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Optimizing Reference Model for Disturbance Rejection Controller for 3-DoF Robot Manipulator

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***Abstract*—In this article, an method is proposed combining optimal control for linear system and disturbances observer to control a 3 degree of freedom (3DoF) robot manipulator. By making the tracking error follow a given stable linear reference model through the observer, an optimal controller LQR will be designed to solve the optimization problem for the reference system, thereby leading to good control quality for the original system. The effectiveness of the method is shown through simulation results performed on Matlab/Simulink.**

***Index Terms*—Adaptive control, Optimal control, Observer, Disturbance rejection, Manipulator.**

I. Introduction

Robotic manipulators are widely used in industry and they play an important role in replacing humans in performing complex jobs that require high accuracy as well as high working frequency [1]. When the dynamic model of the system is known, the model based controller such as PD, PID and some improvements [2], [3], [4] are preferred approaches because of their simplicity and their capacity to apply in practice but the noise resistance of these controllers is not really good. For this reason, the nonlinear control methods including the sliding moed controller [5], [6], [7], the backstepping controller as well as the controllers that combine the nonlinear control method with the linear control method [8], [9], [10] was studied thereby improving the control quality under the influence of external disturbances. Although the stability of system is guaranteed by Lyapunov criterion, the control quality is still limited by the dependence of the control signal on the system model. In the case that the dynamic model of robot is not sufficiently exact, the controllers that rely on the model will no longer retain their effectiveness. Therefore, adaptive control methods have been proposed for the purpose of steer the states of robot to follow trajectory signal without information of system's dynamic model. The common utensils used for adaptive controller design include fuzzy systems [11] and neural networks [12] due to their property of being able to approximate any non-linear functions. By adjusting control parameters according to the change of system and working environment, the adaptive controller is capable of improving control quality in a variety of operating situations. The disturbance observer [13], [14], [15] is also an effective approach to eliminate the influence of external disturbances as well as system's uncertain parameters. The observer can be used just to estimate the disturbances [14] but if only the affect of disturbances is removed, the uncertain parameters can still persist and degrade the control quality. Therefore, by combining all the uncertainties of system and disturbances into an unique total uncertain component and remove it during the operation of system, the observers [13], [15] provide better capacity to deal with the change of uncertain parameters. Besides, the optimal control problem is also a requirement and there are many approaches to solve this problem [16], [17].

The optimal control problem for nonlinear systems in general is still a relatively complex and challenging topic. Algorithms that successfully solve the optimal control problem for linear systems do not seem to be directly applicable to nonlinear systems. For this reason, an approach to solve this problem can be mentioned is to make the tracking error of nominal nonlinear robot system to follow a reference linear model and then apply known optimal control algorithms to this reference model. Inspired by [13], the following paper will present an optimal control method for the robotic system based on the removal of non-linearity and make tracking error converged to zero according to the reference model. The proposed controller will be applied on 3-DoF robot manipulator which has been studied in [14]. However, unlike [14] where the observer is utilized to approximate the disturbances, our proposed observer will be used for linearization purpose by estimate and eliminate the total uncertain component. The selection of the reference model plays an important role to the control quality and an effective reference model can be determined through optimal control methods.

II. Optimal control problem for linear system

Consider a linear system:

$$\dot{\underline{x}} = A\underline{x} + B\underline{u} \tag{1}$$

where $\underline{x}$ is the state vector of system, $\underline{u}$ is the control signal vector, matrix A and B are constant matrices. The optimal control problem for a linear system is to design a linear state feedback controller $\underline{u} = -R\underline{x}$ so that the following cost function reaches the minimum value:

$$J = \frac{1}{2}\int_0^t \left(\underline{x}^T C\underline{x} + \underline{u}^T D\underline{u}\right) d\tau \tag{2}$$

with $C = C^T \geq 0$ and $D = D^T > 0$. From [18], the problem of finding the optimal control signal $\underline{u}$ is equivalent to finding a positive definite symmetry solution L^* of the Riccati equation

$$A^T L + LA + C - LBD^{-1}B^T L = 0 \tag{3}$$

Then, the coefficient matrix of the optimal controller R^* is calculated by

$$R^* = D^{-1}B^T L^* \tag{4}$$

However, finding the exact solution of (3) is relatively difficult and complicated. Therefore, the more commonly used method is approximating the solution of (3) instead of directly solving the Riccati equation. An approximate solution method proposed by Kleiman in [18] goes through the following steps:

