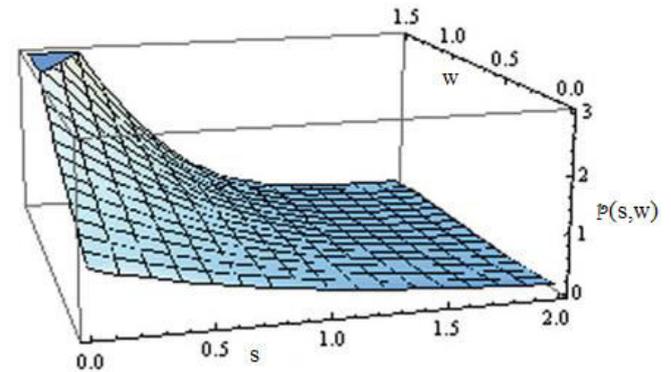


Fig.3 $r = -1, \alpha = 0.9, 0 \leq s \leq 2, 0 \leq w \leq 1.5$

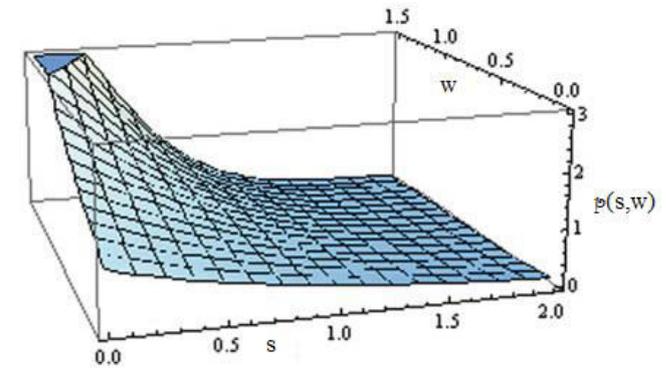


Fig.4 $r = -1, \alpha = 0.95, 0 \leq s \leq 2, 0 \leq w \leq 1.5$

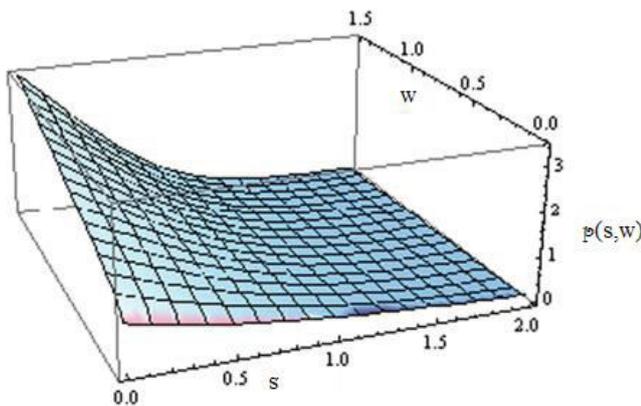


Fig.1 $r = -1, \alpha = 0.5, 0 \leq s \leq 2, 0 \leq w \leq 1.5$

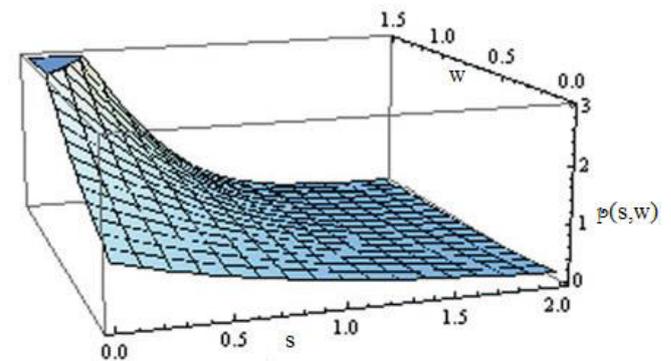


Fig.5 $r = -1, \alpha = 0.98, 0 \leq s \leq 2, 0 \leq w \leq 1.5$

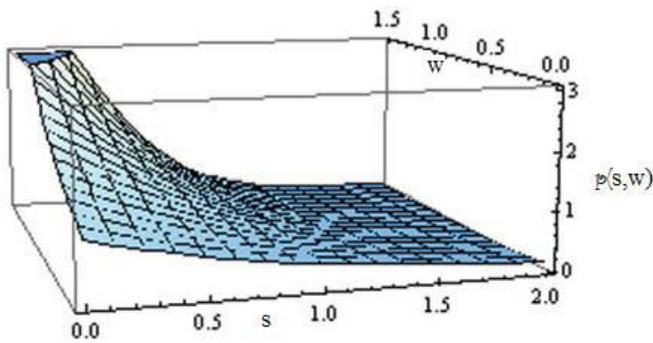


Fig.6 $r = -1, \alpha = 1, 0 \leq s \leq 2, 0 \leq w \leq 1.5$

VII. CONCLUSION

We observed that HDM is a highly useful mathematical tool that may be used to determine an exact or very close to precise solution to a model of time-fractional malignant tumor growth. In malignant tumor growth model, when net death rate of the tumor cells is solely time-dependent, we effectively employed it to produce the precise solution. We also produced other convergent solutions using a fractional parameter α . We discussed the fractional derivative concept's brief history and some of its features.