- Determine R_0 to be the matrix of the state feedback controller so that the system is stable. If A has made the system stable, then R_0 can be chosen including all zero elements.
- Solving the Lyapunov equation

$$(A - BR_k)^T L_k + L_k (A - BR_k) = -C - R_k^T D R_k \tag{5}$$

  To find the solution for L_k with $k = 0, 1, ...$
- Calculate R_{k+1} from L_k using the formula

$$R_{k+1} = D^{-1}B^T L_k \tag{6}$$

Repeat the second and third steps of algorithm until the error satisfies the condition $L_{k+1} - L_k < \epsilon$ for a given arbitrarily small ϵ. With Kleiman's algorithm, it can be proved that the larger the number of iterations, the closer the solution found from the algorithm is to the exact solution of (3), meaning that $\lim\limits_{k\to\infty} L_k = L^*$ then the coefficient matrix of the controller will as close to the optimal coefficient matrix $\lim\limits_{k\to\infty} R_k = R^*$. The proof has been presented in [18] and we obtain the parameter matrix for the optimal controller LQR.

III. Controller design

Since the model of robot manipulator in general and 3-DoF manipulator in particular is nonlinear, it is relatively difficult to directly apply the design of the optimal state feedback controller. Therefore, the Generalized Proportional Integral Observe is used to estimate all the disturbances affections along with the non-linearity of the system, thereby bringing the tracking errors to a linear reference model. From the obtained linear system, the optimal control algorithm will be applied to improve the control quality.

A. Model of 3-DoF Robot Manipulator

The model of 3-DoF manipulator is shown in Fig. 1 with 3 states are 3 rotating joint.

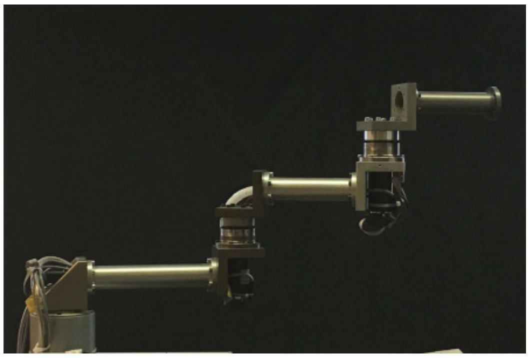

Fig. 1. Model of 3-DoF Robot Manipulator [14]

The dynamic model of 3-DoF robot can be described in Euler-Lagrange form with the following structure

$$M\left(\underline{q}\right)\ddot{\underline{q}} + \underline{\eta}\left(\underline{q}, \dot{\underline{q}}\right) = \underline{\tau} + \underline{\tau}_{ext} \tag{7}$$

where $\underline{q} \in \mathbb{R}^{3\times 1}$ is the vector of movable joint variables, $\dot{\underline{q}}$ and $\ddot{\underline{q}}$ are respectively the first and the second order derivatives of $\underline{q}$, $\underline{\tau} \in \mathbb{R}^{3\times 1}$ is the vector of the control signals and the input disturbances vector is denoted by $\underline{\tau}_{ext}$. Matrix $M \in \mathbb{R}^{3\times 3}$ is the inertia matrix of the system (M is symmetric and positively definite), $\underline{\eta} \in \mathbb{R}^{3\times 1}$ is the lumped vector of centripetal, coriolis and gravitational component. The elements of M and $\underline{\eta}$ will be detailed in Section IV.

B. Linearization Observer

The linearization process is performed based on the elimination of the affect of external disturbances and non-linearity in the system. First, the dynamic model of 3-DoF Robot is represented as

$$\ddot{\underline{q}} = M^{-1}(\underline{\tau} + \underline{\tau}_{ext} - \underline{\eta}) \tag{8}$$

with M and η are are shorthand for $M\left(\underline{q}, \dot{\underline{q}}\right)\ddot{\underline{q}}$ and $\underline{\eta}\left(\underline{q}, \dot{\underline{q}}\right)$. From (8), we have

$$\ddot{\underline{r}} - \ddot{\underline{q}} = -M^{-1}\underline{\tau} + \ddot{\underline{r}} - M^{-1}(\underline{\tau}_{ext} - \underline{\eta}) \tag{9}$$

which lead to

$$\ddot{\underline{\xi}} = K_1\underline{\xi} + K_2\dot{\underline{\xi}} - M^{-1}\underline{\tau} + \underline{f} \tag{10}$$

where $\ddot{\underline{r}}$ is the second order derivative of the reference signal $\underline{r}$, vector $\underline{\xi} = \underline{r} - \underline{q}$ is the tracking error, $\dot{\underline{\xi}}$ and $\ddot{\underline{\xi}}$ are respectively the first and the second order derivatives of ξ, K_1 and K_2 are two 3×3 square parameter matrix which are arbitrarily chosen. The influence of disturbances and non-linearity in the system is represented by total uncertain vector $\underline{f}$

$$\underline{f} = \ddot{\underline{r}} - M^{-1}(\underline{\tau}_{ext} - \underline{\eta}) - K_1\underline{\xi} - K_2\dot{\underline{\xi}} \tag{11}$$

The appearance of $\underline{f}$ in (10) is the cause of the difficulty in controller design because of its uncertainty. Therefore, the observer is proposed with purpose of approximating the value of $\underline{f}$ thereby eliminating the influence of this component on the system. Assume that $\underline{f}$ can be approximated by Taylor expansion with a sufficiently large number of degrees m with m is integer and $\underline{f}^m = \underline{0}$. Let $\underline{\xi}_1 = \underline{\xi}$, $\underline{\xi}_2 = \dot{\underline{\xi}}$, $\underline{\delta}_k = \underline{f}^{(k-1)}$ with $k = 1, 2, ..., m+1$ and denote the corresponding observed values are $\hat{\underline{\xi}}_1$, $\hat{\underline{\xi}}_2$, $\hat{\underline{\delta}}_k$, the structure of the observer is described as follow

$$\begin{cases} \dot{\hat{\underline{\xi}}}_1 = \hat{\underline{\xi}}_2 + \lambda_{m+1}(\underline{\xi}_1 - \hat{\underline{\xi}}_1) \\ \dot{\hat{\underline{\xi}}}_2 = K_1\hat{\underline{\xi}}_1 + K_2\hat{\underline{\xi}}_2 - M^{-1}\underline{\tau} + \hat{\underline{\delta}}_1 + \lambda_m(\underline{\xi}_1 - \hat{\underline{\xi}}_1) \\ \dot{\hat{\underline{\delta}}}_1 = \hat{\underline{\delta}}_2 + \lambda_{m-1}(\underline{\xi}_1 - \hat{\underline{\xi}}_1) \\ \vdots \\ \dot{\hat{\underline{\delta}}}_{m-1} = \hat{\underline{\delta}}_m + \lambda_1(\underline{\xi}_1 - \hat{\underline{\xi}}_1) \\ \dot{\hat{\underline{\delta}}}_m = \lambda_0(\underline{\xi}_1 - \hat{\underline{\xi}}_1) \end{cases} \tag{12}$$

with $\lambda_k \in \mathbb{R}^{3\times 3}$ are diagonal matrices of which all the diagonals are positive. From (12) we deduce

$$\begin{cases} \hat{\underline{\xi}}_1^{(m+2)} = \hat{\underline{\xi}}_2^{(m+1)} + \lambda_{m+1}\tilde{\underline{\xi}}_1^{(m+1)} \\ \hat{\underline{\xi}}_2^{(m+1)} = \begin{pmatrix} K_1\hat{\underline{\xi}}_1^{(m)} + K_2\hat{\underline{\xi}}_2^{(m)} - (M^{-1}\underline{\tau})^{(m)} \\ +\hat{\underline{\delta}}_1^{(m)} + \lambda_m\tilde{\underline{\xi}}_1^{(m)} \end{pmatrix} \\ \hat{\underline{\delta}}_1^{(m)} = \hat{\underline{\delta}}_2^{(m-1)} + \lambda_{m-1}\tilde{\underline{\xi}}_1^{(m-1)} \\ \vdots \\ \ddot{\hat{\underline{\delta}}}_{m-1} = \dot{\hat{\underline{\delta}}}_m + \lambda_1\dot{\tilde{\underline{\xi}}}_1 \\ \dot{\hat{\underline{\delta}}}_m = \lambda_0\tilde{\underline{\xi}}_1 \end{cases} \tag{13}$$