References

[1] Atangana A, Alabaraoye E. Solving a system of fractional partial differential equations arising in the model of HIV infection of CD4+ cells and attractor one-dimensional Keller-Segel equations. *Advances in Difference Equations*. 2013 Dec;2013(1):1-4.

[2] Gandhi H, Tomar A, Singh D. A predicted mathematical cancer tumor growth model of brain and its analytical solution by reduced differential transform method. In *Proceedings of International Conference on Trends in Computational and Cognitive Engineering 2021* (pp. 203-213). Springer, Singapore.

[3] Saadeh R, Qazza A, Amawi K. A New Approach Using Integral Transform to Solve Cancer Models. *Fractal and Fractional*. 2022 Aug 31;6(9):490

[4] Korpınar Z, Inc M, Hınçal E, Baleanu D. Residual power series algorithm for fractional cancer tumor models. *Alexandria Engineering Journal*. 2020 Jun 1;59(3):1405-12.

[5] Iyiola OS, Zaman FD. A fractional diffusion equation model for cancer tumor. *AIP Advances*. 2014 Oct 13;4(10):107121.

[6] Mishra J. A study on the spread of COVID 19 outbreak by using mathematical modeling. *Results in Physics*. 2020 Dec 1;19:103605.

[7] Atangana A, Secer A. The time-fractional coupled-Korteweg-de-Vries equations. In *Abstract and Applied Analysis 2013 Jan 1* (Vol. 2013). Hindawi.

[8] Burgess PK, Kulesa PM, Murray JD, Alvord Jr EC. The interaction of growth rates and diffusion coefficients in a three-dimensional mathematical model of gliomas. *Journal of Neuropathology & Experimental Neurology*. 1997 Jun 1;56(6):704-13.

[9] Moyo S, Leach PG. Symmetry methods applied to a mathematical model of a tumour of the brain. In *Proceedings of Institute of Mathematics of NAS of Ukraine 2004* (Vol. 50, No. Part 1, pp. 204-210).

[10] Ali SM, Bokhari AH, Yousuf M, Zaman FD. A spherically symmetric model for the tumor growth. *Journal of Applied Mathematics*. 2014 Jan 1;2014.

[11] Bokhari AH, Kara AH, Zaman FD. On the solutions and conservation laws of the model for tumor growth in the brain. *Journal of Mathematical analysis and Applications*. 2009 Feb 1;350(1):256-61.

[12] El-Ajou A, Arqub OA, Zhou ZA, Momani S. New results on fractional power series: theories and applications. *Entropy*. 2013 Dec 2;15(12):5305-23.

[13] El-Ajou A, Arqub OA, Momani S. Approximate analytical solution of the nonlinear fractional KdV–Burgers equation: a new iterative algorithm. *Journal of Computational Physics*. 2015 Jul 15;293:81-95.

[14] Khater MM, Attia RA, Lu D. Numerical solutions of nonlinear fractional Wu–Zhang system for water surface versus three approximate schemes. *Journal of Ocean Engineering and Science*. 2019 Jun 1;4(2):144-8.

[15] Atangana A, Baleanu D. Nonlinear fractional Jaulent-Miodek and Whitham-Broer-Kaup equations within Sumudu transform. In *Abstract and applied analysis 2013 Jan 1* (Vol. 2013). Hindawi.

[16] Chen X, Zheng L, Zhang X. Convergence of the homotopy decomposition method for solving nonlinear equations. *Advances in Dynamical Systems and Applications*. 2007;2(1):59-64.

[17] Mishra J, Atangana A. Numerical analysis of a chaotic model with fractional differential operators: from Caputo to Atangana–Baleanu. In *Methods of Mathematical Modelling 2019 Sep 17* (pp. 167-188). CRC Press.

[18] Atangana A, Bulutv H. EXTENSION OF HOMOTOPY DECOMPOSITION METHOD (HDM) TO COUPLED NONLINEAR VAN DER POL TYPE'S EQUATION. *Acta Marisensis. Seria Technologica*. 2013 Jul 1;10(2):54.

[19] Mishra J. Fractional hyper-chaotic model with no equilibrium. *Chaos, Solitons & Fractals*. 2018 Nov 1;116:43-53.

[20] Saadeh R, Qazza A, Amawi K. A New Approach Using Integral Transform to Solve Cancer Models. *Fractal and Fractional*. 2022 Aug 31;6(9):490.