where $\tilde{\underline{\xi}}_1 = \underline{\xi}_1 - \hat{\underline{\xi}}_1$ is observer error. Equation (13) lead to

$$\hat{\underline{\xi}}_1^{\ (m+2)} = \begin{pmatrix} \lambda_{m+1}\tilde{\underline{\xi}}_1^{(m+1)} + K_1\hat{\underline{\xi}}_1^{(m)} + K_2\hat{\underline{\xi}}_2^{(m)} \\ -M^{-1}\underline{\tau}^{(m)} + \lambda_m\tilde{\underline{\xi}}_1^{(m)} + \lambda_{m-1}\tilde{\underline{\xi}}_1^{(m-1)} \\ +... + \lambda_1\dot{\tilde{\underline{\xi}}}_1 + \lambda_0\tilde{\underline{\xi}}_1 \end{pmatrix} \tag{14}$$

From (10) we have

$$(M^{-1}\underline{\tau})^{(m)} = K_1\underline{\xi}_1^{(m)} + K_2\underline{\xi}_1^{(m+1)} + \underline{f}^{(m)} - \underline{\xi}_1^{(m+2)} \tag{15}$$

Substitute (15) into (14) we obtain

$$\underline{f}^{(m)} = \begin{pmatrix} \tilde{\underline{\xi}}_1^{(m+2)} + (\lambda_{m+1} - K_2)\tilde{\underline{\xi}}_1^{(m+1)} \\ +(\lambda_m - K_1 - K_2\lambda_{m+1})\tilde{\underline{\xi}}_1^{(m)} \\ +\lambda_{m-1}\tilde{\underline{\xi}}_1^{(m-1)} + ... + \lambda_1\dot{\tilde{\underline{\xi}}}_1 + \lambda_0\tilde{\underline{\xi}}_1 \end{pmatrix} \tag{16}$$

Since $\underline{f}^m \approx 0$, according to (16), if we can choose the parameter matrices so that the following polynomial $H(s)$ is Hurwitz

$$H(s) = \begin{pmatrix} s^{(m+2)}I_n + (\lambda_{m+1} - K_2)s^{(m+1)} \\ +(\lambda_m - K_1 - K_2\lambda_{m+1})s^{(m)} \\ +\lambda_{m-1}s^{(m-1)} + ... + \lambda_1\dot{s} + \lambda_0 s \end{pmatrix} \tag{17}$$

we can make the observation error $\tilde{\underline{\xi}}_1 \to 0$ when $t \to \infty$. When $\tilde{\underline{\xi}}_1 \to \underline{0}$, we also have $\hat{\underline{\xi}}_1 \to \underline{e}_1$ which lead to $\hat{\underline{\xi}}_2 \to \underline{\xi}_2$ and $\hat{\underline{\delta}}_1 \to \underline{\delta}_1$ is the total uncertain component $\underline{f}$ need to be approximated.

C. Control signal synthesis

From observer the estimation of total uncertainty component is obtained, we denote this value is $\hat{\underline{f}}$. Then the control signal $\underline{\tau}$ for 3-DoF robot manipulator will consist of 2 components satisfying

$$\underline{\tau} = M\left(-\underline{u} + \hat{\underline{f}}\right) \tag{18}$$

With control signal (18), system (10) becomes

$$\ddot{\underline{\xi}} = K_1\underline{\xi} + K_2\dot{\underline{\xi}} + \underline{u} - \hat{\underline{f}} + \underline{f} \tag{19}$$

and when $\hat{\underline{f}}$ approaches the real value $\underline{f}$ we obtain

$$\ddot{\underline{\xi}} = K_1\underline{\xi} + K_2\dot{\underline{\xi}} + \underline{u} \tag{20}$$

Let $\underline{x} = \begin{bmatrix}\underline{\xi} & \dot{\underline{\xi}}\end{bmatrix}^T$, $A = \begin{bmatrix}\Theta_3 & I_3 \\ K_1 & K_2\end{bmatrix}$ and $B = \begin{bmatrix}\Theta_3 \\ I_3\end{bmatrix}$ with Θ_3 and I_3 is the zero matrix and the identity matrix dimension of 3×3. System (20) will be re-expressed in linear form

$$\dot{\underline{x}} = A\underline{x} + B\underline{u} \tag{21}$$

With the linear system (21), we can use Kleiman's LQR optimal controller design algorithm that has been presented in section II to determine the parameter matrix of the optimal control signal $\underline{u}$. From there, combining the output from the observer and the optimal controller LQR, the complete control structure of the system is presented as follows

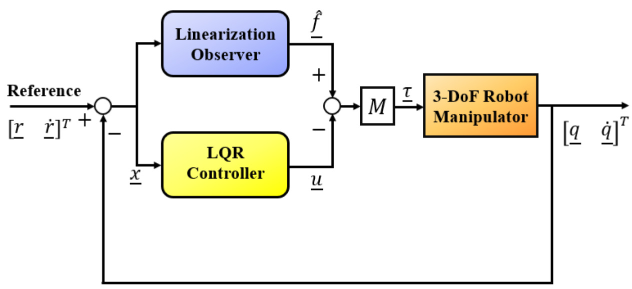

Fig. 2. Structure of controller

IV. Numerical Simulation

To verify the effectiveness of proposed controller, in this Section it will be applied on 3-DoF robot manipulator. First, the dynamic model of robot with the form (7)used for simulation will be presented based on [14], we have:

- The inertia matrix

$$M=\begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix}$$

with

$$\begin{aligned} m_{11} =& \alpha_1+2\beta_1 c_{23}+2\beta_2 c_2+2\beta_3 c_3 \\ m_{12} =& m_{21}=\alpha_2+\beta_1 c_{23}+\beta_2 c_2+2\beta_3 c_3 \\ m_{13} =& m_{31}=\alpha_3+\beta_1 c_{23}+\beta_3 c_3 \\ m_{23} =& m_{32}=\alpha_3+\beta_3 c_3 \\ m_{22} =& \alpha_2+\beta_3 c_3,\ m_{33}=\alpha_3 \end{aligned}$$

- The elements of lumped vector of the coriolis, centrifugal and gravitational components $\underline{\eta}\left(\underline{q},\underline{\dot{q}}\right)=[n_1,n_2,n_3]^T$ in which

$$n_1=\begin{pmatrix} \gamma_1 s_2\dot{q}_1^2+\gamma_2 s_{23}\dot{q}_1^2+\gamma_3 s_2(\dot{q}_1+\dot{q}_2)^2 \\ +\gamma_4 s_3\left(\dot{q}_1+\dot{q}_2\right)+\gamma_5 s_{23}(\dot{q}_1+\dot{q}_2+\dot{q}_3)^2 \\ +\gamma_6 s_3(\dot{q}_1+\dot{q}_2+\dot{q}_3)^2 \end{pmatrix}$$

$$n_2=\begin{pmatrix} \gamma_1 s_2\dot{q}_1^2+\gamma_2 s_{23}\dot{q}_1^2+\gamma_4 s_3(\dot{q}_1+\dot{q}_2)^2 \\ +\gamma_6 s_3(\dot{q}_1+\dot{q}_2+\dot{q}_3)^2 \end{pmatrix}$$

$$n_3=\gamma_2 s_{23}\dot{q}_1^2+\gamma_4 s_3(\dot{q}_1+\dot{q}_2)^2$$

where $c_i=\cos(q_i)$, $s_i=\sin(q_i)$, $c_{ij}=\cos(q_i+q_j)$ and $s_{ij}=\sin(q_i+q_j)$ and the model parameters selected for simulation are given through Table I

TABLE I
THE PARAMETERS OF 3-DOF ROBOT

Parameter	Value	Parameter	Value	Parameter	Value
α_1	1.0425	β_2	0.1742	γ_3	-0.1742
α_2	0.4398	β_3	0.0281	γ_4	0.0281
α_3	0.1788	γ_1	0.1742	γ_5	-0.0405
β_1	0.0405	γ_2	0.0405	γ_6	-0.0281

The controller is designed with two components. The first component is the observer that converts nominal nonlinear system to linear model with the Taylor approximation order of $\underline{f}$ is $m=1$, the parameter matrices of observer are

$$\begin{aligned} &K_1=-diag\left([25;49;36]\right),\ K_2=-diag\left([10;14;12]\right) \\ &\lambda_2=3W_0+K_2,\ \lambda_1=3{W_0}^2+\lambda_2 K_2+K_1 \\ &\lambda_0={W_0}^3,\ W_0=diag\left([80;80;80]\right) \end{aligned}$$

The second component is the LQR controller which is designed by the algorithm of Kleiman for linear system (1) with

$$A=\begin{bmatrix}\Theta_3 & I_3 \\ K_1 & K_2\end{bmatrix},\ B=\begin{bmatrix}\Theta_3 \\ I_3\end{bmatrix},\ C=I_6,\ D=I_3$$

and the parameter matrix for LQR optimal controller is obtained as

$$R=\begin{bmatrix} 1.93 & 0 & 0 & 4.28 & 0 & 0 \\ 0 & 1.01 & 0 & 0 & 3.26 & 0 \\ 0 & 0 & 1.36 & 0 & 0 & 3.71 \end{bmatrix}$$

Three scenarios for simulation will be performed as follows

- Scenario I: The system is not affected by disturbances and the reference is a trapezoidal signal given by

$$\underline{r}(t)=\begin{cases} 0.4t\underline{r}_t, & 0<t<2.5 \\ \underline{r}_t, & 2.5\le t\le 7.5 \\ \left(1-0.4(t-7.5)\right)\underline{r}_t, & t>7.5 \end{cases} \quad (22)$$

where $\underline{r}_t=[1;1.2;0.7]$

- Scenario II: The system is not affected by disturbances and the reference is a cyclic signal given by

$$\underline{r}(t)=\left(1+\sin\frac{\pi}{2.5}t-\frac{\pi}{2}\right)\underline{r}_s \quad (23)$$

with $\underline{r}_s=[0.5;0.8;0.2]$

- Scenario III: The system is affected by sinusoidal disturbances (see Fig. 3) and the reference is cyclic signal (23). The form of disturbances is designed according to [14] because it can perform the waveform of contact force in practice [14]

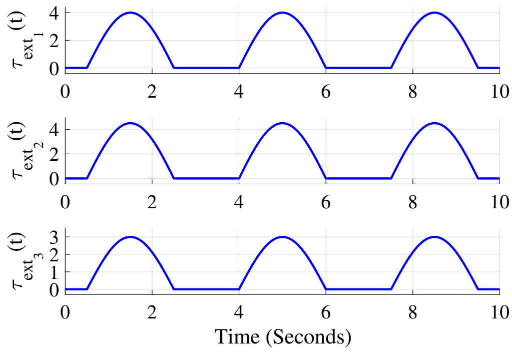

Fig. 3. The disturbances $\tau_{ext}=[\tau_{ext_1};\tau_{ext_2};\tau_{ext_3}]$

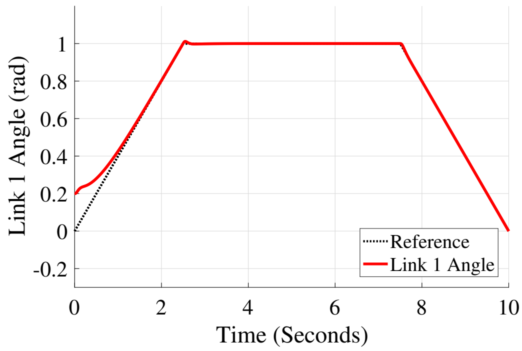

Fig. 4. State response of link 1, Scenario I

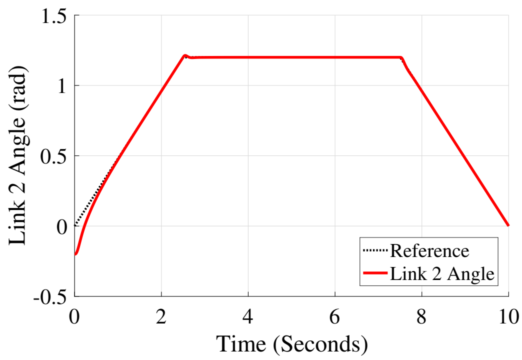

Fig. 5. State response of link 2, Scenario I

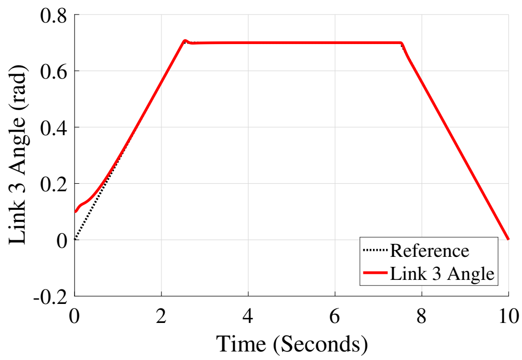

Fig. 6. State response of link 3, Scenario I

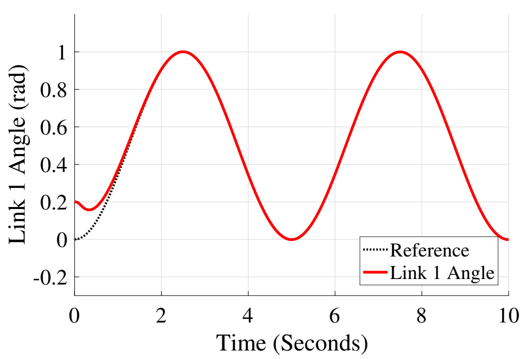

Fig. 7. State response of link 1, Scenario II

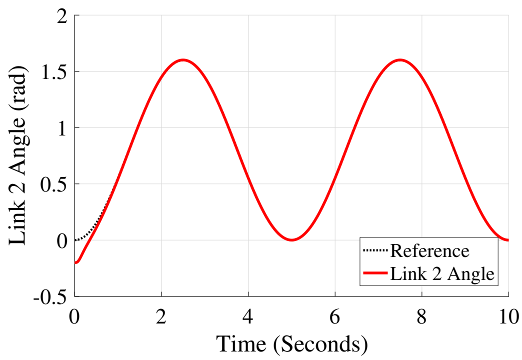

Fig. 8. State response of link 2, Scenario II

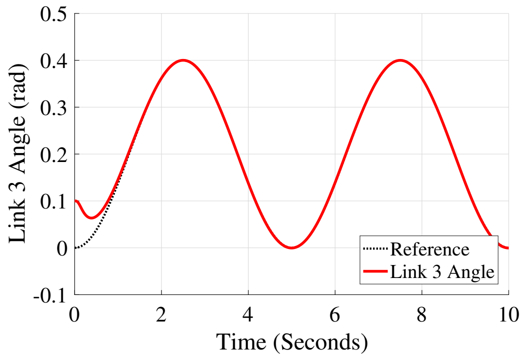

Fig. 9. State response of link 3, Scenario II

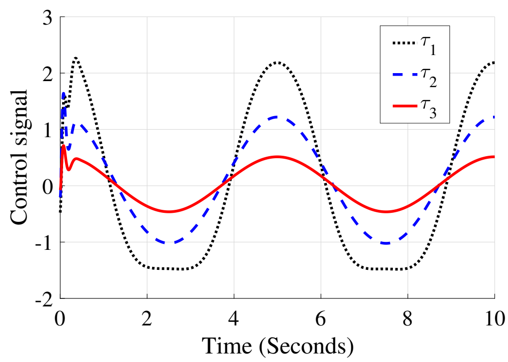

Fig. 10. Control signal $\tau = [\tau_1; \tau_2; \tau_3]$, Scenario II

In the absence of disturbances, the simulation results corresponding to Scenarios I (from Fig. 4 to Fig. 6) show that with desire trajectory is trapezoid, the proposed controller can quickly bring the states of the system to follow the trajectory signal. Furthermore, the proposed control strategy also shows efficiency when the reference is cyclic through the results in Scenarios II (from Fig. 7 to Fig. 10).

On the other hand, under the influence of disturbances, we can observe from Fig. 11 to Fig. 14 in Scenarios III that the proposed controller still retains the quality and efficiency. The angle of all 3 joints quickly track the trajectory signal and there is not too much difference compared to the control quality in Scenario II. Therefore, it can be concluded that the affect of disturbances has been removed by the linearization observer and the LQR optimal controller has completed the remaining job of optimally regulate the tracking error to converge to zero.

V. Conclusions

This paper presents an optimal control method to optimize the references model for an disturbances rejection controller based on linearization observer for 3-DoF robot manipulator. The simulation results have shown the effectiveness

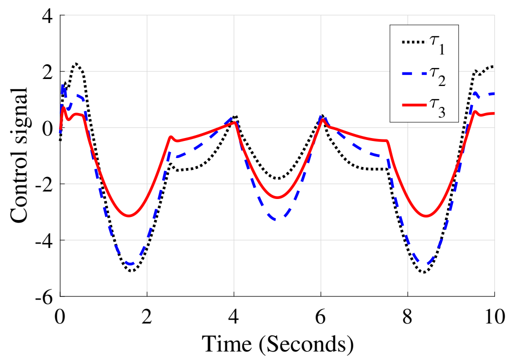

Fig. 14. Control signal $\tau = [\tau_1; \tau_2; \tau_3]$, Scenario III

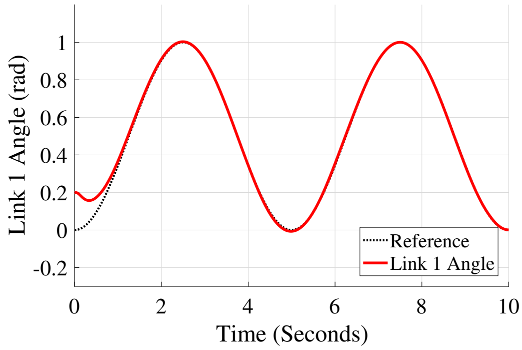

Fig. 11. State response of link 1, Scenario III

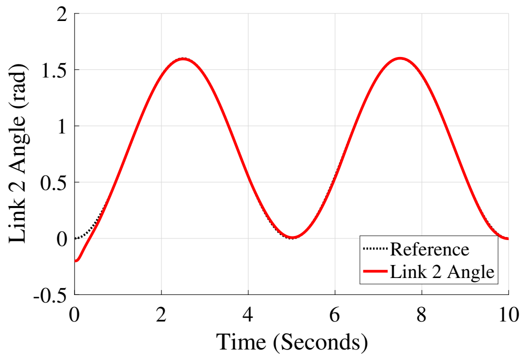

Fig. 12. State response of link 2, Scenario III

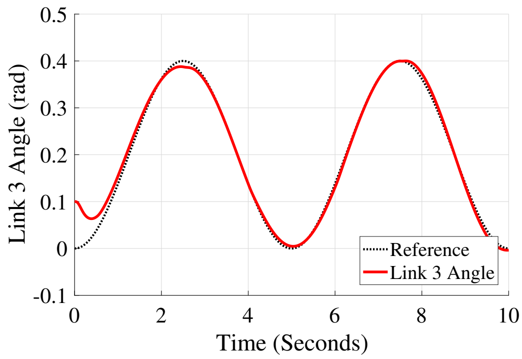

Fig. 13. State response of link 3, Scenario III

of proposed method even when the system is affected by disturbances. By combining all the non-linearity in the system together with the external disturbances into a total uncertain vector and remove it through observer, the tracking errors of nominal nonlinear model are regulated to follow a linear reference model. From there, the LQR optimal controller is designed to bring all the states of reference model to zero, lead to all the states of the original system track the references signal. The linear reference model holds the most important position determining the determining the efficiency of the controller. Therefore, determining the reference model based on the optimal control algorithm provide us an alternative approach instead of choosing arbitrarily.

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